King Fahd University of Petroleum and Minerals
Department of Mathematics

**Math 106**
**Final Exam**
**222**
**May 17, 2023**

# EXAM COVER

**Number of versions: 4**
**Number of questions: 28**

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**Math 106**
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**Net Time Allowed: 180 Minutes**

# MASTER VERSION

1. The function $f(x) = \dfrac{x^2 - 3x - 10}{x^2 - 4}$

   (a) has a point of discontinuity at $x = -2$ _____(correct)
   (b) has no limit at $x = -2$
   (c) is continuous at $x = -2$
   (d) has no limit as $x \to -\infty$
   (e) has no limit as $x \to +\infty$

2. If the total revenue of a product is given $r = q\left(15 - \dfrac{1}{30}q\right)$, the marginal revenue for $q = 15$ is

   (a) 14 _____(correct)
   (b) 15
   (c) $\dfrac{15}{30}$
   (d) $\dfrac{14}{15}$
   (e) 30

3. If the cost function of a certain product is $c(q) = \dfrac{1-q+q^2}{q+1}$. Then the cost of producing the 5-th item is

(a) $\dfrac{22}{25}$ (correct)

(b) $\dfrac{12}{9}$

(c) 6

(d) $\dfrac{14}{25}$

(e) $\dfrac{15}{36}$

4. If $y = \dfrac{1}{1+t}$, $t = 5x - 1$, then $\dfrac{dy}{dx}$ at $x = 1$ is equal to

(a) $\dfrac{-1}{5}$ (correct)

(b) $\dfrac{-1}{4}$

(c) $\dfrac{7}{9}$

(d) 4

(e) $\dfrac{15}{36}$

5. Let $f(x) = \ln \sqrt[4]{\dfrac{1+x^2}{1-x^2}}$ then $f'(x) =$

(a) $\dfrac{x}{1-x^4}$ (correct)

(b) $\dfrac{x}{1-x^2}$

(c) $\dfrac{x}{4(1-x^4)}$

(d) $\dfrac{4}{1-x^4}$

(e) $\dfrac{2x}{1-x^2}$

6. If $xe^y + y = 2x + 9$, then $\dfrac{dy}{dx}$ at $(-9, 0)$ is equal to

(a) $\dfrac{-1}{8}$ (correct)

(b) $\dfrac{2}{5}$

(c) $\dfrac{3}{5}$

(d) $\dfrac{2}{3}$

(e) $\dfrac{-1}{9}$

7. The equation of the tangent line to the curve $f(x) = (1+x)^{(x+1)}$ at the point $(0, 1)$ is

   (a) $y = x + 1$ _____ (correct)
   (b) $y = \frac{1}{2}x - 1$
   (c) $y = 2x$
   (d) $y = x - 1$
   (e) $y = \frac{3}{2}x - 1$

8. The function $f(x) = -\frac{x^4}{4} - x^3$ is

   (a) decreasing on $(-3, 0)$ _____ (correct)
   (b) increasing on $(-3, 0)$
   (c) decreasing on $(-\infty, -3)$
   (d) decreasing on $(-\infty, 0)$
   (e) increasing on $(-3, \infty)$

9. Let the cost function of a product is given by $c = q^2 + 2q + 1$. Then the average cost function

   (a) is concave up for $q > 0$ _____(correct)
   (b) is decreasing for $q > 1$
   (c) is concave down for $q > 0$
   (d) is increasing for all $q > 0$
   (e) has an inflection point at $q = 1$

10. At $x = -2$ the function $y = x^3 - 12x + 1$ has

   (a) a relative maximum _____(correct)
   (b) a relative minimum
   (c) an absolute maximum
   (d) an absolute minimum
   (e) an inflection point

11. The function $f(x) = \dfrac{e^x}{e^x - 1}$ has

(a) $y = 0$ as a horizontal asymptote _____(correct)
(b) $y = x - 1$ as a slant asymptote
(c) $x = 1$ as a vertical asymptote
(d) no asymptotes
(e) no more than two asymptotes

12. $\displaystyle\int (u^e + e^u + 0.7u^3 + 10 + u^{-3})\, du =$

(a) $\dfrac{u^{e+1}}{e+1} + e^u + \dfrac{0.7}{4}u^4 + 10u - \dfrac{u^{-2}}{2} + C$ _____(correct)
(b) $u^e + e^u + \dfrac{0.7}{4}u^4 + 10u - \dfrac{u^{-2}}{2} + C$
(c) $\dfrac{u^{e+1}}{e+1} + \dfrac{e^{u+1}}{u+1} + \dfrac{0.7}{4}u^4 + 10u - \dfrac{u^{-2}}{2} + C$
(d) $\dfrac{u^e}{e} + \dfrac{e^{u+1}}{u+1} + \dfrac{0.7}{4}u^4 + 10u - \dfrac{u^{-4}}{2} + C$
(e) $\dfrac{u^{e+1}}{e} + e^{u+1} + 0.7u^4 + 10u + \dfrac{u^{-4}}{24} + C$

13. $\displaystyle\int \frac{e^x}{e^x + e^{-x}}\,dx =$

(a) $\ln\sqrt{e^{2x}+1}$ _____(correct)

(b) $\ln(e^{2x}+1)$

(c) $\ln(e^{2x}+1)^2$

(d) $\ln(e^x + e^{-x})$

(e) $\ln(e^x+1)$

14. $\displaystyle\int \left[\frac{1}{8x+1} - \frac{1}{e^x(8+e^{-x})^2}\right] dx =$

(a) $\displaystyle\frac{1}{8}\ln|8x+1| - \frac{1}{8+e^{-x}} + C$ _____(correct)

