

King Fahd University of Petroleum and Minerals  
Department of Mathematics  
**Math 106**  
**Final Exam**  
**232**  
**May 30, 2024**  
**Net Time Allowed: 180 Minutes**

**USE THIS AS A TEMPLATE**

Write your questions, once you are satisfied upload this file.

1. Find  $\lim_{x \rightarrow 0} \frac{(x+2)^2 - 4}{x}$  34/10.1

- (a) 4
- (b) 2
- (c) -2
- (d) 3
- (e) 16

2. For products A and B, the joint-cost function for a manufacturer is

$$c = \frac{3}{2} q_A^2 + 3 q_B^2$$

and the demand functions are

$$p_A = 60 - q_A^2 \quad \text{and} \quad p_B = 36 - 2 q_B^2$$

Find the values of  $q_A$  and  $q_B$  that maximize profit. similar to 7/17.6

- (a)  $q_A = 4$  and  $q_B = 2$
- (b)  $q_A = 5$  and  $q_B = 2$
- (c)  $q_A = 4$  and  $q_B = 6$
- (d)  $q_A = 5$  and  $q_B = 3$
- (e)  $q_A = 2$  and  $q_B = 6$

3. Find the slope of the curve  $y = \frac{4x^3}{x^4 + 1}$  at  $(-1, -\frac{1}{2})$  50/11.4

- (a) 2
- (b) 4
- (c)  $-1$
- (d)  $-2$
- (e) 8

4. If  $y = \sqrt[3]{(x^2 - 8)^2}$ , find  $\frac{dy}{dx}$  when  $x = 3$ . 59/11.5

- (a) 4
- (b)  $-3$
- (c)  $-8$
- (d) 3
- (e) 2

5. Find an equation of the tangent line to the curve  $f(x) = \ln(x^2 + 6x + 1)$  when  $x = 0$ ? **similar to 6/12.1**

(a)  $y = 6x$

(b)  $y = 2x + 1$

(c)  $y = 6x + 1$

(d)  $y = (\ln 2)x + 1$

(e)  $y = 2e x + 1$

6. Let  $f(x, y) = \frac{1}{3}(x^3 + 8y^3) - 2(x^2 + y^2) + 1$ . Then  $f$  has **13/17.6**

(a) one relative minimum, one relative maximum, and two saddle points.

(b) only one relative maximum and no saddle point.

(c) only one saddle point and one relative maximum.

(d) two relative minima

(e) two relative maxima

7. For the cost function  $c = q - \frac{1}{q}$ , the average-cost function is increasing when  
similar to 68/13.3

- (a)  $q > 0$
- (b)  $q > 1$
- (c)  $q > 2$
- (d)  $q > 3$
- (e)  $q > 4$

8. Which of the following statements is **FALSE** for the graph of

$$f(x) = \frac{x^2 - 1}{2x^2 - 9x + 4}?$$

16/13.5

- (a) The graph has one vertical asymptote and one horizontal asymptote
- (b) The line  $x = \frac{1}{2}$  is a vertical asymptote
- (c) The line  $x = 4$  is a vertical asymptote
- (d) The graph has two vertical asymptotes and one horizontal asymptote
- (e) The line  $y = \frac{1}{2}$  is a horizontal asymptote

9. If the marginal revenue functions is  $\frac{dr}{dq} = 4000 - 20q - 3q^2$ , find the demand function.  
similar to Example4/14.3

- (a)  $p = 4000 - 10q - q^2$
- (b)  $p = 4000 - q - q^2$
- (c)  $p = 400 - q^2$
- (d)  $p = 2000 - q - q^2$
- (e)  $p = 1000 - 10q - q^2$

10. Find  $\int x^3 e^{4x^4} dx =$  22/14.4

- (a)  $\frac{1}{16} \cdot e^{4x^4} + c$
- (b)  $2xe^{x^2} + c$
- (c)  $x^2e^{x^2} + c$
- (d)  $2x + c$
- (e)  $2x^2 + e^{x^2} + c$

11. Find  $\int \frac{6s^2 + 2s + 1}{s + s^2 + 2s^3} ds =$  **26/14.4**

(a)  $\ln |s + s^2 + 2s^3| + C$

(b)  $\ln |3s^2| + C$

(c)  $\ln(s^3 + 5) - \ln |2s + 1| + C$

(d)  $\ln |6s^2 + 2s + 1| + C$

(e)  $\ln \left| \frac{3s}{s^2 + 5} \right| + C$

12. Find  $\int_0^1 e^{\ln(3x^2 + 1)} dx =$  **similar to 53/14.5**

(a) 2

(b) 3

(c)  $3e^2$

(d)  $e^4$

(e) -3

13. If  $\frac{dr}{dq} = \frac{300}{(q+3)^2}$  is a marginal-revenue function, then the demand function is  
similar to 57/14.5

