1. If $F(x) = \dfrac{x^2+1}{2x-1}$, find $F'(x)$

(a) $\dfrac{2x^2-2x-2}{(2x-1)^2}$ ________________ (correct)

(b) $\dfrac{2x^2+2x-6}{(4x^2+3)^2}$

(c) $\dfrac{8x^2+2x-2}{(4x^2-3)^2}$

(d) $\dfrac{2x^2-2x+6}{(2x-1)^2}$

(e) $\dfrac{2x^2+8x-2}{(2x-1)^2}$

$$F'(x) = \frac{(2x)(2x-1) - (x^2+1)(2)}{(2x-1)^2}$$

$$= \frac{4x^2-2x-2x^2-2}{(2x-1)^2}$$

$$= \frac{2x^2-2x-2}{(2x-1)^2}$$

2. If $z = y^2 - y + 10$, $y = x^2$, and $x = t$, find $\dfrac{dz}{dt}$ when $t = 1$.

(a) 2 ________________ (correct)

(b) 0

(c) 42

(d) 14

(e) 12

$$\frac{dz}{dt} = \frac{dz}{dy}\frac{dy}{dx}\frac{dx}{dt}$$

$$= (2y-1)(2x)(1)$$

when $t=1 \Rightarrow x=1$ and $y=1$

$$\frac{dz}{dt} = (1)(2)(1) = 2$$

3. Find an equation of the tangent line to the curve $f(x) = x^2 + 4$ when $x = 1$?

(a) $y = 2x + 3$ ______________________________ (correct)

(b) $y = 2x$

(c) $y = 3x + 2$

(d) $y = x + 2$

(e) $y = -x$

when $x = 1 \Rightarrow y = 5$

$f'(x) = 2x \Rightarrow f'(1) = 2$

$y - 5 = 2(x - 1)$

$y = 2x - 2 + 5$

$y = 2x + 3$

4. If $f(x) = x^2 - 2x$, then f is

(a) increasing on $(1, \infty)$ ______________________________ (correct)

(b) increasing on $(0, \infty)$

(c) increasing on $(-1, \infty)$

(d) decreasing on $\left(\frac{1}{2}, \infty\right)$

(e) decreasing on $(-\infty, 2)$

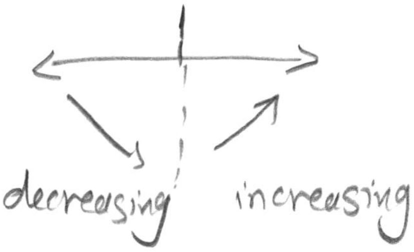
$f'(x) = 2x - 2 = 0$

$2x = 2$

$x = 1$

$f''(x) = 2 > 0 \quad \forall x$

at $x = 1$ $f(x)$ has relative minimum



5. For the cost function $c = q^3 + 2q - 1$, the average-cost function is

(a) concave up on $(1, \infty)$ ________________ (correct)

(b) concave up on $(0, \infty)$

(c) concave up on $(0, 2)$

(d) concave up on $(0, 1)$

(e) concave down on $(2, \infty)$

$$\bar{c} = \frac{c}{q} = q^2 + 2 - \frac{1}{q}$$

$$\bar{c}' = 2q + \frac{1}{q^2}$$

$$\bar{c}'' = 2 - \frac{2}{q^3} = 0$$

$$2 = \frac{2}{q^3} \Rightarrow q^3 = 1 \Rightarrow q = 1$$

0 $q=1$ ∞

c'' − +

∩ ∪

6. Which of the following statements is **TRUE** for the graph of $f(x) = \dfrac{4x^2}{x^2 - x - 2}$?

(a) it has a horizontal asymptote at $y = 4$ ________________ (correct)

(b) it has a vertical asymptote at $x = 4$

(c) it has horizontal asymptote at $y = 5$ and $y = 6$

(d) it has no vertical asymptote

(e) it has no horizontal asymptote

$$f(x) = \frac{4x^2}{x^2 - x - 2} = \frac{4x^2}{(x-2)(x+1)}$$

V.A at $x = 2$ and $x = -1$

$\lim_{x \to \pm\infty} f(x) = 4 \Rightarrow$ H.A at $y = 4$

7. $\displaystyle\int \frac{2x}{x^2+4}\,dx =$

(a) $\ln(x^2+4)+c$ ________________ (correct)

(b) $\ln|2x|+c$

(c) $\ln(x^2+4)-\ln|2x|+c$

(d) $\dfrac{x^2}{2}+2x+c$

(e) $\ln\left|\dfrac{2x}{x^2+4}\right|+c$

$u = x^2+4$

$du = 2x\,dx$

$\displaystyle\int \frac{2x}{x^2+4}\,dx = \int \frac{du}{u} = \ln|u|+c$

$= \ln(x^2+4)+c$

8. $\displaystyle\int x^2 e^{3x^3}\,dx =$

(a) $\dfrac{e^{3x^3}}{9}+c$ ________________ (correct)

