221-106Ex2

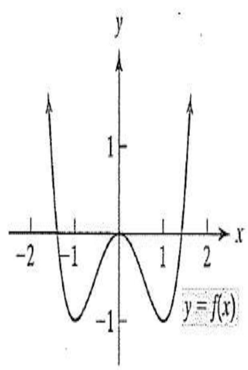
1. If $f(x) = (2x-5)^5$ then rate of change of $f'(x)$ is

(a) $80(2x-5)^3$
(b) $40(2x-5)^3$
(c) $80(2x-5)^7$
(d) $10(2x-5)^4$
(e) $20(2x-5)^3$

$f'(x) = 5 \times 2\,(2x-5)^4 = 10\,(2x-5)^4$

$f''(x) = 80(2x-5)^3$

2. The graph of a function $f(x)$ is given by



Which of the following statements is not true about $f(x)$?

(a) It has absolute maximum at $(0,0)$
(b) It is decreasing on $(-\infty,-1)$ and $(0,1)$
(c) It is increasing on $(-1,0)$ and $(1,\infty)$
(d) It has relative minima at $(-1,-1)$ and $(1,-1)$
(e) It has relative maximum at $(0,0)$

3. If $c = 3q - 3q^2 + q^3$ is a cost function, when is the marginal cost increasing?

(a) $q > 1$
(b) $q > -1$
(c) $q < 1$
(d) $q < 0$
(e) $q > 6$

$c' = 3 - 6q + 3q^2 \Rightarrow c'' = -6 + 6q$

	0		1		∞
c''		$-$	ϕ	$+$	

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4. The absolute maximum of $f(x) = \dfrac{x}{x^2+1}$ on $[0, 2]$ is

(a) $\frac{1}{2}$
(b) $\frac{2}{5}$
(c) 2
(d) $\frac{4}{5}$
(e) $\frac{5}{2}$

$$f'(x) = \frac{1 \times (x^2+1) - 2x \times x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = 0$$

the only solution in $[0, 2]$ is 1

$\text{Max}\{f(0), f(1), f(2)\} = \text{Max}\{0, \frac{1}{2}, \frac{2}{5}\}$

5. If $f(x) = x^4 - 8x^2 - 6$, which of the following statement is false about $f(x)$

(a) $f(x)$ is decreasing on $\left(\frac{-2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right)$

(b) concave up on $\left(-\infty, \frac{-2\sqrt{3}}{3}\right)$

(c) concave up on $\left(\frac{2\sqrt{3}}{3}, \infty\right)$

(d) concave down on $\left(\frac{-2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right)$

(e) inflection points when $x = \frac{-2\sqrt{3}}{3}$ or $\frac{2\sqrt{3}}{3}$

$$f'(x) = 4x^3 - 16x$$
$$= 4x(x^2 - 4)$$
$$f'(x) = 0 \Rightarrow x = 0, -2, 2$$
$$f''(x) = 12x^2 - 16$$
$$= 4\left[3x^2 - 4\right] = 0$$
$$\Rightarrow x = \pm \frac{2\sqrt{3}}{3}$$

	-2	$-2\sqrt{3}/3$	0	$\frac{2\sqrt{3}}{3}$	2
$f''(x)$	$+$ | $+$	0 $-$	| $-$	0 $+$	| $+$
$f'(x)$	$-$ 0	$+$ | $+$	0 $-$	| $-$	0 $+$
$f(x)$					

6. Which of the following statements is true for demand equation $p = \frac{100}{q+2}$ graph?

(a) It is decreasing and concave up when $q > 0$

(b) It is decreasing and concave down when $q < 0$

(c) It is increasing and concave up when $q > 0$

(d) It is always concave up for any value of q

(e) It is always concave down for any value of q

$$p' = \frac{-100}{(q+2)^2} < 0$$

$$p'' = \frac{200}{(q+2)^3} > 0$$

7. Let $f(x) = x^4 - 24x^2 + x - \sqrt{12}$. Then, f has

(a) two inflection points
(b) one inflection point
(c) no inflection point
(d) three inflection points
(e) four inflection points

$f'(x) = 4x^3 - 48x + 1 \Rightarrow f''(x) = 12x^2 - 48 = 12(x-2)(x+2)$

x		-2		2	
$f''(x)$	$+$	ϕ	$-$	ϕ	$+$

8. Let $f(x) = e^{-x^2}$. Then which of the following statements about f is true.

(a) f is concave up on $\left(\frac{1}{\sqrt{2}}, +\infty,\right)$
(b) f is concave down on $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$
(c) f is concave up on $(-\infty, \infty)$
(d) f is concave down on $\left(-\infty, \frac{1}{\sqrt{2}}\right)$
(e) f is concave up on $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

