King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 102
Exam II
223
July 31, 2023
Net Time Allowed: 120 Minutes

MASTER VERSION

1. If the volume of the solid generated by revolving the region bounded by the graphs of $y = \dfrac{2}{1+x}$, $y = 0$, $x = 0$, $x = 4$ about the line $y = 4$ is $k\left(\ln 5 - \dfrac{1}{5}\right)$, then $k =$

Question 19, page 461

(a) $16\,\pi$ (correct)

(b) $8\,\pi$

(c) $4\,\pi$

(d) $2\,\pi$

(e) π

2. $\displaystyle\int_{\sqrt{3}}^{2} \frac{\sqrt{x^2-3}}{x}\,dx =$

Example 4, page 544

(a) $1 - \dfrac{\sqrt{3}}{6}\pi$ (correct)

(b) $\sqrt{3} - \dfrac{\pi}{6}$

(c) $\pi - \dfrac{\sqrt{3}}{2}$

(d) $2 - \sqrt{3}\,\pi$

(e) $2\sqrt{3} + \dfrac{\pi}{2}$

3. If the region enclosed by $y^2 - 4y = -x$ and $x = 0$ is revolving about the x-axis, then the formula for the volume is

Question 27, page 470

(a) $2\pi \int_0^4 y(4y - y^2)\, dy$ (correct)

(b) $2\pi \int_0^2 (y^2 - 4y)\, dy$

(c) $\pi \int_0^4 (2 + \sqrt{4 - x})^2\, dx$

(d) $2\pi \int_0^4 (2 + \sqrt{4 - x})^2\, dx$

(e) $2\pi \int_0^2 4 - (y - 2)^2\, dy$

4. The volume of the solid, generated by revolving the region bounded by the graph of $y = x^3$, $y = 0$, $x = 2$ about the line $x = 4$, is

Question 29, page 461

(a) $\frac{96\pi}{5}$ (correct)

(b) $\frac{88\pi}{5}$

(c) $\frac{116\pi}{5}$

(d) $\frac{32\pi}{5}$

(e) $\frac{\pi}{5}$

5. The arc length of the graph of $y = \ln(\sin x)$ over $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ equals to

Question 15, page 481

(a) $\int_{\pi/4}^{3\pi/4} \csc x \, dx$ (correct)

(b) $\int_{\pi/4}^{3\pi/4} \sec x \, dx$

(c) $\int_{\pi/4}^{3\pi/4} \tan x \, dx$

(d) $\int_{\pi/4}^{3\pi/4} \cot x \, dx$

(e) $\int_{\pi/4}^{3\pi/4} \ln(\cos^2 x) \, dx$

6. The area of the surface, generated by revolving the curve $y = \sqrt{9 - x^2}$ on $-1 \leq x \leq 1$ about the x-axis, is

Similar to Question 44, page 482

(a) $12\,\pi$ (correct)

(b) $24\,\pi$

(c) $36\,\pi$

(d) $72\,\pi$

(e) $112\,\pi$

7. $\int x^n \ln x \, dx =$

Question 69, page 530

(a) $\dfrac{x^{n+1}}{(n+1)^2}\left[(n+1)\ln x - 1\right] + C$ (correct)

(b) $\dfrac{x^{n+1}}{(n+1)^2}\left[x - 1\right] + C$

(c) $\dfrac{x^{n}}{n+1}\left[(n+1)\ln x - 1\right] + C$

(d) $\dfrac{x^n \ln x}{n+1} + C$

(e) $\dfrac{x^{n+1}}{(n+1)^2}\left[\ln x - 1\right] + C$

8. $\displaystyle\int_2^4 x \sec^{-1} x \, dx =$

Question 51, page 530

(a) $8\sec^{-1} 4 + \dfrac{3\sqrt{3} - 3\sqrt{15} - 4\pi}{6}$ (correct)

(b) $8\sec^{-1} 4 - \sec^{-1} 2$

(c) $\dfrac{\sqrt{3} - \sqrt{5} - \pi}{3} - \sec^{-1} 2$

(d) $\dfrac{\sqrt{3} - \sqrt{5} - 2\pi}{3} + \sec^{-1} 2$

(e) $4\sec^{-1} 4 + 6\sec^{-1} 2 - \dfrac{\sqrt{3} - \sqrt{5} - 2\pi}{3}$

9. $\int (\ln x)^2\,dx =$

Question 7, page 529

(a) $x[(\ln x)^2 - 2\ln x + 2] + C$ (correct)

