

1. Let

$$f(x) = \begin{cases} \sqrt{2-x} & \text{if } x < 2, \\ x^3 + k(x+1) & \text{if } x \geq 2. \end{cases}$$

Determine the value of k for which $\lim_{x \rightarrow 2} f(x)$ exists.

exact problem 64. Section 10.2

$$(a) -\frac{8}{3}$$

(b) 0

(c) -8

(d) $\frac{5}{3}$

(e) $-\frac{4}{3}$

$$\lim_{x \rightarrow 2} f(x) \text{ exists} \Leftrightarrow \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^-} f(x)$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (x^3 + k(x+1)) = 8 + 3k$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \sqrt{2-x} = 0$$

$$8 + 3k = 0$$

$$k = -\frac{8}{3}$$

2. If $\lim_{t \rightarrow \infty} \frac{4t^n + t^{n-1} - 2}{7t^m - 2t^{m-1} + 28} = \frac{4}{7}$, then $m - n =$

similar to problem 25. Section 10.2

$$(a) 0$$

(b) 1

(c) -1

(d) 2

(e) -2

if $m = n$

$$\lim_{t \rightarrow \infty} \frac{4t^n + t^{n-1} - 2}{7t^m - 2t^{m-1} + 28} = \frac{4}{7}$$

then $m - n = 0$

3. If $a, b > 0$ and the function

$$f(x) = \begin{cases} \frac{a}{x^2} & \text{if } x < -3 \\ bx + 1 & \text{if } x \geq -3 \end{cases}$$

is continuous at $x = -3$, then $\sqrt{a + 27b} =$

similar to problem 34. Section 10.3

(a) 3

(b) 1

(c) 9

(d) 6

(e) 18

$f(x)$ is continuous at $x = -3 \Leftrightarrow \lim_{x \rightarrow -3} f(x) = f(-3)$

$$f(-3) = b(-3) + 1 = -3b + 1$$

$$\lim_{x \rightarrow -3^+} f(x) = \lim_{x \rightarrow -3^+} (bx + 1) = -3b + 1$$

$$\lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^-} \left(\frac{a}{x^2} \right) = \frac{a}{9}$$

$$\Rightarrow \frac{a}{9} = -3b + 1 \Rightarrow a + 27 = 9$$

$$\sqrt{a + 27} = 3$$

4. The curve of $f(x) = x^3 - 6x^2 + 12x + 12$ has

similar to problem 35. Section 11.1

(a) one horizontal tangent line

(b) one vertical tangent line

(c) three horizontal tangent lines

(d) two horizontal tangent lines

(e) two vertical tangent lines

the curve of $f(x)$ has horizontal ~~and~~ tangent at $x = x_0$

when $f'(x) = 0$

$$f'(x) = 3x^2 - 12x + 12$$

$$3x^2 - 12x + 12 = 0$$

$$x^2 - 4x + 4 = 0$$

$$(x - 2)^2 = 0$$

$$x - 2 = 0$$

$x = 2 \Rightarrow$ one H.T.

$f(x)$ is continuous poly function
No V.T.

5. $\lim_{t \rightarrow 5} \frac{\sqrt{t+4} - 3}{t-5} =$

similar to problem 64. Section 10.1

(a) $\frac{1}{6}$

(b) $\frac{1}{9}$

(c) $\frac{1}{3}$

(d) ∞

(e) 0

$$= \lim_{t \rightarrow 5} \frac{\sqrt{t+4} - 3}{t-5} \cdot \frac{\sqrt{t+4} + 3}{\sqrt{t+4} + 3}$$

$$= \lim_{t \rightarrow 5} \frac{t+4-9}{(\sqrt{t+4} + 3)(t-5)}$$

$$= \lim_{t \rightarrow 5} \frac{1}{\sqrt{t+4} + 3} = \frac{1}{\sqrt{9} + 3} = \frac{1}{6}$$

6. The average cost \bar{c} for producing q units of a product is $\bar{c} = 0.4q + 4 + \frac{9}{q}$. The marginal cost when producing one unit is

similar to problem 32 and problems 19-22. Section 11.3

(a) 4.8

(b) 4.2

(c) 4.4

(d) 4.1

(e) 4.6

$$\begin{aligned} C &= q\bar{c} \\ &= q(0.4q + 4 + \frac{9}{q}) \\ &= 0.4q^2 + 4q + 9 \end{aligned}$$

$$\frac{dC}{dq} = 0.8q + 4$$

$$\frac{dC}{dq}(1) = 0.8 + 4 = 4.8$$

7. If the demand function for a product is $p(q) = \frac{q+5}{q+3}$, then the marginal-revenue when $q = 1$ is

similar to problem 62. Section 11.4

(a) $\frac{11}{8}$

(b) $\frac{1}{8}$

(c) $\frac{11}{2}$

(d) $\frac{21}{2}$

(e) $\frac{21}{8}$

$$\begin{aligned} \text{revenue } r &= pq \\ &= \left(\frac{q+5}{q+3}\right)q \end{aligned}$$

$$= \frac{q^2+5q}{q+3}$$

$$\frac{dr}{dq} = \frac{(2q+5)(q+3) - (q^2+5q)(1)}{(q+3)^2}$$

$$\text{at } q=1 \quad \frac{dr}{dq}(1) = \frac{(7)(4) - (6)(1)}{(4)^2} = \frac{28-6}{16} = \frac{22}{16} = \frac{11}{8}$$