(b) $\displaystyle\frac{1}{8}\ln|8x+1| + \frac{1}{8+e^{-x}} + C$

(c) $\displaystyle 8\ln|8x+1| - \frac{1}{8+e^{x}} + C$

(d) $\displaystyle\frac{1}{8}\ln|8x+1| + \frac{1}{8+e^{x}} + C$

(e) $\displaystyle 8\ln|8x+1| - \frac{1}{8+e^{-x}} + C$

15. The marginal-cost function for a manufacturer's product is given by

$$\frac{dc}{dq} = \frac{9}{10}\sqrt{q}\sqrt{0.04q^{3/2}+4}$$

where $c$ is the total cost in dollars when $q$ units are produced. If fixed costs are \$360, then the total cost of producing 25 units is

(a) \$550 _____(correct)
(b) \$360
(c) \$380
(d) \$1530
(e) \$560

16. $\int_0^1 |2x-1|\,dx =$

(a) $\frac{1}{2}$ _____(correct)
(b) 1
(c) $\frac{1}{4}$
(d) $\frac{3}{4}$
(e) $\frac{5}{4}$

17. The area between the graph of $f(x) = x^2$ and $g(x) = x + 12$ is given by

(a) $\int_{-3}^{4} (12 + x - x^2)\, dx$ _____(correct)

(b) $\int_{3}^{-4} (12 + x - x^2)\, dx$

(c) $\int_{-3}^{4} (x^2 - x + 12)\, dx$

(d) $\int_{-3}^{4} (x^2 - x - 12)\, dx$

(e) $\int_{-2}^{6} (12 + x - x^2)\, dx$

18. The area bounded by $f(x) = \sqrt{2x - 1}$, the $x$-axis and $x = 1$ and $x = 5$ is

(a) $\frac{26}{3}$ _____(correct)

(b) $\frac{31}{6}$

(c) $\frac{25}{6}$

(d) $\frac{27}{7}$

(e) $\frac{28}{3}$

19. If $f(x, y) = \tan(xy)$ then $f_{xx} =$

(a) $2y^2 \sec^2 xy \tan xy$ _____(correct)

(b) $y^2 \sec xy \tan xy$

(c) $2 \sec^2 xy \tan xy$

(d) $y \sec xy \tan xy$

(e) $2y^2 \sec xy \tan^2 xy$

20. $\int_1^4 \sqrt{x} \ln(x^9)\, dx =$

(a) $96 \ln(2) - 28$ _____(correct)

(b) $8 \ln(4) - 84$

(c) $96 \ln(4) - 28$

(d) $84 \ln(2) - 28$

(e) $84 \ln(2) - 48$

21. $\int x^2 e^x \, dx =$

(a) $e^x (x^2 - 2x + 2) + C$ _____(correct)
(b) $e^x (x^2 - x + 2) + C$
(c) $e^x (x^2 - 2x + 1) + C$
(d) $e^x (2x^2 - 2x + 2) + C$
(e) $e^x (x^2 - x + 1) + C$

22. Given that $\int \frac{du}{(a^2 + u^2)^{\frac{3}{2}}} = \frac{u}{a^2\sqrt{a^2 + u^2}} + C$, then $\int \frac{dx}{(25 + 9x^2)^{\frac{3}{2}}} =$

(a) $\frac{x}{25\sqrt{25 + 9x^2}} + C$ _____(correct)
(b) $\frac{3x}{25\sqrt{25 + 9x^2}} + C$
(c) $\frac{9x^2}{25\sqrt{25 + 9x^2}} + C$
(d) $\frac{1}{\sqrt{3}} \left( \frac{3x}{25\sqrt{25 + 9x^2}} \right) + C$
(e) $\frac{1}{3} \left( \frac{3x}{25\sqrt{25 + 9x^2}} \right) + C$

23. If $f(r,s,t) = rst\,(r^2 + s^3 + t^4)$, then $f_s(1,-1,2) =$

(a) 26 _____(correct)
(b) 22
(c) 24
(d) 16
(e) 28

24. Let $f(x,y) = x^4y^3 + 3x^3y^2 - 7x - 7$. Then $f_{xyx}(1,1) + f_{xxy}(1,1) =$

(a) 144 _____(correct)
(b) 36
(c) 72
(d) 288
(e) 180

25. $z = \dfrac{\ln(x^2+5)}{y}$, then $\dfrac{\partial^2 z}{\partial y \partial x}(1,2) =$

(a) $\dfrac{-1}{12}$ (correct)

(b) $\dfrac{-2}{9}$

(c) $\dfrac{4}{9}$

(d) $\dfrac{1}{12}$

(e) $\dfrac{-1}{6}$

26. If $g(r,s,t) = e^{s+t}\,(r^2+7s^3)$ then $g_t(2,0,1) =$

(a) $4\,e$ (correct)

(b) $4$

(c) $11\,e$

(d) $e^4$

(e) $1$

27. If $f(x,y) = y - y^2 - 3x - 6x^2$, then it is false that

(a) there is a relative minimum at $\left(-\frac{1}{4}, \frac{1}{2}\right)$ _____(correct)
(b) $\left(-\frac{1}{4}, \frac{1}{2}\right)$ is a critical point
(c) there is a relative maximum at $\left(-\frac{1}{4}, \frac{1}{2}\right)$
(d) $D\left(-\frac{1}{4}, \frac{1}{2}\right) > 0$
(e) $f_{xx}(x,y) < 0$

28. Let $f(x,y) = xy - x + y$ then $f$ has

(a) a saddle point at $(-1,1)$ _____(correct)
(b) 3 critical points
(c) a relative maximum at $(-1,1)$
(d) a saddle point at $(1,-1)$
(e) a relative minimum at $(1,-1)$