King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 102

Final Exam

231

December 20, 2023

Net Time Allowed: 180 Minutes

MASTER VERSION

Example 4/ Section 8.8

1. $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\,dx =$

(a) $\frac{\pi}{2}$ ________________ (correct)

(b) $\frac{\pi}{4}$

(c) $-\frac{\pi}{2}$

(d) $-\frac{\pi}{4}$

(e) 0

Question 15 / Section 8.8

2. $\displaystyle\int_{0}^{2} \frac{1}{(x-1)^2}\,dx$

(a) diverges ________________ (correct)

(b) is equal to 1

(c) is equal to -2

(d) is equal to 2

(e) is equal to 0

Question 74 / Section 9.1

3. The sequence $\left\{\sqrt{n}\ln\left(1+\frac{1}{n}\right)\right\}$

(a) converges to 0 ________________ (correct)

(b) converges to 1

(c) converges to 2

(d) converges to e

(e) diverges

Question 36 / Section 9.2

4. $\displaystyle\sum_{n=0}^{\infty}(0.3)^n =$

(a) $\frac{10}{7}$ ________________ (correct)

(b) $\frac{11}{7}$

(c) $\frac{12}{7}$

(d) $\frac{9}{7}$

(e) $\frac{8}{7}$

Question 35 / Section 9.5

5. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$. The least number of terms required to approximate the sum of the series with an error less than 0.001 is
(Hint: Use Alternating Series Remainder Theorem)

(a) 10 ________________ (correct)

(b) 8

(c) 12

(d) 14

(e) 6

Question 47 / Section 9.6

6. Using the Root test for the series $\sum_{n=1}^{\infty} a_n$ where $a_n = \frac{n}{3^n}$, we have $\lim_{n\to\infty} \sqrt[n]{|a_n|} =$

(a) $\frac{1}{3}$ ________________ (correct)

(b) $\frac{2}{3}$

(c) 1

(d) $\frac{4}{3}$

(e) $\frac{1}{2}$

Question 83 / Section 5.4

7. If $F(x) = \int_0^{\sin x} \sqrt{t}\, dt$, then $F'\left(\frac{\pi}{6}\right) =$

(a) $\frac{\sqrt{6}}{4}$ ________________ (correct)

(b) $\sqrt{6}$

(c) $\frac{\sqrt{6}}{2}$

(d) $\frac{\sqrt{6}}{3}$

(e) $\frac{\sqrt{2}}{2}$

Question 91 / Section 5.5

8. The area of the region bounded by the graphs of the equations $y = xe^{-x^2/4}$, $y = 0$, $x = 0$ and $x = \sqrt{6}$ is

(a) $2 - 2e^{-3/2}$ ________________ (correct)

(b) $1 - e^{-3/2}$

(c) $2 + 2e^{-3/2}$

(d) $-2 - 2e^{-3/2}$

(e) $e^{-3/2}$

Question 49 / Section 5.9

9. $\int \cosh^2(x-1)\,\sinh(x-1)\,dx =$

(a) $\frac{1}{3}\cosh^3(x-1)+c$ ________ (correct)

(b) $\frac{1}{3}\sinh^3(x-1)+c$

(c) $\cosh^3(x-1)+c$

(d) $\sinh^3(x-1)+c$

(e) $\tanh^3(x-1)+c$

Question 11 / Section 8.4

10. $\int \frac{x}{2}\sqrt{4+x^2}\,dx =$

(a) $\frac{1}{6}(4+x^2)^{3/2}+c$ ________ (correct)

(b) $\frac{1}{4}(4+x^2)^{3/2}+c$

(c) $\frac{1}{6}(4+x^2)^{1/2}+c$

(d) $\frac{1}{4}(4+x^2)^{1/2}+c$

(e) $\frac{1}{6}(2+x^2)^{3/2}+c$

Example 3 / Section 8.5

11. If $\dfrac{2x^3-4x-8}{(x^2-x)(x^2+4)} = \dfrac{2}{x} - \dfrac{2}{x-1} + \dfrac{Cx+D}{x^2+4}$, then $C+D=$

(a) 6 ______ (correct)