(a)  $p = \frac{100}{q+3}$

(b)  $p = \frac{300}{(q+3)^3}$

(c)  $p = \frac{100}{(q+3)^2}$

(d)  $p = \frac{-100}{(q+3)^2}$

(e)  $p = \frac{200q}{q+3}$

14.  $\int_{-1}^2 (3x^2 - 4x + 6) dx =$  similar to Example 1/14.7

(a) 21

(b) 34

(c) -8

(d) 49

(e) -58



15.  $\int_{1/2}^3 \frac{1}{x^2} dx =$  **15/14.7**

(a)  $\frac{5}{3}$

(b) 3

(c)  $\frac{1}{2}$

(d) 1

(e)  $\frac{3}{2}$

16. Find the area of the region bounded by the curve  $y = x^2 - 1$  and  $y = x - 1$ . **similar to 41/14.9**

(a)  $\frac{1}{6}$

(b)  $\frac{1}{2}$

(c) 2

(d)  $\frac{2}{3}$

(e)  $\frac{3}{8}$

17. Find  $\int_{-1}^0 x e^x dx =$  Illustration after the integration by parts formula/15.1

(a)  $\frac{2}{e} - 1$

(b)  $\frac{1}{e} - 2$

(c)  $-1$

(d)  $e - 1$

(e)  $e + 1$

18. Find  $\int_1^e 9x^2 \ln x dx$  by using integration by parts. similar to 6/15.1

(a)  $1 + 2e^3$

(b)  $\frac{1}{2}e + \frac{1}{2}$

(c)  $e^2$

(d)  $-4e^2$

(e)  $-2e$

19. Let  $f(x, y, z) = z^2(3x^2 - 4xy^3)$  then  $f_{xyyz}(10, 1, -1) =$  similar to 12/17.4

- (a) 48
- (b) 26
- (c) 52
- (d) 78
- (e) 46

20. Use the following formula  $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln | u + \sqrt{u^2 - a^2} | + C,$   
to find the integral  $\int \frac{dx}{\sqrt{9x^2 - 4}} =$  similar to 33/15.3

- (a)  $\frac{1}{3} \ln | 3x + \sqrt{9x^2 - 4} | + C$
- (b)  $\ln | 3x - \sqrt{9x^2 + 4} | + C$
- (c)  $\ln | x - \sqrt{x^2 - 4} | + C$
- (d)  $\frac{1}{2} \ln | 3x - \sqrt{9x^2 + 4} | + C$
- (e)  $\frac{1}{\sqrt{9x^2 - 4}} + C$

21. Let  $g(x, y) = 3x^2 + 5x \cos(y) + y \sin(x)$  then  $g_x(0, 0) + g_y\left(\frac{\pi}{2}, 0\right) =$  **Handout and 17.1**

- (a) 6
- (b) 3
- (c) 5
- (d) 4
- (e) 9

22. If  $g(x, y, z) = e^{3x} \sqrt{y + 18z}$ , then  $g_z(0, -9, 1) =$  **similar to 29/17.1**

- (a) 3
- (b)  $\frac{1}{8}$
- (c)  $\frac{1}{2}$
- (d)  $e$
- (e) 9

23. If  $y = (x - 2) \tan x$ , find  $y'(0) =$  **similar to 3/Handout**

- (a)  $-2$
- (b)  $2$
- (c)  $-1$
- (d)  $\pi$
- (e)  $-\pi$

24. Let  $f(x, y) = 3x^2 - y^3 + 18x + 3y + 9$ . Which of the following statements is **FALSE**?  
**similar to 12/17.6**

- (a)  $f$  has a relative maximum at  $(-3, 1)$
- (b)  $f$  has a relative minimum at  $(-3, -1)$
- (c)  $f$  has a saddle point at  $(-3, 1)$
- (d)  $f_{xx}(-3, -1) = 6$
- (e)  $f_{yy}(-3, -1) = 6$

25. Let  $f(x, y) = 2xy - 9x - 7y$ . If  $f$  has a critical point at  $(c, d)$ , then  $c + d =$  **similar to 4/17.6**

- (a) 8
- (b) 7
- (c) 9
- (d) 1
- (e) 2

26. If  $z^2 - 3x^2 + y^2 = -1$ , find  $\frac{\partial^2 z}{\partial x^2}$  at  $x = 1, y = 1, z = 1$   
(Hint: use implicit differentiation) **similar to 23/17.4**

- (a) -6
- (b) 2
- (c) 4
- (d) -4
- (e) 8

27.  $\int \sin(2x) dx =$  1/ Handout (integral part)

(a)  $-\frac{1}{2} \cos(2x) + C$

(b)  $\sin(2x) + C$

(c)  $2 \cos(2x) + C$

(d)  $2x \cos(2x) + C$

(e)  $-\frac{1}{2} \sin(2x) + C$

28. If  $f(x) = 3x^5 - 5x^3$ , then  $f(x)$  has relative maximum when  $x =$  23/13.1

(a)  $-1$

(b)  $1$

(c)  $0$

(d)  $3$

(e)  $-5$