1. Let $f(x) = x^3 - 6x^2 + 9x$. If (a, b) is the **inflection point** of the graph of f, then $a + b =$

(a) 3

(b) 1

$\boxed{\text{(c) } 4}$

(d) 2

(e) 0

$f'(x) = 3x^2 - 12x + 9$

$f''(x) = 6x - 12$

$f''(x) = 0 \Rightarrow x = 2$

$- - - - \quad \underset{2}{\times} \quad + + + +$

$(2, f(2))$ is the inflection pt.

$f(2) = 8 - 6(4) + 9(2) = 2$

$(a, b) = (2, 2) \Rightarrow a + b = 4$

2. If $y = x^2 - 3$, then **the equation of the tangent** line at the point $(1, -2)$ is

(a) $y = 2x$

(b) $y = 2x + 4$

(c) $y = x - 4$

$\boxed{\text{(d) } y = 2x - 4}$

(e) $y = 2x - 3$

$y' = 2x$

$y'(1) = 2 \Rightarrow$ Slope $= 2$

$y - (-2) = 2(x - 1)$

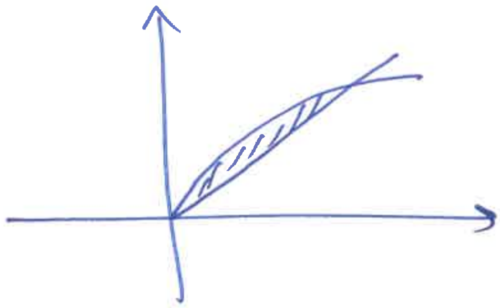
$\boxed{y = 2x - 4}$

3. The **area** of the region bounded by the graphs $y = \sqrt{x}$ and $y = x$ is

(a) $\frac{1}{2}$
(b) 2
(c) $\frac{7}{6}$
$\boxed{\text{(d) } \frac{1}{6}}$
(e) 1

$\sqrt{x} = x$

$\Rightarrow x = 0, 1$



$$A = \int_0^1 (\sqrt{x} - x)\,dx$$

$$= \left.\frac{2x^{3/2}}{3}\right]_{x=0}^{x=1} - \left.\frac{x^2}{2}\right]_{x=0}^{x=1}$$

$$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$$

4. Let $f(x, y) = x^2 - 4x + 2y^2 + 4y + 7$. If f has a critical point at (a, b), then $a + b =$

(a) 2
(b) -2
(c) -1
(d) 0
$\boxed{\text{(e) } 1}$

$f_x = 2x - 4 = 0 \Rightarrow x = 2$

$f_y = 4y + 4 = 0 \Rightarrow y = -1$

$(a, b) = (2, -1)$

$a + b = 1$

5. If $f(x) = x^3 - 6x^2 + 12x - 6$, then

(a) f has a relative maximum at $x = 2$.
$\boxed{\text{(b)}}$ f has no relative maximum or relative minimum.
(c) f is decreasing on $(-\infty, 2)$.
(d) f has a relative minimum at $x = 2$.
(e) f is concave up on $(-\infty, 2)$.

$f'(x) = 3x^2 - 12x + 12 = 3(x-2)^2$ ++ + $\overset{0}{\underset{2}{X}}$ ++ + f'

$\Rightarrow$ f has no relative max. or relative min.

6. $\displaystyle\int \frac{s^2}{s^3+5}\, ds =$

$\boxed{\text{(a) } \frac{1}{3}\ln|s^3+5| + c}$
(b) $\ln|s| + c$
(c) $\frac{1}{3}\ln|s^2+5| + c$
(d) $3\ln|s^3+5| + c$
(e) $\ln|s^3+5| + c$

Let $u = s^3 + 5$

$\Rightarrow du = 3s^2\, ds \Rightarrow ds = \dfrac{du}{3s^2}$

$$\int \frac{s^2}{s^3+5}\, ds = \int \frac{\cancel{s^2}}{u} \frac{du}{3\cancel{s^2}}$$

