King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 106
Major Exam I
221
October 02, 2022
Net Time Allowed: 120 Minutes

MASTER VERSION

1. Use the graph of $f(x)$ to estimate $\lim_{x\to 1} f(x)$ (10.1, Q2)

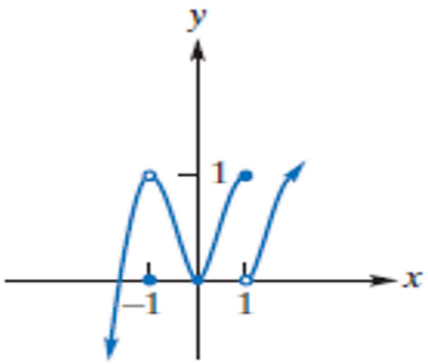
(a) does not exist _____(correct)

(b) 1

(c) -1

(d) 0

(e) $-\infty$



2. The value of $\lim_{x\to 3^-}\left(3-\frac{1}{x-3}\right)$ is (10.2, Q.4)

(a) ∞ _____(correct) $3-\frac{1}{0^-}=+\infty$

(b) -2

(c) $-\infty$

(d) 2

(e) 0

3. The value of $\lim_{x\to 0} \dfrac{(x+2)^2-4}{x}$ is

(10.1, Q. 3 e)

(a) 4 (correct)

(b) -4

(c) ∞

(d) $-\infty$

(e) 0

$= \lim_{x\to 0} \left[\dfrac{x^2+4x+4-4}{x} = \dfrac{x(x+4)}{x}\right]$

$= 0 + 4 = 4$

4. The value of $\lim_{x\to 0^-} f(x)$ for the function depicted in the graph below equals

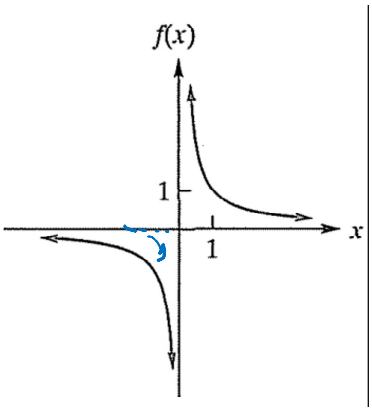
(a) $-\infty$ (correct)

(b) ∞

(c) 0

(d) 1

(e) -1



5. The sum of all points of discontinuity of

$$f(x) = \frac{x^2 + 6x + 9}{x^2 + 2x - 15}$$

equals

(a) -2 (correct)
(b) 0
(c) 1
(d) -13
(e) -15

Discontinuous at $x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$\Rightarrow x = -5, 3$ & $-5+3 = -2$

6. The slope of the tangent line to the curve $y = x^2 + 4$ at the point $(-2, 8)$ is

(a) -4 (correct)
(b) 16
(c) 0
(d) 8
(e) 32

$y' = 2x$

$y'(-2) = -4$

(11.9, Q.19)

7. An equation of the tangent line to the curve $y = x^2 + 2x + 3$ at the point $(1, 6)$ is

(a) $y - 4x - 2 = 0$ (correct)

(b) $y + 4x + 2 = 0$

(c) $y + 4x + 23 = 0$

(d) $y + 4x - 12 = 0$

(e) $y - 4x + 23 = 0$

$y' = 2x + 2$

$y'(1) = 4$, $y(1) = 6$

(11.1, 0.7)

Eq is $y - 6 = 4(x - 1)$

$y - 4x - 6 + 4 = 0$

$y - 4x - 2 = 0$

8. The equation of tangent line to the curve $y = \dfrac{1}{x^2}$ at $x = 2$ is

(11.2, 0.81)

(a) $y = -\dfrac{1}{4}x + \dfrac{3}{4}$ (correct)

(b) $y = -\dfrac{1}{2}x + \dfrac{3}{2}$

(c) $y = \dfrac{1}{4}x - \dfrac{3}{4}$

(d) $y = \dfrac{1}{2}x - 1$

(e) $y = -\dfrac{1}{4}x + 1$

$y(2) = \frac{1}{4}$, $y' = -\frac{2}{x^3}$

$y'(2) = -\frac{2}{2^3} = -1/4$

$y - 1/4 = -\frac{1}{4}(x - 2)$

$\Rightarrow y = -\frac{1}{4}x + \frac{2}{4} + 1/4$

$\Rightarrow = -1/4\,x + 3/4$

9. If the position function of an object moving along a number line is given by $s = f(t) = 3e^t + 1$, where t is in seconds and s is in meters, then the average velocity over the interval $[1, 1.2]$ equals