(b) $\dfrac{e^{3x^3}}{3}+c$

(c) $e^{3x^3}+c$

(d) $9e^{3x^3}+c$

(e) $\dfrac{e^{3x^3}}{6}+c$

$u = 3x^3$

$du = 9x^2\,dx$

$\displaystyle\int x^2 e^{3x^3}\,dx = \frac{1}{9}\int e^u\,du = \frac{1}{9}e^u + c$

$= \dfrac{e^{3x^3}}{9}+c$

9. $\int_0^2 (3x^2 - x + 1)\, dx =$ $\left[\frac{3x^3}{3} - \frac{x^2}{2} + x\right]_0^2$

(a) 8 ______________________________ (correct)

(b) 4

(c) 0

(d) 12

(e) 16

$= 8 - 2 + 2$

$= 8$

10. $\int_2^{e+1} \frac{1}{x-1}\, dx =$ $\left[\ln|x-1|\right]_2^{e+1} = \ln|e+1-1| - \ln|2-1|$

$= \ln e - \ln 1$

$= 1 - 0$

$= 1$

(a) 1 ______________________________ (correct)

(b) $\ln(e) - \ln(2)$

(c) $1 - \ln(e+2)$

(d) 0

(e) $\ln(e+1) - \ln(2)$

11. Using integration by parts, find $\int_1^e 2x \ln x \, dx$

(a) $\dfrac{e^2+1}{2}$ ———————————————— (correct)

(b) 0

(c) $\dfrac{e^2+1}{3}$

(d) e^2+1

(e) e^2+2

$u = \ln x \qquad dv = 2x$

$du = \frac{1}{x} dx \qquad v = x^2$

$$\int_1^e 2x \ln x \, dx = \left[x^2 \ln x\right]_1^e - \int_1^e x \, dx$$

$$= e^2 \ln e - e \ln(1) - \left[\frac{x^2}{2}\right]_1^e$$

$$= e^2 - \frac{e^2}{2} + \frac{1}{2}$$

$$= \frac{1}{2}e^2 + \frac{1}{2} = \frac{e^2+1}{2}$$

12. Find the area of the region bounded by the curves $y = x^2$ and $y = x$.

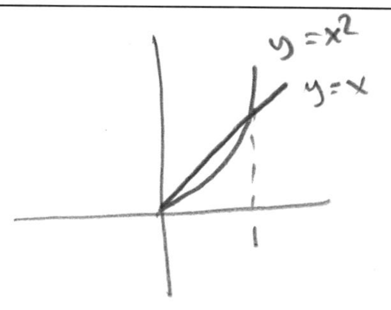
(a) $\frac{1}{6}$ ———————————————— (correct)

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

(e) $\frac{1}{5}$



$$A = \int_0^1 (x - x^2) \, dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

13. $\int \sin(2x)\, dx =$ $\frac{1}{2}\int 2\sin(2x)\,dx = -\frac{1}{2}\cos(2x) + C$

(a) $-\frac{1}{2}\cos(2x) + c$ ______ (correct)

(b) $-\frac{1}{2}\sin(2x) + c$

(c) $2\cos(2x) + c$

(d) $2x\cos(2x) + c$

(e) $2\sin(2x) + c$

14. Determine $\int \frac{5x-2}{x^2-x}\, dx$ by using partial fractions.

(a) $2\ln|x| + 3\ln|x-1| + c$ ______ (correct)

(b) $\ln|x| + \ln|x-1| + c$

(c) $5\ln|x| - \ln|x-1| + c$

(d) $-2\ln|x| + \ln|x-1| + c$

(e) $\ln|5x-2| + \ln|x^2-x| + c$

$$\frac{5x-2}{x^2-x} = \frac{5x-2}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$= \frac{A(x-1)+Bx}{x(x-1)}$$

$$5x-2 = A(x-1) + Bx$$

$$x=1 \Rightarrow \boxed{3 = B}$$

$$x=0 \Rightarrow -2 = -A \Rightarrow \boxed{A=2}$$

$$\int\left(\frac{2}{x} + \frac{3}{x-1}\right)dx = 2\ln|x| + 3\ln|x-1| + C$$

15. The function $f(x) = 3x^5 - 5x^3$ has a relative maximum at $x =$

(a) -1 ____(correct)

