King Fahd University of Petroleum and Minerals
Department of Mathematics
MATH 106
Major Exam II
213
21 July 2022
Net Time Allowed: 120 minutes

MASTER VERSION

1.   The solpe of tangent line to the curve $y = 2^{2x^2}$ at $x = 1$ is

    (a) $16\ln(2)$ (correct)

    (b) $\ln(2)$

    (c) $4\ln(2)$

    (d) $8\ln(2)$

    (e) $32\ln(2)$

2.   The average cost $\bar{c}$ for producing q units of the product is given by $\bar{c} = \frac{1}{q}(10,000e^{\frac{q}{900}}) + q$. At a production level of 900 units, the **rate of change of marginal cost** is

    (a) $\frac{1}{81}(e + 162)$ (correct)

    (b) $\frac{10}{81}(e + 162)$

    (c) $\frac{10}{9}(e + 162)$

    (d) $\frac{1}{81}(e + 2)$

    (e) $\frac{1}{81}(e^9 + 162)$

3.   If $\dfrac{d}{dx}(c^{2x} - (2x)^c)\Big|_{x=\frac{1}{2}} = 0$ then the value of the positive constant c is

(a) e (correct)

(b) $e^{\frac{1}{2}}$

(c) e^2

(d) $e^{-\frac{1}{2}}$

(e) $-e$

4.   If $y = x^{x^3}$ then $\dfrac{dy}{dx} =?$

(a) $x^{x^3}(x^2 + 3x^2 \ln x)$ (correct)

(b) $x^{x^3}(x^2 + \ln x)$

(c) $x^{x^3}(x^3 + x^3 \ln x)$

(d) $x^{x^3}(3x^2 \ln x)$

(e) $x^{x^3}(x^2 + 3x^2)$

5. $\dfrac{d}{dx} x^{x^x} =$

   (a) $x^{x^x} x^x \left(\dfrac{1}{x} + (1 + \ln x) \ln x \right)$ (correct)

   (b) $x^{x^x} x^x \left(\dfrac{1}{x} + (1 + \ln x) \right)$

   (c) $x^{x^x} \left(\dfrac{1}{x} + (1 + \ln x) \ln x \right)$

   (d) $x^{x^x} x^x \left((1 + \ln x) \ln x \right)$

   (e) $x^x \left(\dfrac{1}{x} + (1 + \ln x) \ln x \right)$

6.   If $y^2 - 6xy = 4$ then $\left. \dfrac{d^2 y}{dx^2} \right|_{(1,6)} =$

   (a)   6   (correct)

   (b) -6

   (c)   1

   (d) $\dfrac{1}{6}$

   (e) $-\dfrac{1}{6}$

7.   The equation of the tangent line to $y^2 e^{2x} = 3y + x^2$ at $(0, 3)$ is

    (a) $y = -6x + 3$ (correct)

    (b) $y = 6x + 3$

    (c) $y = \dfrac{3}{18}x + 3$

    (d) $y = -6x - 3$

    (e) $y = \dfrac{3}{18}x - 3$

8.   The third derivative of $y = e^{-5x} + 8\ln(2x^4)$ is

    (a) $-125e^{-5x} + 64x^{-3}$ (correct)

    (b) $25e^{-5x} - 32x^{-2}$

    (c) $-125e^{-5e} - 64x^{-2}$

    (d) $125e^{-5x} - 64x^{-3}$

    (e) $-125e^{-5x} - 64x^{-4}$

9.   Suppose that the demand equation for a monopolist's product is $p = 400 - 2q$ and the average cost function is $\bar{c} = 0.2q + 4 + \frac{400}{q}$, where q is the number of units and both p and $\bar{c}$ expressed in dollars per unit. Then which of the following statement is **True**?

(a)   The maximum profit is at $q = 90$.   (correct)

(b)   The minimum profit is at $q = 90$.

(c)   The profit is decreasing on $0 < q < 80$.

(d)   The profit is increasing on $80 < q < \infty$.

(e)   The maximum profit is at $q = 80$.

