King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 102
Final Exam
223
August 17, 2023
Net Time Allowed: 180 Minutes

MASTER VERSION

1. Which one of the following is False about the sequence with the nth term $a_n = \dfrac{\sin\sqrt{n}}{\sqrt{n}}$?

Question# 18, page# 680

(a) diverges (correct)

(b) converges

(c) not monotonic

(d) bounded

(e) a_n approaches 0 as $n \to \infty$

2. Which one of the following statements is False when the integral test is applied to $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{n^2+1}$?

Question# 13, page# 613

(a) The series diverges (correct)

(b) The series converges

(c) $f(x) = \dfrac{\arctan x}{x^2+1}$ is positive for $x \geq 1$

(d) $f(x) = \dfrac{\arctan x}{x^2+1}$ is decreasing for $x \geq 1$

(e) $f(x) = \dfrac{\arctan x}{x^2+1}$ is continuous for $x \geq 1$

3. $\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$

Question# 66, page# 615

(a) converges to $-\ln 2$ _____ (correct)

(b) converges to $\ln 3 - \ln 2$

(c) converges to $\ln 2 - \ln 3$

(d) converges to $\ln 2$

(e) diverges

4. How many of these series

(i) $\sum_{n=1}^{\infty} \frac{\ln n}{n+1}$

(ii) $\sum_{n=0}^{\infty} \frac{1}{n!}$

(iii) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^3}$

(iv) $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n} - 1}$

is (are) divergent series?

Question# 9+, page# 620

(a) 2 _____ (correct)

(b) 0

(c) 1

(d) 3

(e) 4

5. $\sum_{n=1}^{\infty} \frac{x^{2n}}{2n}$

Question# 68, page# 630

(a) converges for $-1 < x < 1$ _____(correct)

(b) converges for $-1 \leq x < 1$

(c) converges for $x = 1$ or $x = -1$

(d) converges for $x > 1$ or $x < -1$

(e) diverges for all $x \in R$

6. Which of the following series converges conditionally?

Question# 46+, page# 630

(a) $\sum_{n=0}^{\infty} \left(\frac{\cos n\pi}{n+1} \right)$ _____(correct)

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$

(c) $\sum_{n=1}^{\infty} \frac{\cos\left(n\frac{\pi}{3}\right)}{n^2}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{e^{n^2}}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$

7. Applying the ratio test to the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\left(\frac{\pi}{2}\right)^n}{n^2}$, we get $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$

~Question# 28, page# 637

(a) > 1 and then the series diverges _____(correct)

(b) < 1 and then the series converges

(c) $= 1$ and then the ratio test is inconclusive

(d) $= \infty$

(e) $= 0$

8. If $a_1 = 1$, and $a_{n+1} = \frac{\sin n + 1}{\sqrt{n}}\, a_n$, then $\sum_{n=1}^{\infty} a_n$

Question# 79, page# 638

(a) converges by the ratio test _____(correct)

(b) diverges by the ratio test

(c) the ratio test is inconclusive

(d) converges by the integral test

(e) diverges by the integral test

9. $\displaystyle\int \frac{x-8}{x^2-x-6}\,dx =$

Question# 37, page# 583

(a) $\ln\left|\dfrac{(x+2)^2}{x-3}\right|+C$ (correct)

(b) $\ln\left|\dfrac{x+2}{(x-3)^2}\right|+C$

(c) $\ln\left|\dfrac{x+2}{x-3}\right|+C$

(d) $\ln\left|\dfrac{x-3}{x+2}\right|+C$

(e) $\ln\left|\dfrac{(x-3)^2}{x+2}\right|+C$

10. If $\displaystyle\int_3^4 x^3\sqrt{x^2-9}\,dx = k\left[\frac{\tan^3\theta}{3}+\frac{\tan^5\theta}{5}\right]_0^{\sec^{-1}\left(\frac{4}{3}\right)}$, then $k =$

Question# 34, page# 583

(a) 243 (correct)

(b) 139

(c) 97

(d) 175

(e) 39

11. If $P_3(x)$ is the third Maclaurin polynomial for $f(x) = x^2e^x$, then $p_3(1) =$

Question# 16, page# 648

(a) 2 _____ (correct)