(b) 0

(c) 2

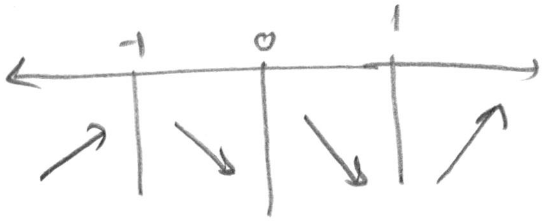
(d) 3

(e) -4

$f'(x) = 15x^4 - 15x^2 = 0$

$x^2(x^2-1) = 0$

$x = 0 \quad x = 1 \quad x = -1$



16. Given $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left|u + \sqrt{u^2-a^2}\right| + c$, then $\displaystyle\int \frac{dx}{\sqrt{(3x)^2-4}} =$

(a) $\dfrac{1}{3}\ln\left|3x + \sqrt{9x^2-4}\right| + c$ ____________________________________(correct)

(b) $\dfrac{1}{6}\ln\left|3x - \sqrt{9x^2-4}\right| + c$

(c) $\dfrac{1}{3}\ln\left|3x + \sqrt{3x^2-2}\right| + c$

(d) $3\ln\left|x + \sqrt{3x^2-2}\right| + c$

(e) $\dfrac{1}{6}\ln\left|3x + \sqrt{3x^2-2}\right| + c$

$u = 3x \qquad du = 3\,dx$

$\dfrac{1}{3}\displaystyle\int \frac{3\,dx}{\sqrt{(3x)^2-4}} = \frac{1}{3}\ln\left|3x + \sqrt{9x^2-4}\right| + C$

17. Let $h(x, y, z) = z^2(3y + yx^2)$ then $h_{xyz}(1, 0, 1) =$

(a) 4 ____(correct)

(b) 0

(c) 3

(d) 1

(e) 2

$h_x = z^2(2yx)$

$h_{xy} = z^2(2x)$

$h_{xyz} = (2z)(2x)$

at $x=1$ $y=0$ $z=1$ $\Rightarrow h_{xyz}(1,0,1) = 4$

18. If $f(x, y) = 10e^x + 3y^2$, find $f_y(x, y)$.

(a) $6y$ ____(correct)

(b) $6e^x y$

(c) $10xe^x + 6y$

(d) y

(e) $3y^2$

$f_y = 6y$.

19. If $y = \tan(7x^2)$, find $\dfrac{dy}{dx}$.

(a) $14x\sec^2(7x^2)$ ______ (correct)
(b) $14\sec^2(7x^2)$
(c) $\cot^2(7x^2)$
(d) $14x\cos(14x)$
(e) $\sec^2(14x)$

$u = 7x^2$

$\frac{du}{dx} = 14x$

$y = \tan(7x^2)$

$\frac{dy}{dx} = (14x)\sec^2(7x^2)$

20. Let $f(x, y) = x^2 + y^2 - xy$. If f has a critical point at (a, b), then $a + b =$

(a) 0 ______ (correct)
(b) 2
(c) 1
(d) -1
(e) -2

$f_x = 2x - y = 0 \Rightarrow 2x = y$

$f_y = 2y - x = 0 \Rightarrow x = 2y$

$x = 2(2x) = 4x$

$\Rightarrow x = 0 \Rightarrow y = 0$

critical Point $(0, 0)$

$a + b = 0$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	D_{18}	C_{18}	E_{19}	B_{16}
2	A	D_{6}	B_{7}	C_{7}	C_{3}
3	A	E_{5}	B_{2}	B_{20}	D_{20}
4	A	D_{16}	B_{12}	A_{16}	C_{12}
5	A	D_{20}	D_{20}	E_{15}	A_{13}
6	A	E_{1}	D_{14}	B_{18}	A_{9}
7	A	A_{13}	D_{1}	E_{14}	B_{7}
8	A	E_{12}	E_{9}	E_{9}	C_{6}
9	A	E_{10}	D_{15}	C_{2}	A_{1}
10	A	B_{11}	B_{4}	A_{13}	C_{17}
11	A	C_{9}	D_{8}	E_{8}	A_{2}
12	A	A_{15}	B_{17}	E_{12}	A_{8}
13	A	B_{14}	A_{13}	E_{4}	D_{10}
14	A	B_{7}	E_{16}	B_{1}	D_{19}
15	A	D_{2}	A_{6}	E_{6}	B_{18}
16	A	E_{19}	E_{11}	E_{11}	E_{15}
17	A	D_{8}	C_{5}	A_{10}	A_{14}
18	A	D_{4}	E_{10}	C_{17}	A_{11}
19	A	D_{17}	D_{19}	D_{3}	E_{5}
20	A	C_{3}	A_{3}	A_{5}	C_{4}