10.   The graph of $y = (x^2 - 1)^4$ is

(a)   increasing on $(-1, 0)$ and $(1, 2)$ (correct)

(b)   decreasing on $(-1, 0)$ and $(1, 2)$

(c)   increasing on $(-2, 0)$ and $(0, 2)$

(d)   relative minimum at $x = 0$

(e)   relative maximum at $x = 1$

11.   On the interval $[0, 2]$, the function $y = 3x^4 - 4x^3$ has

    (a)   an absolute maximum at $x = 2$ and an absolute minimum at $x = 1$.

    (correct)

    (b)   an absolute minimum at $x = 1$ and no absolute maximum.

    (c)   an absolute maximum at $x = 0$ and an absolute minimum at $x = 1$.

    (d)   an absolute maximum at $x = 2$ and an absolute minimum at $x = 0$.

    (e)   no absolute maximum and no absolute minimum.

12.   Consider the function $f(x) = 4 + x^2 - x^3$ on the interval $[-1, 1]$. Then $f(x)$ has an absolute maximum at $x =$

    (a) -1 (correct)

    (b) 0

    (c) 1

    (d) $\dfrac{-1}{2}$

    (e) $\dfrac{1}{2}$

13.   If $f(x) = x^3 - 7x^2 + 2x - 5$ then f is concave down on the interval

(a) $\left(-\infty, \dfrac{7}{3}\right)$ (correct)

(b) $(-\infty, 3)$

(c) $\left(\dfrac{2}{3}, \infty\right)$

(d) $\left(\dfrac{7}{3}, \infty\right)$

(e) $(-\infty, \infty)$

14.   The function $f(x) = \dfrac{x^5}{20} + \dfrac{x^4}{12} + x - 3$ has how many inflection points?

(a) one   (correct)

(b) two

(c) three

(d) four

(e) none

15.   Vertical asymptote(s) of the graph of $f(x) = \dfrac{2x^2 + 3x + 1}{x^2 - 5}$ is (are)

(a) $x = -\sqrt{5}$ and $x = \sqrt{5}$ (correct)

(b) $x = -5$ and $x = 5$

(c) $x = -\sqrt{2}$ and $x = \sqrt{2}$

(d) $x = \dfrac{-1}{5}$ and $x = \dfrac{1}{5}$

(e) $x = \dfrac{-1}{2}$ and $x = \dfrac{1}{2}$

16.   The graph of $f(x) = \dfrac{9x^2 - 16}{2(3x + 4)^2}$ has

(a)   one vertical asymptote and one horizontal asymptote.   (correct)

(b)   only one horizontal asymptote.

(c)   only one vertical asymptote

(d)   only two vertical asymptotes.

(e)   only two horizontal asymptotes.

17.   A manufacturer found that the total cost c of producing q units of a product is given by $c = 0.02q^2 + 2q + 800$. The average cost will be a minimum at

  (a) $q = 200$ units   (correct)

  (b) $q = 100$ units

  (c) $q = 400$ units

  (d) $q = 20$ units

  (e) $q = 40$ units

18.   The demand equation for a monopolist's product is $p = 200 - 0.98q$, where p is the price per unit (in dollars) of producing q units. If the total cost c (in dollars) of producing q units is given by $c = 0.02q^2 + 2q + 8000$. Find the level of production at which profit is maximized?

  (a)   99 units   (correct)

  (b)   90 units

  (c)   100 units

  (d)   80 units

  (e)   110 units

19.   If $f(x) = 2e^x - 8^x$ then $f'(1)$ is

    (a) $2e - 8\ln(8)$ (correct)

    (b) $2 - 8\ln(8)$

    (c) $-8\ln(8)$

    (d) $2e^2 - \ln(8)$

    (e) $\ln(8)$

20.   If $f(x) = 4x^{\frac{3}{5}} - \dfrac{1}{8}x^{-2} - x^{\frac{1}{2}}$ then $f'(1) =$

    (a) $\dfrac{43}{20}$ (correct)

    (b) $\dfrac{53}{20}$

    (c) $\dfrac{53}{12}$

    (d) $\dfrac{43}{12}$

    (e) $-\dfrac{53}{12}$