

King Fahd University of Petroleum and Minerals

Department of Mathematics

**Math 106**

**Final Exam**

**221**

**December 31, 2022**

**Net Time Allowed: 180 Minutes**

**USE THIS AS A TEMPLATE**

**Write your questions, once you are satisfied upload this file.**

$$1. \lim_{x \rightarrow -4} \frac{x+4}{x^2 - 16} =$$

(a)  $-\frac{1}{8}$

(b) 0

(c)  $\infty$

(d)  $\frac{1}{4}$

(e)  $-\frac{1}{4}$

2. If a manufacturer's average-cost equation is

$$\bar{c} = 0.002q^2 - 0.5q + 60 + \frac{7000}{q},$$

find the marginal cost function when 25 units are produced?

(a) 38.75

(b) 37.75

(c) 36.75

(d) 35.75

(e) 34.75

3. If  $F(x) = \frac{4x^2 + 3}{2x - 1}$ , find  $F'(x)$

(a)  $\frac{8x^2 - 8x - 6}{(2x - 1)^2}$

(b)  $\frac{8x^2 + 8x - 6}{(4x^2 + 3)^2}$

(c)  $\frac{8x^2 + 8x - 6}{(4x^2 - 3)^2}$

(d)  $\frac{8x^2 - 8x + 6}{(2x - 1)^2}$

(e)  $\frac{8x^2 + 8x - 6}{(2x - 1)^2}$

4. If  $z = 2y^2 - 4y + 5$ ,  $y = 6x - 5$ , and  $x = 2t$ , then  $\frac{dz}{dt}$  when  $t = 1$  is equal to

(a) 288

(b) 0

(c) 48

(d) 144

(e) 12

5. An equation of the tangent line to the curve  $f(x) = x^2e^x$  when  $x = -2$  is

- (a)  $y = \frac{4}{e^2}$
- (b)  $y = \frac{8}{e^2}x + \frac{20}{e^2}$
- (c)  $y = -\frac{8}{e^2}x + \frac{20}{e^2}$
- (d)  $y = \frac{4}{e^2}x + \frac{8}{e^2}$
- (e)  $y = -\frac{4}{e^2}x + \frac{8}{e^2}$

6. If  $f(x) = -x \ln x + 2x$ , then  $f$  is

- (a) increasing on  $(0, e)$
- (b) increasing on  $(0, \infty)$
- (c) increasing on  $(e, \infty)$
- (d) decreasing on  $\left(\frac{1}{e}, \infty\right)$
- (e) decreasing on  $\left(0, \frac{1}{e}\right)$

7. For the cost function  $c = q^3 + 2q - 1$ , the average-cost function is

- (a) concave up on  $(1, \infty)$
- (b) concave up on  $(0, \infty)$
- (c) concave up on  $(0, 2)$
- (d) concave up on  $(0, 1)$
- (e) concave down on  $(2, \infty)$

8. Identify the true statement. The graph of  $f(x) = \frac{2x^2}{x^2 + x - 6}$  has

- (a) a horizontal asymptote  $y = 2$
- (b) a vertical asymptote  $x = 3$
- (c) horizontal asymptotes  $x = 2$  and  $x = -3$
- (d) no vertical asymptote
- (e) no horizontal asymptote

9. If the marginal revenue functions is  $\frac{dr}{dq} = 275 - 2q - 0.3q^2$ , then the demand function is

- (a)  $p = 275 - q - 0.1q^2$
- (b)  $p = 275q - q^2 - 0.1q^3$
- (c)  $p = \frac{275}{q} - 2 - 0.3q^2$
- (d)  $p = -2 - 0.6q$
- (e)  $p = \frac{275}{q} + 2 - 0.3q^2$

10.  $\int x^2(3x^3 + 7)^3 dx =$

- (a)  $\frac{(3x^3 + 7)^4}{36} + c$
- (b)  $\frac{(9x^2 + 7)^4}{4} + c$
- (c)  $\frac{(3x^3 + 7)^4}{12} + c$
- (d)  $\frac{x^3(3x^3 + 7)^4}{12} + c$
- (e)  $\frac{x^3(3x^3 + 7)^4}{36} + c$

$$11. \int \frac{2x}{x^2 + 5} dx =$$

- (a)  $\ln(x^2 + 5) + c$
- (b)  $\ln|2x| + c$
- (c)  $\ln(x^2 + 5) - \ln|2x| + c$
- (d)  $\frac{x^3}{3} + 5x + c$
- (e)  $\ln \left| \frac{2x}{x^2 + 5} \right| + c$

$$12. \int \frac{(3x+2)(x-4)}{x-3} dx =$$

- (a)  $\frac{3}{2}(x-3)^2 + 8(x-3) - 11 \ln|x-3| + c$
- (b)  $\frac{3}{2}(x-3)^2 - 8(x-3) - 11 \ln|x-3| + c$
- (c)  $\frac{3}{2}(x-3)^2 + 8(x-3) + 5 \ln|x-3| + c$
- (d)  $\frac{3}{2}(x-3)^2 - 8(x-3) + 5 \ln|x-3| + c$
- (e)  $\frac{3}{2}(x-3)^2 + 11 \ln|x-3| + c$

$$13. \int \frac{(\sqrt{x} + 2)^2}{3\sqrt{x}} dx =$$

- (a)  $\frac{2}{9}(\sqrt{x} + 2)^3 + c$
- (b)  $\frac{2}{3}(\sqrt{x} + 2)^3 + c$
- (c)  $\frac{1}{9}(\sqrt{x} + 2)^3 + c$
- (d)  $\frac{1}{3}(\sqrt{x} + 2)^3 + c$
- (e)  $\frac{2}{3}(\sqrt{x} + 2)^3 + \frac{1}{9}(\sqrt{x} + 2)^2 - \frac{1}{3}\sqrt{x} + c$

$$14. \int_2^{e+1} \frac{1}{x-1} dx =$$

- (a) 1
- (b)  $\ln(e) - \ln(2)$
- (c)  $1 - \ln(e+2)$
- (d) 0
- (e)  $\ln(e+1) - \ln(2)$