(a) $15(e^{6/5} - e)$ _____(correct)

(b) 0

(c) 0.2

(d) 0.6

(e) $5(1 + 3e^{6/5})$

Avg vel $= \frac{f(1.2) - f(1)}{1.2 - 1} = \frac{3e^{6/5} - 3e}{6/5 - 1}$

$= \frac{5 \cdot 3(e^{6/5} - e)}{6 - 5} = 15(e^{6/5} - e)$

(11.3, Q.9)

10. Sociologists studied the relation between income and number of years of education for members of a particular urban group. They found that a person with x years of education before seeking regular employment can expect to receive an average yearly income of y dollars per year, where

$$y = 5x^{5/2} + 5900, \quad 4 \le x \le 16.$$

The rate of change of income with respect to number of years of education when $x = 9$ equals

(a) 337.5 _____(correct)

(b) 67.5

(c) 164025

(d) 0.535161

(e) 0.107032

$y'(x) = \frac{25}{2} x^{3/2}$

$y'(9) = \frac{25}{2} \cdot (9)^{3/2} = \frac{25}{2} \cdot (3^2)^{3/2}$

$= \frac{13.5}{2} \cdot 25 = 337.5$

11. If $y = (x^2 + 5x - 7)(6x^2 - 5x + 4)$, then $\dfrac{dy}{dx}$ is

(11.4, Q.9)

(a) $24x^3 + 75x^2 - 126x + 55$ (correct)
(b) $24x^3 - 75x^2 - 126x + 55$
(c) $24x^3 + 175x^2 - 126x + 55$
(d) $24x^3 - 175x^2 - 126x + 55$
(e) $24x^3 + 75x^2 + 126x + 55$

$y' = (2x+5)(6x^2 - 5x + 4) + (x^2 + 5x - 7)(12x - 5)$

$= 12x^3 + (30 - 10)x^2 + (-25 + 8)x + 20 + 12x^3 + (-5 + 60)x^2 + (-35 - 84)x + 35$

$= 24x^3 + 75x^2 - 126x + 55$

(11.4, exp.8)

12. If the demand equation for a manufacturer's product is $p = \dfrac{1000}{q+5}$ where p is in dollars, then the marginal revenue when $q = 45$ is

(a) 2 (correct)
(b) 45
(c) 20
(d) 1
(e) 0

$R = pq = \dfrac{1000}{q+5} \cdot q$

$\dfrac{dR}{dq} = \dfrac{1000(q+5) - 1000q}{(q+5)^2} = \dfrac{5000}{(q+5)^2}$

$\left.\dfrac{dR}{dq}\right|_{q=45} = \dfrac{5000}{(50)^2} = \dfrac{50 \times 100}{(50)^2} = 2$

13. If $z = u^2 + \sqrt{u} + 9$ and $u = 2s^2 - 1$, then dz/ds when $s = -1$.

(H.W 2, Q.7)

(a) -10 (correct)

(b) 10

(c) 1

(d) 11

(e) -1

$\frac{dz}{ds} = \frac{dz}{du} \cdot \frac{du}{ds}$

$= \left[2u + \frac{1}{2\sqrt{u}}\right] \cdot 4s$

at $s = -1$, $u = 1$.

$\left.\frac{dz}{ds}\right|_{s=-1} = (2 + \frac{1}{2})(-4) = \frac{5}{2}(-4) = -10$

14. If $y = \ln((-5x + 2)^{10})$, then $\frac{dy}{dx} =$

(12.1, Ex P.3)

(a) $\frac{50}{-2 + 5x}$ (correct)

(b) $\frac{10 \ln(2 - 5x)^9}{2 - 5x}$

(c) $\frac{50}{2 + 5x}$

(d) $\frac{50 \log(2 - 5x)^9}{2 - 5x}$

(e) $\frac{5}{-2 + 5x}$

$y = 10 \ln(-5x + 2)$

$y' = 10 \cdot \frac{1}{-5x+2} \cdot (-5) = \frac{-50}{-5x+2} = \frac{50}{-2+5x}$

15. The equation of the tangent line to the curve $y = x\ln(x) - x$ at the point where $x = e$ is

(12.1, Q.46)