(b) 8

(c) 10

(d) 4

(e) 2

Question 83 / Section 9.1

12. Given that the sequence $\sqrt{2}$, $\sqrt{2+\sqrt{2}}$, $\sqrt{2+\sqrt{2+\sqrt{2}}}$, ... is a convergent sequence, its limit is equal to

(a) 2 ______ (correct)

(b) $\sqrt{2}$

(c) $\sqrt{3}$

(d) 3

(e) $2+\sqrt{2}$

Question 38 / Section 9.2

13. $\sum_{k=1}^{\infty} \frac{1}{9k^2+3k-2} =$

(a) $\frac{1}{6}$ ______ (correct)

(b) $\frac{1}{5}$

(c) $\frac{1}{7}$

(d) $\frac{1}{4}$

(e) $\frac{1}{3}$

Question 47 / Section 9.3

14. Using the integral test, and for positive values of p, the series $\sum_{n=1}^{\infty} \frac{n}{(1+n^2)^p}$ converges if

(a) $p > 1$ ______ (correct)

(b) $0 < p < 1$

(c) $p \geq 1$

(d) $0 < p \leq 1$

(e) $p > 0$

Question 27 / Section 9.7

15. If $P_3(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3$ is the 3rd Taylor polynomial for the function $f(x) = \frac{2}{x}$, centered at $x = 1$, then $b + c + d =$

(a) -2 ______________________________ (correct)

(b) -4

(c) 4

(d) 2

(e) 0

Example 4 / Section 5.2

16. The upper sum for the region bounded by the graph of $f(x) = x^2$ and the x-axis between $x = 0$ and $x = 2$ in terms of n (the number of subintervals) is

(a) $\frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2}$ ______________________________ (correct)

(b) $\frac{5}{3} + \frac{2}{n} + \frac{2}{3n^2}$

(c) $\frac{8}{3} - \frac{4}{n} + \frac{4}{3n^2}$

(d) $\frac{5}{3} - \frac{2}{n} + \frac{2}{3n^2}$

(e) $\frac{8}{3} + \frac{4}{3n^2}$

Question 29 / Section 5.7

17. $\displaystyle\int \frac{dx}{1+\sqrt{2x}} =$

(a) $\sqrt{2x} - \ln(1+\sqrt{2x}) + c$ ________ (correct)

(b) $\sqrt{2x} + \ln(1+\sqrt{2x}) + c$

(c) $\ln(1+\sqrt{2x}) + c$

(d) $\sqrt{2x} + c$

(e) $\sqrt{x} - \ln(1+\sqrt{x}) + c$

Question 40 / Section 5.8

18. $\displaystyle\int \frac{2\,dx}{\sqrt{4x-x^2}} =$

(a) $2\arcsin\left(\frac{x-2}{2}\right) + c$ ________ (correct)

(b) $\arcsin\left(\frac{x-2}{2}\right) + c$

(c) $\arcsin\left(\frac{x+2}{2}\right) + c$

(d) $2\arcsin\left(\frac{x+2}{2}\right) + c$

(e) $4\arcsin(x-2) + c$

Question 16 / Section 7.4

19. The arc length of the graph of the function $y = \ln(\cos x)$ over the interval $\left[0, \frac{\pi}{3}\right]$ is

(a) $\ln(2+\sqrt{3})$ ________________ (correct)

(b) $\ln(1+\sqrt{3})$

(c) $\ln(2+\sqrt{2})$

(d) $\ln(2-\sqrt{3})$

(e) $\ln(1-\sqrt{3})$

Example 5 (b) / Section 9.2 and Questions 11, 26 /Section 9.4

20. Which of the following series is convergent?

I. $\sum_{n=0}^{\infty} \frac{1}{n!}$

II. $\sum_{n=1}^{\infty} \sin\frac{1}{n}$

III. $\sum_{n=0}^{\infty} \frac{n!}{2(n!)+1}$

(a) I only ________________ (correct)