(b) $(\ln x)^2 - 2\ln x - x + C$

(c) $\dfrac{(\ln x)^2}{2} + C$

(d) $\dfrac{(\ln x)^2 - \ln x - x}{2} + C$

(e) $x(\ln x)^2 - x^2 + C$

10. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3 x\,dx =$

Question 15, page 538

(a) $\dfrac{2}{3}$ (correct)

(b) $\dfrac{1}{3}$

(c) $\dfrac{1}{2}$

(d) 1

(e) $\dfrac{3}{4}$

11. $\int \csc^4(3x)\,dx =$

Question 51, page 539

(a) $-\frac{1}{3}\cot(3x) - \frac{1}{9}\cot^3(3x) + C$ (correct)

(b) $\frac{1}{5}\csc^5(3x) + C$

(c) $-\frac{1}{2}\tan(3x) - \frac{1}{3}\tan^3(3x) + C$

(d) $-\frac{1}{2}\csc(3x) - \frac{1}{6}\csc^3(3x) + C$

(e) $\frac{1}{2}\cot x - \frac{1}{3}\cot^3 x + C$

12. The volume of the solid, generated by revolving the region bounded by the graphs of the equations $y = \tan x$, $y = 0$, $x = \frac{-\pi}{4}$, $x = \frac{\pi}{4}$ about the x-axis, is

Question 75, page 539

(a) $\pi\left(2 - \frac{\pi}{2}\right)$ (correct)

(b) $2\pi\left(2 - \frac{\pi}{4}\right)$

(c) $2\pi\,(1 - \pi)$

(d) $\pi\left(1 - \frac{\pi}{4}\right)$

(e) $2\pi\left(1 - \frac{\pi}{2}\right)$

13. If $\displaystyle\int \frac{x^2}{x^4-2x^2-8}\,dx = \frac{1}{6}\left[\ln\left|\frac{x-2}{x+2}\right| + \alpha\,\tan^{-1}\frac{x}{\sqrt{2}}\right] + C$, then $\alpha =$

Question 17, page 557

(a) $\sqrt{2}$ (correct)
(b) $\sqrt{3}$
(c) $2\sqrt{2}$
(d) $3\sqrt{2}$
(e) $2\sqrt{3}$

14. $\displaystyle\int \frac{\sin x}{\cos x + \cos^2 x}\,dx =$

Question 25, page 557

(a) $\ln|1+\sec x| + C$ (correct)
(b) $\ln|2+\csc x| + C$
(c) $\ln|6+\tan x| + C$
(d) $6\ln|\sin x + \cos x| + C$
(e) $\dfrac{6}{\cos^2 x + \cos^3 x} + C$

15. $\int_1^\infty (1-x)e^{-x}\,dx$

Example 3, page 574

(a) converges to $\frac{-1}{e}$ _____ (correct)

(b) converges to $\frac{1}{e}$

(c) converges to e

(d) converges to $-e$

(e) diverges

16. $\int_3^5 \frac{dx}{\sqrt{x^2-9}} =$

Question 43, page 579

(a) $\ln 3$ _____ (correct)

(b) $\ln 5$

(c) $\ln 2$

(d) $\ln 5 - \ln 3$

(e) 1

17. $\int \frac{x}{2}\sqrt{x^2+4}\,dx =$

Question 11, page 547

(a) $\frac{1}{6}(x^2+4)^{\frac{3}{2}}+C$ (correct)

(b) $\frac{1}{3}(x^2+4)^{\frac{1}{2}}+C$

(c) $\frac{2}{3}(x^2+4)^{\frac{-3}{2}}+C$

(d) $2x\sqrt{x^2+4}+C$

(e) $\frac{\sqrt{x^2+4}}{2x}+C$

18. Consider the solid formed by revolving the region bounded by $y=\sqrt{x}$, $y=0$, $x=2$, and $x=4$ about the x-axis. The values of x, in the interval $[2,4]$ that divide the solid into three parts of equal volume, are

Similar to Question 58, page 462

(a) $2\sqrt{2}$ and $2\sqrt{3}$ (correct)

(b) $\sqrt{5}$ and 3

(c) $2\sqrt{2}$ and $3\sqrt{5}$

(d) $2\sqrt{3}$ and $3\sqrt{3}$

(e) $\sqrt{6}$ and $\sqrt{5}$