8. The relative rate of change of $y = 2x^2 + 5$ at $x = 10$ is

exact problem 35. Section 11.3

(a) $\frac{8}{41}$

(b) $\frac{8}{5}$

(c) $\frac{3}{11}$

(d) $\frac{1}{9}$

(e) $\frac{5}{22}$

$$\text{relative rate of change} = \frac{y'}{y}$$

$$y' = 4x$$

$$\frac{y'}{y} = \frac{4x}{2x^2+5}$$

$$\frac{y'}{y} \Big|_{x=10} = \frac{40}{205} = \frac{8}{41}$$

9. If the demand equation is $p = 100 + aq + bq^2$ for some real numbers a and b such that

$$\frac{dq}{dp} = \begin{cases} 1 & \text{if } q = 0, \\ \frac{1}{2} & \text{if } q = \frac{1}{2}. \end{cases}$$

Then, $a + b =$

similar to problems 31-34. Section 12.4

(a) 2

(b) 0

(c) -2

(d) $-\frac{1}{2}$

(e) $\frac{1}{2}$

$$p = 100 + aq + bq^2$$

$$\text{take } \frac{d}{dp} \Rightarrow$$

$$1 = a \frac{dq}{dp} + 2bq \frac{dq}{dp}$$

$$\text{at } q=0, \frac{dq}{dp} = 1 \Rightarrow 1 = a(1) + 0 \Rightarrow \boxed{a=1}$$

$$\text{at } q=\frac{1}{2}, \frac{dq}{dp} = \frac{1}{2} \Rightarrow 1 = (1)\left(\frac{1}{2}\right) + 2b\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$\frac{1}{2} = \frac{1}{2}b \Rightarrow \boxed{b=1}$$

$$a+b=2$$

10. A consumption function is given by

$$C = \frac{2(3\sqrt{I^3} + 2)}{I + 16}$$

The marginal propensity to save when $I = 4$ is

similar to example 9. Section 11.4

$$\frac{dc}{dI} = \frac{9I^{3/2-1}(I+16) - 2(3\sqrt{I^3} + 2)}{(I+16)^2}$$

$$\text{when } I=4$$

$$\left. \frac{dc}{dI} \right|_{I=4}$$

$$= \frac{9\sqrt{4}(20) - 2(3(2)^3 + 2)}{(20)^2} = \frac{360 - 52}{400} = \frac{77}{100}$$

the marginal
propensity
to consume

$$\text{the marginal propensity to Save} = 1 - \frac{77}{100} = \frac{23}{100}$$

(a) $\frac{23}{100}$

(b) $\frac{77}{100}$

(c) $\frac{11}{100}$

(d) $\frac{33}{100}$

(e) $\frac{73}{100}$

$$11. \frac{d}{dx} (x \ln x - x) =$$

exact problem 11. Section 12.1

(a) $\ln x$

(b) $\frac{\ln x}{x}$

(c) $2 + \ln x$

(d) $x \ln x$

(e) $\ln x - 2$

$$x\left(\frac{1}{x}\right) + \ln x - 1$$

$$1 + \ln x - 1$$

$$\ln x$$

$$12. \text{ The derivative of } y = e^{1+\sqrt{x}} + 5^{2x^3} \text{ is}$$

exact problem 15+19. Section 12.2

(a) $\frac{e^{1+\sqrt{x}}}{2\sqrt{x}} + (6x^2)5^{2x^3} \ln 5$

(b) $\sqrt{x}e^{1+\sqrt{x}} + (2x^3 - 1)5^{2x^3} \ln 5$

(c) $\sqrt{x}e^{\sqrt{x}} + (2x^3 - 1)5^{2x^3-1} \ln 5$

(d) $\frac{e^{\sqrt{x}}}{2\sqrt{x}} + (6x^2)5^{2x^3-1} \ln 5$

(e) $\frac{e^{1+\sqrt{x}}}{2\sqrt{x}} + (6x^2 - 1) \frac{5^{2x^3-1}}{\ln 5}$

$$y' = e^{1+\sqrt{x}} \cdot \frac{d}{dx}(1+\sqrt{x}) + 5^{2x^3} \frac{d}{dx}(2x^3) \cdot \ln 5$$

$$= e^{1+\sqrt{x}} \frac{1}{2\sqrt{x}} + (6x^2)(5^{2x^3}) \ln 5$$

13. If $(1 + e^{3x})^2 = 3 + \ln(x + y)$, then $y'(0, 1) =$

exact problem 23. Section 12.4

(a) 11

(b) 12

(c) 10

(d) 9

(e) 8

$$2(1+e^{3x}) \cdot e^{3x} \cdot 3 = 3 + \frac{1+y'}{x+y}$$

$$[6(1+e^{3x})e^{3x} - 3](x+y) - 1 = y'$$

at $x=0, y=1$

$$y' = (6(1+e^0)e^0 - 3)(1) - 1$$

$$= 18(2) - 1$$

$$= 12 - 1 = 11$$

14. If $y = 2u^3 + 3u^2 + 5u - 1$ and $u = 3x + 1$, then for $x = 0$, $dy/dx =$

exact problem 8. Section 11.5

(a) 51

(b) 41

(c) 62

(d) 71

(e) 17

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (6u^2 + 6u + 5)(3)$$

at $x=0 \Rightarrow u=1$

$$\left. \frac{dy}{dx} \right|_{u=1} = (17)(3) = 51$$

15. The slope of the curve $y = (x^2 - 7x - 8)^3$ at the point ~~(8, 0)~~ equal to
exact problem 57. Section 11.5 $x=8$

(a) 0

(b) 1

(c) -1

(d) -7

(e) ∞

$$y' = 3(x^2 - 7x - 8)^2 (2x - 7)$$

$$\text{at } x=8$$

$$y' = 3(\underbrace{64 - 56 - 8}_{=0})^2 (16 - 7)$$
$$= 0$$