King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 102
Major Exam I
232
February 18, 2024
Net Time Allowed: 120 Minutes

USE THIS AS A TEMPLATE

Write your questions, once you are satisfied upload this file.

1. $\sum_{i=1}^{7} i(i+3)^2 =$

   (a) 1876
   (b) 1678
   (c) 1786
   (d) 1668
   (e) 1787

2. Using the midpoint rule with $n = 3$, the approximate area of the region bounded by the graph of $f(x) = \sin \pi x$ and the x-axis over $\left[0, \frac{3}{2}\right]$ is equal to

   (a) $\frac{3}{4}\sqrt{2}$
   (b) $\frac{\sqrt{2}}{4}$
   (c) $\frac{\sqrt{2}}{2}$
   (d) $\frac{3\sqrt{2}}{2}$
   (e) 0

3. When evaluating the limit of $\sum_{i=1}^{n} \left(1 + \frac{3}{c_i}\right) \triangle x$, as $\| \triangle x \| \to 0$ over $[1, 5]$, where c_i is any point in the i^{th} subinterval, the value is equal to:

   (a) $4 + \ln(125)$
   (b) $4\ln(5)$
   (c) $\frac{-12}{5}$
   (d) $\frac{8}{5}$
   (e) $\frac{3}{5}$

4. $\lim_{n\to\infty} \frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{n}}{n^{3/2}}$ equals to

   (a) $\frac{2}{3}$
   (b) $\frac{3}{2}$
   (c) $\sqrt{3}$
   (d) 0
   (e) ∞

5. Given that $\int_2^6 x^3 dx = A$ and $\int_2^6 x\,dx = B$, the integral of $\int_2^6 \left(6|x| - \frac{1}{8}x^3\right) dx =$

(a) $6B - \frac{1}{8}A$

(b) $-6B - \frac{1}{8}A$

(c) $12B - \frac{1}{8}A$

(d) $2B - \frac{1}{8}A$

(e) $-12B + \frac{1}{8}A$

6. The constant c, that is guaranteed by the mean value theorem of Integral of $f(x) = 5 - \frac{1}{x}$ over $[1, 4]$ is equal to

(a) $\frac{3}{\ln 4}$

(b) $\frac{1}{\ln 4 - 10}$

(c) $\frac{1}{2\ln 2}$

(d) $\frac{1}{2\ln 2 + 10}$

(e) $\frac{1}{\ln 2}$

7. If $F(x) = \int_0^{2x} \cos t^4 \, dt$, then $F'(x) =$

(a) $2\cos(16x^4)$
(b) $-2\cos(16x^4)$
(c) $32\cos(x^4)$
(d) $\cos(16x^4)$
(e) $-\cos(16x^4)$

8. $\int_9^1 \frac{\sqrt{2}}{\sqrt{x}(1+\sqrt{x})^2} \, dx =$

(a) $\frac{-\sqrt{2}}{2}$
(b) $\frac{-1}{2}$
(c) $\frac{\sqrt{2}}{2}$
(d) $\frac{1}{2}$
(e) $8\sqrt{2}$

9. $\int x\sqrt{x+6}\,dx =$

(a) $\frac{2}{5}(x+6)^{3/2}(x-4)+C$

(b) $\frac{2}{5}(x-4)^{3/2}(x+6)+C$

(c) $\frac{2}{3}(x+6)^{2/3}(x-4)+C$

(d) $\frac{3}{2}(x-6)^{1/2}(x-1)^{3/2}+C$

(e) $\frac{4}{5}(x+6)^{3/2}(x+16)+C$

10. $\int \frac{x^4+x-4}{x^2+2}\,dx =$

(a) $\frac{1}{3}x^3-2x+\ln\sqrt{x^2+2}+C$

(b) $\frac{1}{2}x^2-x+2\ln\sqrt{x^2+2}+C$

(c) $\frac{1}{3}x^3+\ln|x|+\ln(x^2-4)+C$

(d) $\frac{2}{3}x^3+\ln|x|+\ln(x^2+2)+C$

(e) $\frac{1}{3}x^3+2x+\ln(x^2-4)+C$

11. $\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x)\, dx =$

(a) $1 - \cos 1$
(b) 0
(c) $\pi - 1$
(d) 1
(e) -1

12. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x^3 + x^4 \tan x)\, dx =$

(a) 0
(b) $\dfrac{\pi}{2}$
(c) $-\dfrac{\pi}{2}$
(d) $\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$
(e) $\left(\dfrac{\pi}{4}\right)^4 \tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$

13. $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} (\csc 2\theta - \cot 2\theta)\, d\theta =$

(a) $\ln \sqrt{1 + \dfrac{\sqrt{2}}{2}}$

(b) $\dfrac{1}{2} \ln(\sqrt{2} - 1)$

(c) $\dfrac{1}{4} \ln(\pi - 1)$

(d) $\dfrac{\sqrt{2}}{2} \ln(\pi + 1)$

(e) $\dfrac{1}{2} \ln \left(\dfrac{\sqrt{2} + \pi}{\sqrt{2} - \pi} \right)$

14. $\int \dfrac{1}{x\sqrt{x^4 - 4}}\, dx =$

(a) $\dfrac{1}{4} \sec^{-1} \left(\dfrac{x^2}{2} \right) + C$

(b) $\dfrac{1}{2} \sin^{-1} \left(\dfrac{x^2}{4} \right) + C$

(c) $\dfrac{1}{2} \sec^{-1} \left(\dfrac{x^2}{4} \right) + C$

(d) $\dfrac{1}{4} \sin^{-1} \left(\dfrac{x^2}{2} \right) + C$

(e) $\dfrac{1}{2} \sin^{-1} \left(\dfrac{x^2}{2} \right) + C$

15. $\displaystyle\int \frac{x}{x^4+25}\,dx =$

(a) $\dfrac{1}{10}\tan^{-1}\left(\dfrac{x^2}{5}\right)+C$

(b) $\dfrac{1}{25}\tan^{-1}\left(\dfrac{x^2}{25}\right)+C$

(c) $\dfrac{1}{5}\tan^{-1}\left(\dfrac{x^2}{10}\right)+C$

(d) $\dfrac{1}{10}\tan^{-1}\left(\dfrac{x}{5}\right)+C$

(e) $\dfrac{1}{5}\tan^{-1}\left(\dfrac{x}{10}\right)+C$

16. The area of the region bounded between $y_1=(x-1)^3$ and $y_2=x-1$ is equal to

(a) $\dfrac{1}{2}$

(b) $\dfrac{1}{4}$

(c) 1

(d) $2\sqrt[3]{2}$

(e) $\dfrac{1}{2}\sqrt[3]{2}$

17. If A is the exact region bounded between the two curves $f(x) = \cos x$ and $g(x) = 2 - \cos x$ over the interval $[0, 2\pi]$, then

(a) $12 < A < 13$
(b) $10 < A < 11$
(c) $9 < A < 10$
(d) $5 < A < 6$
(e) $7 < A < 8$

18. $\displaystyle\int_0^{\ln 2} 2e^{-x} \cosh x \, dx =$

(a) $\dfrac{3}{8} + \ln 2$
(b) $\dfrac{2 + \ln 2}{3}$
(c) $2 \ln 2 + e$
(d) $\dfrac{1 + \ln 2}{e}$
(e) $e^{-1} \ln 2$