$$= \frac{1}{3}\ln|u| + c$$

$$= \frac{1}{3}\ln|s^3+5| + c$$

7. $\int \sin(2x)\, dx =$

Let $u = 2x \Rightarrow du = 2dx$

$\Rightarrow dx = \frac{du}{2}$

(a) $\frac{1}{2}\cos(2x) + c$

(b) $-\frac{1}{2}\cos(x) + c$

(c) $-\frac{1}{2}\cos(2x) + c$

(d) $\frac{1}{2}\cos(x) + c$

(e) $2\cos(2x) + c$

$\int \sin u \frac{du}{2} = \frac{1}{2}\int \sin u\, du$

$= \frac{-1}{2}\cos(2x) + c$

8. If $f(x, y) = 6xy^2$, then $f_{xy}(-1, 1) =$

(a) 12

(b) −12

(c) −6

(d) 6

(e) 24

$f_x = 6y^2$

$f_{xy} = 12y$

$f_{xy}(-1,1) = 12$

9. $\int x\sqrt{x^2+5}\ dx =$

(a) $\boxed{\dfrac{(x^2+5)^{\frac{3}{2}}}{3}+c}$

(b) $\dfrac{(x^2+5)^{\frac{1}{2}}}{3}+c$

(c) $\dfrac{(x^2+5)^{\frac{5}{2}}}{3}+c$

(d) $\dfrac{x^2+5}{3}+c$

(e) $\dfrac{2(x^2+5)^{\frac{3}{2}}}{3}+c$

Let $u = x^2+5 \Rightarrow du = 2x\,dx$

$\Rightarrow dx = \dfrac{du}{2x}$

$\int x\sqrt{u}\,\dfrac{du}{2x} = \frac{1}{2}\int \sqrt{u}\,du$

$= \dfrac{1}{2}\dfrac{u^{3/2}}{3/2} + C$

$= \dfrac{1}{3}(x^2+5)^{3/2} + C$

10. If $f(x) = \dfrac{8}{x}$, then $f''(2) =$

(a) -2

(b) 4

(c) 8

(d) -4

$\boxed{\text{(e) } 2}$

$f(x) = 8x^{-1}$

$f'(x) = -8x^{-2}$

$f''(x) = 16x^{-3}$

$f''(2) = \dfrac{16}{8} = 2$

11. $\lim_{x \to 5^-} \frac{1}{x-5} =$ $\frac{1}{0^-} = -\infty$

(a) $\boxed{-\infty}$
(b) 1
(c) 0
(d) -1
(e) ∞

12. If $z = u^2 + \sqrt{u} + 9$ and $u = 2s^2 - 1$, then $\frac{dz}{ds}$ when $s = -1$ is

(a) 10
(b) -5
(c) -12
(d) 5
(e) $\boxed{-10}$

$$\frac{dz}{ds} = \frac{dz}{du} \cdot \frac{du}{ds}$$

if $s=-1 \Rightarrow u=1$

$$= \left(2u + \frac{1}{2\sqrt{u}}\right)(4s)$$

$$= \left(2 + \frac{1}{2}\right)(-4)$$

$$= -10$$

13. $\int_{-1}^{1} (3x^2 - 2x + 1)\, dx =$

(a) 5
$\boxed{\text{(b) } 4}$
(c) -2
(d) 3
(e) -1

$$x^3 - x^2 + x \Big]_{x=-1}^{x=1}$$

$$= 1 - 1 + 1 - (-1 - 1 - 1)$$

$$= 1 + 3$$

$$= 4$$

14. If $c(q) = 0.1q^2 + 3q + 2$, is the cost function, then **the marginal cost** at $q = 3$ is

(a) 3.2
$\boxed{\text{(b) } 3.6}$
(c) 4.8
(d) 3
(e) 5.6

$$\frac{dc}{dq} = 0.2q + 3$$

$$\left.\frac{dc}{dq}\right|_{q=3} = 3.6$$

15. If $f(x, y) = 2x^2 + 3xy$, then $f_x(1, 1) =$

(a) 3
(b) 4
$\boxed{\text{(c) } 7}$
(d) 6
(e) 5

$f_x = 4x + 3y$

$f_x(1,1) = 7$

16. $\int 3x^2 \ln x \, dx =$

$\boxed{\text{(a) } x^3 \ln x - \frac{x^3}{3} + c}$

(b) $x \ln x - \frac{x^3}{3} + c$

(c) $x^3 \ln x - x^3 + c$

(d) $3x^3 \ln x - \frac{x^3}{3} + c$

(e) $x^2 \ln x - \frac{x^3}{3} + c$

Let $u = \ln x$, $dv = 3x^2 dx$

$du = \frac{1}{x} dx$, $v = x^3$

$\int u \, dv = uv - \int v \, du$

$= x^3 \ln x - \int x^3 \left(\frac{1}{x}\right) dx$

$= x^3 \ln x - \frac{x^3}{3} + c$

17. The **horizontal asymptote** of the graph of the function $f(x) = \dfrac{2x^3+9}{3-x^3}$, is

(a) $y = 0$
(b) $y = -1$
(c) $\boxed{y = -2}$
(d) $y = 2$
(e) $y = 3$

$y = \dfrac{2}{-1} = -2$

18. $\displaystyle\lim_{x\to 3} \frac{x^2-9}{x^2-x-6} =$

$\displaystyle\lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x+2)} = \frac{6}{5}$

(a) $\frac{3}{4}$
(b) does not exist
(c) $\boxed{\frac{6}{5}}$
(d) 6
(e) 1

19. Given $\int \frac{u\,du}{(a+bu)^2} = \frac{1}{b^2}\left(\ln|a+bu| + \frac{a}{a+bu}\right) + c$, then $\int \frac{x}{(2+3x)^2}\,dx =$

(a) $\frac{1}{9}\left(\ln|3+2x| + \frac{2}{3+2x}\right) + c$

(b) $\frac{1}{9}\left(\ln|2+3x| + \frac{1}{3x}\right) + c$

(c) $\frac{1}{9}\left(\ln|2+3x| + \frac{2}{2+3x}\right) + c$

(d) $\ln|2+3x| + \frac{2}{2+3x} + c$

(e) $\frac{1}{4}\left(\ln|2+3x| + \frac{2}{2+3x}\right) + c$

Let $u = x \Rightarrow du = dx$

$a = 2, \; b = 3$

$\Rightarrow \int \frac{x}{(2+3x)^2}\,dx = \int \frac{u}{(2+3u)^2}\,du$

$= \frac{1}{9}\left(\ln|2+3u| + \frac{2}{2+3u}\right) + c$

$= \frac{1}{9}\left(\ln|2+3x| + \frac{2}{2+3x}\right) + c$

20. If $y = \tan(x^3)$, then $\frac{dy}{dx} =$

(a) $3x^2\sec^2(x^2)$

(b) $3x^2\sec(x)$

(c) $3x^2\sec^2(x^3)$

(d) $x^2\sec^2(x^2)$

(e) $x^2\sec^2(x^3)$

$\frac{dy}{dx} = 3x^2\sec^2(x^3)$.