(b) I and II only

(c) I and III only

(d) I, II and III

(e) II and III only

Questions 15, 21, 22 / Section 9.5

21. Which of the following series is convergent?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(n+1)}$

II. $\displaystyle\sum_{n=1}^{\infty} \sin\frac{(2n-1)\pi}{2}$

III. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}\cos n\pi$

(a) III only ________________ (correct)

(b) I and III only

(c) I and II only

(d) I only

(e) I, II and III

Questions 52, 54 / Section 9.5

22. Consider the following series

I. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{4/3}}$

II. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+4}}$.

Then

(a) I converges absolutely but II converges conditionally ________________ (correct)

(b) I and II converge absolutely

(c) I and II converge conditionally

(d) I converges absolutely but II diverges

(e) I diverges but II converges absolutely

Question 25 / Section 9.8

23. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^n}{n9^n}$ is

(a) $(-5, 13]$ ———————— (correct)

(b) $[-5, 13]$

(c) $[-5, 13)$

(d) $(-5, 13)$

(e) $(-\infty, \infty)$

Question 68 / Section 9.10

24. $\int_0^{1/2} \arctan(x^2)\, dx =$

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+3)(2n+1)2^{4n+3}}$ ———————— (correct)

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n-3)(2n-1)2^{4n+3}}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+3)(2n+1)2^{4n-3}}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(4n+3)(2n+1)2^{4n+3}}$

(e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(4n+3)(2n+1)2^{4n+3}}$

Question 39 / Section 9.9

25. The power series representation for the function $f(x) = \dfrac{1+x}{(1-x)^2}$ is

(a) $\sum_{n=0}^{\infty}(2n+1)\,x^n,\ |x|<1$ ________________ (correct)

(b) $\sum_{n=0}^{\infty}(n+1)\,x^n,\ |x|<1$

(c) $\sum_{n=0}^{\infty}(2n-1)\,x^n,\ |x|<1$

(d) $\sum_{n=0}^{\infty}(2n+1)\,x^{n+1},\ |x|<1$

(e) $\sum_{n=0}^{\infty}(n-1)\,x^n,\ |x|<1$

Question 23 / Section 7.3

26. The volume of the solid generated by revolving the region bounded by the graphs of the equations $y = 2x - x^2$ and $y = 0$ about the line $x = 4$ is

(a) $8\,\pi$ ________________ (correct)

(b) $6\,\pi$

(c) $4\,\pi$

(d) $10\,\pi$

(e) $12\,\pi$

Question 23 / Section 8.2

27. $\displaystyle\int \frac{xe^{2x}}{(2x+1)^2}\,dx =$

(a) $\dfrac{e^{2x}}{4(2x+1)} + c$ ——————————(correct)

(b) $\dfrac{e^{2x}}{2(2x+1)} + c$

(c) $\dfrac{e^{2x}}{4(x+1)} + c$

(d) $\dfrac{e^{x}}{4(2x+1)} + c$

(e) $\dfrac{e^{x}}{2(2x+1)} + c$

Example 3 / Section 8.7

28. $\displaystyle\int_0^2 \frac{x}{1+e^{-x^2}}\,dx =$

(a) $\dfrac{1}{2}\left(\ln\left(\dfrac{1+e^4}{2}\right)\right)$ ——————————(correct)

(b) $\dfrac{1}{2}\left(\ln\left(\dfrac{1+e^{-4}}{2}\right)\right)$

(c) $\dfrac{1}{2}\ln(1+e^4)$

(d) $\dfrac{1}{2}\ln(1+e^{-4})$

(e) $\ln\left(\dfrac{1+e^4}{2}\right)$