(a) $y = x - e$ (correct)

(b) $y = x + e$

(c) $y = -x - e$

(d) $y = -x + e$

(e) $y = e$

$y' = \ln(x) + 1 - 1 = \ln(x)$

$y'|_{x=e} = 1$, $y(e) = e\ln(e) - e = 0$

$y - 0 = 1(x - e)$

$\Rightarrow y = x - e$

16. The slope of the tangent line to the curve of $y = e^{1-\sqrt{x}}$ at $x = 1$ is

(a) $-\frac{1}{2}$ (correct)

(b) $-e^{-\frac{1}{2}}$

(c) $e^{\frac{1}{2}}$

(d) 0

(e) $\frac{1}{2}$

(12.2, Q.19)

$\frac{dy}{dx} = -\frac{1}{2\sqrt{x}} \cdot e^{1-\sqrt{x}}$

$\frac{dy}{dx}\Big|_{x=1} = -\frac{1}{2} \cdot e^{0} = \boxed{-1/2}$

17. Let $f(x) = 5^{x^2 \ln x}$ then $f'(1)$ is

(12.2, Q.30)

(a) $\ln(5)$ (correct)

(b) 0

(c) 5

(d) e^5

(e) 10

$f'(x) = 5^{x^2 \cdot \ln(x)} \cdot (x^2 \ln(x))' \cdot \ln(5)$

$= 5^{x^2 \ln(x)} (2x \ln(x) + x) \ln(5)$

$f'(1) = 5^{\ln(1)} = 1 \; (0 + 1) \ln(5)$

$= \ln(5)$

18. If $xe^y + y = 13$, then $\dfrac{dy}{dx}$ by implicit differentiation is equal to

(12.4, Q.21)

(a) $-\dfrac{e^y}{xe^y + 1}$ (correct)

(b) $-\dfrac{e^y}{1 - xe^y}$

(c) $\dfrac{e^y}{x - 1}$

(d) $\dfrac{e^x}{y + 1}$

(e) $\dfrac{e^y}{xe^y + 1}$

$x e^y \cdot y' + e^y + y' = 0$

$\Rightarrow \; y' = \dfrac{-e^y}{xe^y + 1}$

19. If $q - p = \ln q + \ln p$, find $\dfrac{dq}{dp}$, by taking $q = f(p)$.

(12.4, Q.19)

(a) $\dfrac{(1+p)q}{p(q-1)}$ (correct)

(b) $\dfrac{(1+p)}{(q-1)}$

(c) $\dfrac{(1-p)}{p(q-1)}$

(d) $\dfrac{(1+p)q}{p(q+1)}$

(e) $\dfrac{(1-p)q}{p(q-1)}$

$\frac{dq}{dp} - 1 = \frac{1}{q}\cdot\frac{dq}{dp} + \frac{1}{p}$

$\Rightarrow (1-\frac{1}{q})\frac{dq}{dp} = 1 + 1/p$

$\Rightarrow \frac{dq}{dp} = \frac{(p+1)/p}{\frac{q-1}{q}} = \frac{q(p+1)}{p(q-1)}$

20. The derivative of $y = (1+e^x)^{\ln x}$ is

(13.5, exp.4)

(a) $(1+e^x)^{\ln x}\left(\dfrac{\ln(1+e^x)}{x} + \dfrac{e^x \ln x}{1+e^x}\right)$ (correct)

(b) $(-1+e^x)^{\ln x}\left(\dfrac{\ln(-1+e^x)}{x} + \dfrac{e^x \ln x}{-1+e^x}\right)$

(c) $\dfrac{1+e^x}{x} + e^x \ln x$

(d) $\dfrac{2(1+e^x)}{x} + e^x \ln x^2$

(e) $\dfrac{1}{(1+e^x)x} - \dfrac{e^x \ln x}{(1+e^x)^2}$

$\ln(y) = \ln(x)\cdot\ln(1+e^x)$

diff $\Rightarrow \frac{1}{y}\cdot y' = \frac{1}{x}\ln(1+e^x) + \ln(x)\cdot\frac{e^x}{1+e^x}$

$\Rightarrow y' = y\,[\quad"\quad]$

$= (1+e^x)^{\ln(x)}\left[\frac{\ln(1+e^x)}{x} + \frac{e^x\ln(x)}{1+e^x}\right]$