King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 106

Final Exam

213

August 11, 2022

Net Time Allowed: 180 Minutes

MASTER VERSION

1. $\lim_{x \to 2^-} \dfrac{x+2}{x(x^2-4)} =$

   (a) $-\infty$ ________________ (correct)
   (b) ∞
   (c) $\dfrac{1}{2}$
   (d) $-\dfrac{1}{2}$
   (e) 0

2. $\lim_{x \to -3} \dfrac{x^4-81}{x^2+8x+15} =$

   (a) -54 ________________ (correct)
   (b) 54
   (c) ∞
   (d) $-\infty$
   (e) 0

3. Given that $f(x) = x^3 - 4x^2$. If

$$\frac{f(x+h) - f(x)}{h} = Ax^2 + Bxh + Ch^2 + Dx + Eh$$

Then $A + B + C + D + E =$

(a) -5 ______ (correct)
(b) 5
(c) 6
(d) -6
(e) 0

4. For what values of a, $\lim_{x \to -1} \frac{x^2 + 2x + a}{x^2 - 2x - 3}$ does exist.

(a) 1 ______ (correct)
(b) -1
(c) 2
(d) -2
(e) 0

5. For what value of the constant c is the function

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

continuous on $(-\infty, \infty)$?

(a) $\frac{2}{3}$ ______________________________ (correct)

(b) $-\frac{2}{3}$

(c) $\frac{3}{2}$

(d) $-\frac{3}{2}$

(e) 2

6. Find the equation of tangent line to the curve $y = \sqrt{1 + xe^{-2x}}$ at $(0, 1)$

(a) $2y = x + 2$ ______________________________ (correct)

(b) $2y = x + 1$

(c) $y = x + 1$

(d) $y = x + 2$

(e) $2y = 2x + 1$

7. Find the equation of the tangent line to the curve $y = x^2 e^{-1/x}$ at $\left(1, \dfrac{1}{e}\right)$

(a) $y = \dfrac{3x-2}{e}$ ________________ (correct)

(b) $y = \dfrac{3}{e}x - 2$

(c) $y = \dfrac{e}{3}x - 2$

(d) $y = \dfrac{x-2}{3e}$

(e) $y = \dfrac{e(x-2)}{3}$

8. If $F(x) = f(g(x))$, where $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$, $g'(5) = 6$ then $F'(5) =$

(a) 24 ________________ (correct)

(b) -24

(c) -12

(d) 12

(e) 25

9. The average cost $\bar{c}$ for producing q units of a product is given by $\bar{c} = 0.00002q^2 - 0.01q + 6 + \dfrac{20,000}{q}$. Find the marginal cost for $q = 200$.

   (a) 4.4 ________________ (correct)
   (b) 5.4
   (c) 6.4
   (d) 4.6
   (e) 4.0

10. If $y = 2^x x^2$ then $\dfrac{dy}{dx} =$

   (a) $x(2^x)(2 + x\ln 2)$ ________________ (correct)
   (b) $(2^x)(2 + x\ln 2)$
   (c) $x(2 + x\ln 2)$
   (d) $2x(2 + \ln 2)$
   (e) $x(2^x + \ln 2)$

11. If $y = e^{-x} \ln x$, then find $\left.\dfrac{dy}{dx}\right|_{x=1} =$

(a) $\dfrac{1}{e}$ ______ (correct)

(b) $-\dfrac{1}{e}$

(c) $-e$

(d) e

(e) 1

12. If $x\sqrt{y+1} = y\sqrt{x+1}$, find $\left.\dfrac{dy}{dx}\right|_{(3,3)} =$

(a) 1 ______ (correct)

(b) -1

(c) 2

(d) -2

(e) 0

13. If $y = (\ln x)^{e^x}$. find $\left.\dfrac{dy}{dx}\right|_{x=e}$

(a) e^{e-1} ____ (correct)

(b) e^{1-e}

(c) e^{e+1}

(d) e^{2e-1}

(e) e^{e-2}

14. If $y = \left(\dfrac{3}{x^2}\right)^x$ find $\left.\dfrac{dy}{dx}\right|_{x=1}$

(a) $3(-2 + \ln(3))$ ____ (correct)