15.  $\int_{-71}^{71} \frac{t}{\ln e^t} dt =$

- (a) 142
- (b) 0
- (c)  $7(t - 7)$
- (d)  $\ln(1 + 2t)$
- (e)  $\ln(\ln(142))$

16. The area of the region bounded by the curves  $y = 4x - x^2 + 8$  and  $y = x^2 - 2x$  is equal to

- (a)  $\frac{125}{3}$
- (b)  $\frac{112}{3}$
- (c) 0
- (d)  $\frac{13}{3}$
- (e)  $\frac{112}{125}$

17.  $\int \frac{2x+7}{e^{3x}} dx =$

(a)  $-\frac{6x+23}{9e^{3x}} + c$

(b)  $\frac{6x+23}{9e^{3x}} + c$

(c)  $-\frac{2x+7}{3e^{3x}} + c$

(d)  $\frac{2x+7}{3e^{3x}} + c$

(e)  $-\frac{x-7}{3e^{3x}} + c$

18. By using partial fractions the  $\int \frac{5x-2}{x^2-x} dx =$

(a)  $2\ln|x| + 3\ln|x-1| + c$

(b)  $\ln|x| + \ln|x-1| + c$

(c)  $5\ln|x| - \ln|x-1| + c$

(d)  $-2\ln|x| + \ln|x-1| + c$

(e)  $\ln|5x-2| + \ln|x^2-x| + c$

19. By using partial fractions the  $\int \frac{6x^2 + 13x + 6}{(x+2)(x+1)^2} dx =$

- (a)  $4 \ln|x+2| + 2 \ln|x+1| + \frac{1}{x+1} + c$
- (b)  $4 \ln|x+2| + 3 \ln|x+1| + c$
- (c)  $4 \ln|x+2| - 2 \ln|x+1| + \ln(x+1)^2 + c$
- (d)  $4 \ln|x+1| - 2 \ln|x+2| - \frac{1}{x+1} + c$
- (e)  $\ln|x+2| + 2 \ln|x+1| + c$

20. Given  $\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2u} + c$ , then  $\int \frac{dx}{x^2\sqrt{2-3x^2}} =$

- (a)  $-\frac{1}{2} \frac{\sqrt{2-3x^2}}{x} + c$
- (b)  $-\frac{\sqrt{3}}{2} \frac{\sqrt{2-3x^2}}{x} + c$
- (c)  $-\frac{1}{2} \frac{\sqrt{2-x^2}}{x} + c$
- (d)  $-\frac{1}{2\sqrt{3}} \frac{\sqrt{2-x^2}}{x} + c$
- (e)  $-\frac{1}{\sqrt{3}} \sqrt{\frac{2-3x^2}{x}} + c$

21. Let  $h(r, s, t, u) = rst^2u \ln(1 + rstu)$  then  $h_t(1, 1, 0, 1) =$

- (a) 0
- (b) 2
- (c)  $\ln(2)$
- (d) 1
- (e) 3

22. If  $f(x, y) = 7x^2 + 3y^2$ , then  $f_y(x, y)$  is equal to

- (a)  $6y$
- (b)  $14x$
- (c) 0
- (d)  $3y$
- (e)  $14x + 6y$

23. If  $f(x, y) = y^2e^x + \ln(xy)$ , then  $f_{xyy}(1, 2)$  is equal to

- (a)  $2e$
- (b)  $4e$
- (c)  $4e + 1$
- (d)  $4e^2$
- (e)  $e$

24. Let  $f(x, y) = xy - \frac{1}{x} - \frac{1}{y}$ . Then  $f$  has

- (a) a relative minimum
- (b) a relative maximum
- (c) a relative minimum and a relative maximum
- (d) two relative minimums
- (e) two relative maximums

25. Let  $f(x, y) = 18x + 20y - 2x^2 - 4y^2 - xy$ . If  $f$  has a relative maximum at the point  $(c, d)$ , then  $c + d =$

- (a) 6
- (b) 5
- (c) 4
- (d) 3
- (e) 7

26. If  $xe^y + y = 13$ , then by the implicit differentiation  $\frac{dy}{dx} =$

- (a)  $\frac{-e^y}{1 + xe^y}$
- (b)  $\frac{e^y}{1 - xe^y}$
- (c)  $\frac{1 + xe^y}{e^y}$
- (d)  $\frac{e^y}{1 + xe^y}$
- (e)  $(x + e^y)e^y$

27. If  $y = \tan(7x^2 - 8x + 2)$ , then  $\frac{dy}{dx} =$

- (a)  $(14x - 8) \sec^2(7x^2 - 8x + 2)$
- (b)  $\sec^2(7x^2 - 8x + 2)$
- (c)  $\cot^2(7x^2 - 8x + 2)$
- (d)  $(14x - 8) \csc^2(7x^2 - 8x + 2)$
- (e)  $(14x - 8) \tan(7x^2 - 8x + 2)$

28. If  $f(x) = 3x^2 + 12x + 14$ , then  $f$  has a relative maximum at

- (a) no point in the domain of  $f$
- (b) 2
- (c) -2
- (d) 38
- (e) 14