$f'(x) = -2xe^{-x^2} \Rightarrow f''(x) = -2e^{-x^2} + 4x^2e^{-x^2} = 4(x^2 - \frac{1}{2})e^{-x^2}$

x		$-1/\sqrt{2}$		$1/\sqrt{2}$	
$f''(x)$	$+$	ϕ	$-$	ϕ	$+$

9. Which of the following statements is true for the graph of $f(x) = \dfrac{2x^3+1}{3x(2x-1)(4x-3)}$

(a) vertical asymptotes at x is equal to $0, \frac{1}{2}$ and $\frac{3}{4}$

(b) It has no horizontal asymptote

(c) horizontal asymptotes at y is equal to $0, \frac{1}{2}$ and $\frac{3}{4}$

(d) vertical asymptote is $x = \frac{1}{12}$

(e) horizontal asymptote is $y = \frac{2}{3}$

f has denominator zero at x is equal to 0, 1/2 and 3/4.

10. The demand equation is $p = -10q + 40$. At what price will the revenue be maximized?

(a) 20

(b) 10

(c) 30

(d) 40

(e) 50

$R = pq = (-10q+40)q = -10q^2 + 40q$

$R' = -20q + 40$; $R' = 0 \Leftrightarrow q = 2$

Then at q=2, p=-20+40=20

11. A manufacturer's total cost function is $c = \dfrac{q^2}{4} + 3q + 400$. The quality q, for which average cost is minimum, is equal to

(a) 40
(b) 80
(c) -40
(d) 20
(e) 160

$$\bar{c} = \frac{c}{q} = \frac{1}{4}q + 3 + \frac{400}{q}$$

$$\frac{d\bar{c}}{dq} = \frac{1}{4} - \frac{400}{q^2} = \frac{q^2 - 1600}{4q^2} = 0 \Rightarrow q = 40 \text{ as } q > 0$$

	0		40	
$(\bar{c})'$		$-$	0	$+$
$\bar{c}$		↘		↗

12. A manufacturer can produce at most 120 units of a certain product each year. The demand equation for the product is $p = q^2 - 100q + 3200$ and the manufacturer's average-cost function is

$$\bar{c} = \frac{2}{3}q^2 - 40q + \frac{10000}{q}.$$

The profit-maximizing output is q and the corresponding maximum profit is p then

(a) $q = 120,\ p = 86,00$
(b) $q = 40,\ p = \dfrac{130000}{3}$
(c) $q = 80,\ p = \dfrac{98000}{3}$
(d) $q = 0,\ p = 10,000$
(e) $q = 1,\ p = 10,000$

$$P = pq - \bar{c}q$$
$$= q^3 - 100q^2 + 3200q - \frac{2}{3}q^3 + 40q^2 - 10,000$$
$$= \frac{1}{3}q^3 - 60q^2 + 3200q - 10,000$$
$$P' = q^2 - 120q + 3200 = (q - 40)(q - 80)$$
$$\text{Max}\{P(0), P(40), P(80), P(120)\}$$
$$= \text{Max}\{10,000,\ 63333\tfrac{1}{3},\ 52666\tfrac{2}{3},\ \boxed{86000}\}$$

13. By using differential, an approximation of $\sqrt[4]{17}$ is

(a) $\frac{65}{32}$
(b) $\frac{67}{32}$
(c) $\frac{69}{32}$
(d) $\frac{71}{32}$
(e) $\frac{63}{32}$

Let $f(x) = \sqrt[4]{x}$. Then $f'(x) = \frac{1}{4} \cdot \frac{1}{\sqrt[4]{x}^3}$

$$f(17) - f(16) \simeq f'(16) \cdot (17-16) = \frac{1}{4\sqrt[4]{16}^3} = \frac{1}{32}$$

$$\Rightarrow \sqrt[4]{17} \simeq \sqrt[4]{16} + \frac{1}{32} = 2 + \frac{1}{32} = \frac{65}{32}$$

14. Let $f(x) = x^3 - 12x + 1$, then $f(x)$ has relatively maximum at $x = a$ relatively minimum at $x = b$,then $a + b$ is equal to