(b) $3(2 + \ln(3))$

(c) $2(3 + \ln(3))$

(d) $2(-3 + \ln(3))$

(e) $3(-2 + \ln(2))$

15. The function $f(x) = 2x^3 + x^2 + 2x$ has how many critical points?

(a) 0 ________________ (correct)
(b) 1
(c) 2
(d) 3
(e) 4

16. On the interval $[-2, 3]$ the function

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

has an

(a) absolute maximum at $x = -1$ and absolute minimum at $x = 2$ ________ (correct)
(b) absolute maximum at $x = 2$ and absolute minimum at $x = -2$
(c) absolute maximum at $x = 3$ and absolute minimum at $x = -2$
(d) absolute maximum at $x = 2$ and absolute minimum at $x = -1$
(e) absolute maximum at $x = 2$ and absolute minimum at $x = 3$

17. The graph of $f(x) = 4x^3 + 3x^2 - 6x + 1$ is

(a) increasing on $(-\infty, -1)$ and $\left(\frac{1}{2}, \infty\right)$ ________________ (correct)

(b) decreasing on $(-\infty, -1)$ and $\left(\frac{1}{2}, \infty\right)$

(c) increasing on $(-\infty, -1)$ and $\left(-1, \frac{1}{2}\right)$

(d) decreasing on $\left(-1, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \infty\right)$

(e) only increasing on $\left(-1, \frac{1}{2}\right)$

18. If $f(x) = x^4 - 2x^2 + 3$ then f is concave up on the interval

(a) $\left(-\infty, -\frac{\sqrt{3}}{3}\right)$ and $\left(\frac{\sqrt{3}}{3}, \infty\right)$ ________________ (correct)

(b) $(-\infty, \infty)$

(c) $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

(d) $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$

(e) $(-\infty, 1/3)$ and $(1/3, \infty)$

19. The graph of

$$f(x) = \frac{x^4 + 1}{1 - x^4}$$

has

(a) two vertical asymptotes and one horizontal asysmptote ________ (correct)
(b) only one vertical asymptote
(c) only two horizontal asymptote
(d) one vertical and one horizontal asymptote
(e) one vertical and two horizontal asymptotes

20. The demand equation for a product q is $p = \dfrac{10}{\sqrt{q}}$ using differentials, approximate the price when 24 units are demanded

(a) $\dfrac{51}{25}$ ________ (correct)
(b) $\dfrac{50}{25}$
(c) $\dfrac{25}{50}$
(d) $\dfrac{25}{51}$
(e) $\dfrac{51}{20}$

21. The marginal revenue function is given by

$$\frac{dr}{dq} = 275 - q - 0.3q^2.$$

Find the price when 10 units of product are demanded

(a) $p = 260$ ________________ (correct)
(b) $p = 275$
(c) $p = 250$
(d) $p = 290$
(e) $p = 265$

22. $\displaystyle\int \frac{6x^2 - 11x + 5}{3x - 1}\,dx =$

(a) $x^2 - 3x + \frac{2}{3}\ln|3x - 1| + c$ ________________ (correct)
(b) $x^2 - 3 - \frac{2}{3}\ln|3x + 1| + c$
(c) $x^2 + 3x - \frac{2}{3}\ln|3x + 1| + c$
(d) $x^2 + 3x + \frac{2}{3}\ln|3x - 1| + c$
(e) $x^2 - 3x - \frac{2}{3}\ln|3x - 1| + c$

23. $\displaystyle\int_0^1 x^2 \sqrt[3]{7x^3+1}\,dx =$

(a) $\dfrac{15}{28}$ ____ (correct)

(b) $-\dfrac{15}{28}$

(c) $\dfrac{28}{15}$

(d) $-\dfrac{28}{15}$

(e) $\dfrac{16}{28}$

24. Find the area of region bounded by $y = x^2 - 4x$, $x = 2$, $x = 6$ and x-axis

(a) 16 ____ (correct)