(b) 1

(c) 3

(d) 4

(e) 5

12. If $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n\,5^n}$, then the interval of convergence for $f(x)$ is

Question# 50, page# 659

(a) $(0, 10]$ _____ (correct)

(b) $(0, 10)$

(c) $[0, 10)$

(d) $(-5, 5)$

(e) $(-5, 5]$

13. Which one of the following statements is True?

Question# 73+, page# 660

(a) If $f(x) = \sum_{n=0}^{\infty} a_n x^n$ converges for $|x| < 2$, then $\int_0^1 f(x)\,dx = \sum_{n=0}^{\infty} \frac{a_n}{n+1}$. (correct)

(b) If the power series $\sum_{n=1}^{\infty} a_n x^n$ converges for $x = 2$, then it also converges for $x = -2$.

(c) It is possible to find a power series whose interval of converges is $[0, \infty)$.

(d) If the interval of converges for $\sum_{n=0}^{\infty} a_n x^n$ is $(-1, 1)$, then the interval of convergence for $\sum_{n=0}^{\infty} a_n (x-1)^n$ is $(-2, 0)$.

(e) The radius of convergence for $\sum_{n=1}^{\infty} \frac{1}{n}$ is 1.

14. $\frac{1}{10} \sum_{n=1}^{\infty} n \left(\frac{9}{10}\right)^n =$

Question# 42, page# 667

(a) 9 (correct)

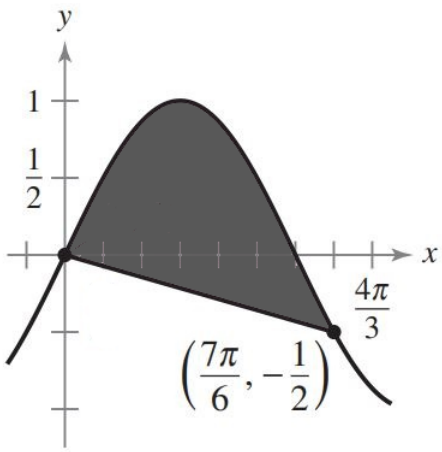
(b) $\frac{1}{10}$

(c) 10

(d) 90

(e) $\frac{9}{10}$

15. If the area, between the graph of $y = \sin x$ and line segment joining the point $(0, 0)$ and $\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$ (shown in the figure) equals to $\frac{\sqrt{3}}{2} + \frac{7\pi}{24} + k$, then $k =$



Question# 81, page# 453

(a) 1 (correct)

(b) 2

(c) 3

(d) 4

(e) 5

16. The coefficient of x^5 in the power series of $f(x) = (\cos x)\ \ln(1 + x)$ is

Question# 49, page# 677

(a) $\frac{3}{40}$ (correct)

(b) $\frac{5}{24}$

(c) $\frac{-1}{6}$

(d) $\frac{3}{8}$

(e) $\frac{1}{5}$

17. $\int_0^1 e^{-x^3}\,dx =$

Question# 63, page# 678

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(3n+1)\cdot n!}$ (correct)

(b) $\sum_{n=1}^{\infty} \frac{1}{(3n+1)^n}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(3n)(2n+1)!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(3n+1)(2n+1)!}$

(e) $\sum_{n=0}^{\infty} \frac{1}{(3n+1)(2n+1)}$

18. $\sum_{n=0}^{\infty} \left[\left(\frac{3}{4}\right)^n - \frac{1}{(n+1)(n+2)}\right] =$

Question# 34, page# 681

(a) 3 (correct)

(b) 2

(c) 1

(d) 4

(e) 5

19. $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\sqrt{\frac{i^2}{n^2}} \left(\frac{2i-1}{n^2} \right) \right] =$

Example# 1, page# 306

(a) $\frac{2}{3}$ (correct)

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) $\frac{5}{2}$

(e) 2

20. If $G(x) = \int_0^x \left[s \int_0^s f(t)\, dt \right] ds + \int_0^{2x} t\, dt$, then $G''(0) =$

~Question# 113, page# 331

(a) 4 (correct)

(b) 0

(c) $G'(0)$

(d) 1

(e) 2

21. $\int_3^2 (3-x)7^{(3-x)^2}\,dx =$

Question# 56, page# 342

(a) $\dfrac{-3}{\ln 7}$ (correct)