(a) 0
(b) 2
(c) 4
(d) -4
(e) -2

$$f'(x) = 3x^2 - 12 = 3(x-2)(x+2)$$

x		-2		2	
$f'(x)$	$+$	0	$-$	0	$+$
	$\nearrow$		$\searrow$		$\nearrow$

$\Rightarrow a = -2$ and $b = 2$

$\Rightarrow$ sum $= 0$

15. If $q = \sqrt{2500 - q^2}$ then $\dfrac{dp}{dq}$ is equal to

(a) $-\dfrac{\sqrt{2500 - p^2}}{p}$

(b) $-\dfrac{p}{\sqrt{2500 - p^2}}$

(c) $-\dfrac{\sqrt{2500 - p^2}}{2p}$

(d) $\dfrac{\sqrt{2500 - p^2}}{2p}$

(e) $\dfrac{p}{(2500 - p^2)^{3/2}}$

$$\frac{dp}{dq} = \frac{1}{\frac{dq}{dp}} = \frac{1}{\frac{-2p}{2\sqrt{2500-p^2}}} = -\frac{\sqrt{2500-p^2}}{p}$$

16. If $\dfrac{dr}{dq} = 4 - \dfrac{q}{5}$, then the demand function is

(a) $p = 4 - \dfrac{q}{10}$

(b) $p = 4 - \dfrac{q}{5}$

(c) $p = 49 - \dfrac{q^2}{10}$

(d) $p = 49 - \dfrac{q^2}{5}$

(e) $p = 49 - \dfrac{q^2}{5} + 10$

$$r = 4q - \frac{q^2}{10} \Rightarrow p = \frac{r}{q} = 4 - \frac{q}{10}$$

17. $\displaystyle\int\left(-\frac{\sqrt[3]{x^2}}{5}-\frac{7}{2\sqrt{x}}+6x\right)\,dx=$

$$\int\left(-\frac{x^{2/3}}{5}-\frac{7}{2}x^{-1/2}+6x\right)dx$$

$$=-\frac{x^{\frac{2}{3}+1}}{5\left(\frac{2}{3}+1\right)}-\frac{7}{2}\,\frac{x^{-\frac{1}{2}+1}}{\left(-\frac{1}{2}+1\right)}+\frac{6x^2}{2}+C$$

$$=-\frac{3}{25}x^{5/3}-7x^{1/2}+3x^2+C$$

(a) $\dfrac{-3x^{5/3}}{25}-7x^{1/2}+3x^2+c$

(b) $\dfrac{3x^{1/3}}{5}-\dfrac{7}{4}x^{1/2}+6+c$

(c) $\dfrac{-2}{15\sqrt[3]{x}}+\dfrac{7}{4}x^{-2/3}+c$

(d) $\dfrac{3x^{5/3}}{25}+7x^{1/2}-3x^2+c$

(e) $-\dfrac{3x^{1/2}}{5}+\dfrac{7}{4}x^{-1/2}-6+c$

18. $\displaystyle\int \pi e^x\,dx=$

$$\pi e^x+c$$

(a) πe^x+c

(b) $\dfrac{\pi^2e^x}{2}+c$

(c) $\dfrac{\pi e^x}{x}+c$

(d) $\dfrac{(\pi e^x)^2}{2}+c$

(e) $\pi e^{2x}+c$

19. An oblique asymptote for the graph of the function, $f(x) = \dfrac{3-x^4}{x^3+x^2}$ is

(a) $y = -x+1$
(b) $y = 2x+1$
(c) $y = -1$ or $y = 0$
(d) does not exist
(e) $x = \sqrt[4]{3}$

$$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{3-x^4}{x^4+x^3} = \lim_{x\to\infty} \frac{-x^4}{x^4} = -1$$

$$f(x)-x = \frac{3-x^4}{x^3+x^2} + x = \frac{3-x^4+x^4+x^3}{x^3+x^2} = \frac{x^3+3}{x^3+x^2} \longrightarrow 1$$

$$\boxed{y = -x+1}$$

20. If $y'' = x+1$, $y'(1) = 2$ and $y(1) = 5$, then

(a) $y = \dfrac{x^3}{6} + \dfrac{x^2}{2} + \dfrac{x}{2} + \dfrac{23}{6}$
(b) $y = x^3 + 3x^2 + 3x + 23$
(c) $y = \dfrac{x^3}{6} + \dfrac{x^2}{2} - \dfrac{x}{2} - 23$
(d) $y = x+1$
(e) $y = x^4 - \frac{x^2}{2} + x + 7$

$$y' = \frac{x^2}{2} + x + C$$

$$y'(1) = \frac{1}{2} + 1 + C = 2 \Rightarrow C = \frac{1}{2}$$

$$\Rightarrow y' = \frac{x^2}{2} + x + \frac{1}{2}$$

$$y = \frac{x^3}{6} + \frac{x^2}{2} + \frac{1}{2}x + D$$

$$y(1) = \frac{1}{6} + \frac{1}{2} + \frac{1}{2} + D = 5 \Rightarrow D = 4 - \frac{1}{6} = \frac{23}{6}$$

$$\Rightarrow y = \frac{x^3}{6} + \frac{x^2}{2} + \frac{x}{2} + \frac{23}{6}$$