(b) -16

(c) $\dfrac{32}{3}$

(d) $-\dfrac{32}{3}$

(e) $\dfrac{16}{3}$

25. Evaluate $\displaystyle\int_1^2 x^4(\ln x)^2\,dx =$

(a) $\dfrac{62}{125} + \dfrac{32}{5}(\ln 2)^2 - \dfrac{64}{25}\ln(2)$ ________________ (correct)

(b) $\dfrac{62}{125} - \dfrac{32}{5}(\ln 2)^2 + \dfrac{64}{25}\ln(2)$

(c) $\dfrac{62}{125} + \dfrac{5}{32}(\ln 2)^2 + \dfrac{25}{64}\ln(2)$

(d) $\dfrac{62}{125} - \dfrac{5}{32}(\ln 2)^2 - \dfrac{25}{64}\ln(2)$

(e) $\dfrac{62}{125} - \dfrac{5}{32}(\ln 2) - \dfrac{25}{64}\ln(2)^2$

26. Use $\displaystyle\int u^2\sqrt{u^2 \pm a^2}\,du = \frac{u}{8}(2u^2 \pm a^2)\sqrt{u^2 \pm a^2} - \frac{a^4}{8}\ln|u + \sqrt{u^2 \pm a^2}| + c$
to find $\displaystyle\int 7x^2\sqrt{3x^2 - 6}\,dx =$

(a) $\dfrac{7}{3\sqrt{3}}\left[\dfrac{3\sqrt{3}x}{4}(x^2-1)\sqrt{3x^2-6} - \dfrac{9}{2}\ln|\sqrt{3}x + \sqrt{3x^2-6}|\right] + c$ ________ (correct)

(b) $\dfrac{7}{3\sqrt{3}}\left[\dfrac{\sqrt{3}}{4}(x^2-1)\sqrt{3x^2+6} - \dfrac{9}{2}\ln|\sqrt{3}x + \sqrt{3x^2+6}|\right] + c$

(c) $\dfrac{7}{3\sqrt{3}}\left[\dfrac{3\sqrt{3}}{4}x\sqrt{3x^2+6} - \dfrac{9}{2}\ln|\sqrt{3}x - \sqrt{3x^2+6}|\right] + c$

(d) $\dfrac{7}{3\sqrt{3}}\left[\dfrac{3\sqrt{3}}{4}x(x^2+1)\sqrt{3x^2+6} - \dfrac{9}{2}\ln|\sqrt{3}x + \sqrt{3x^2+6}|\right] + c$

(e) $\dfrac{7}{3\sqrt{3}}\left[\dfrac{3\sqrt{3}x}{4}(x^2-1)\sqrt{3x^2+6} - \dfrac{9}{2}\ln|x + \sqrt{3x^2+6}|\right] + c$

27. If $Z = e^{\sqrt{x^2+y^2}}$ find $\dfrac{\partial^2 z}{\partial y^2}$

(a) $\dfrac{Z(x^2 + y^2\sqrt{x^2+y^2})}{(x^2+y^2)^{3/2}}$ ____________________(correct)

(b) $\dfrac{x^2 + y^2\sqrt{x^2+y^2}}{(x^2+y^2)^{3/2}}$

(c) $\dfrac{Z(x^2\sqrt{x^2+y^2} + y^2)}{(x^2+y^2)^{3/2}}$

(d) $\dfrac{x^2\sqrt{x^2+y^2} + y^2}{(x^2+y^2)^{3/2}}$

(e) $\dfrac{Z(x^2\sqrt{x^2+y^2})}{(x^2+y^2)^{3/2}} + y^2$

28. A candy company produces two varieties of candy A and B for which the constant average costs of production are 60 SR and 70 SR respectively. The demand functions for A and B are given by

$$q_A = 5(P_B - P_A) \text{ and } q_3 = 500 + 5(P_A - 2P_B).$$

Find the selling prices P_A and P_B that maximize the company's profit

(a) $P_A = 80$ and $P_B = 85$ ____________________(correct)

(b) $P_A = 85$ and $P_B = 80$

(c) $P_A = 90$ and $P_B = 95$

(d) $P_B = 95$ and $P_B = 90$

(e) $P_A = 70$ and $P_B = 75$