(b) $\dfrac{2}{\ln 7}$

(c) $\dfrac{-6}{\ln 7}$

(d) $\dfrac{1}{\ln 7}$

(e) $\dfrac{4}{\ln 7}$

22. The average value of $f(x) = \frac{\ln x^2}{x}$ over $[1, e]$ is

Question# 75, page# 363

(a) $\dfrac{1}{e-1}$ (correct)

(b) $\dfrac{1}{e}$

(c) 1

(d) $\dfrac{1}{e^2}$

(e) e

23. $\displaystyle\int_0^1 \frac{dx}{2\sqrt{3-x}\,\sqrt{x+1}} =$

Question# 46, page# 370

(a) $\dfrac{\pi}{12}$ (correct)

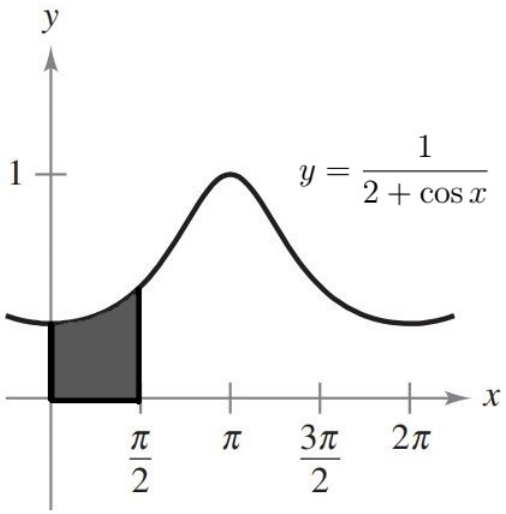
(b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{4}$

(d) $\dfrac{\pi}{6}$

(e) $\dfrac{\pi}{2}$

24. The area of region bounded by the graph of $y = \dfrac{1}{2+\cos x}$ (as shown in the figure) and $y = 0$ for $0 \le x \le \dfrac{\pi}{2}$ is



Question# 5, page# 585

(a) $\dfrac{\sqrt{3}}{9}\pi$ (correct)

(b) $\dfrac{\pi}{\sqrt{3}}$

(c) $3\sqrt{3}\pi$

(d) $\dfrac{\sqrt{3}}{2}\pi$

(e) $\dfrac{\sqrt{3}}{27}\pi$

25. If the region bounded by $y = x^2$, $x = 0$, and $y = 9$ in the 1st quadrant is revolved about the line $x = 3$, then the volume of the resulted solid is

Question# 55, page#462

(a) $\dfrac{135\pi}{2}$ (correct)

(b) $\dfrac{123\pi}{3}$

(c) $\dfrac{113\pi}{5}$

(d) $\dfrac{27\pi}{4}$

(e) $\dfrac{143\pi}{6}$

26. The volume of solid, that is resulted by revolving the region bounded by $y = \dfrac{10}{x^2}$, $y = 0$, $x = 1$ and $x = 5$ about the line $y = 10$, is given by

Question# 10, page# 471

(a) $100\pi \displaystyle\int_1^5 \left(\frac{2}{x^2} - \frac{1}{x^4}\right) dx$ (correct)

(b) $2\pi \displaystyle\int_1^5 \frac{100}{x} - \frac{2}{x^4}\, dx$

(c) $\pi \displaystyle\int_1^5 \left(100 - \frac{1}{x^4}\right) dx$

(d) $200\pi \displaystyle\int_1^5 \left(\frac{1}{x} - \frac{1}{x^3}\right) dx$

(e) $10\pi \displaystyle\int_1^5 \left(\frac{1}{x} - \frac{1}{x^3}\right) dx$

27. The area of the surface, generated by revolving the graph of $y = 3x$ over $[0, 3]$ about the x-axis, is

Question# 42, page# 482

(a) $27\sqrt{10}\,\pi$ (correct)

(b) $3\sqrt{10}\,\pi$

(c) $9\sqrt{10}\,\pi$

(d) $18\sqrt{10}\,\pi$

(e) $6\sqrt{10}\,\pi$

28. $\displaystyle\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx =$

Question# 32, page# 370

(a) $\dfrac{\pi}{4}$ (correct)

(b) $\dfrac{\pi}{2}$

(c) $\dfrac{\pi}{3}$

(d) $\left(\dfrac{\pi}{2}\right)^2$

(e) π