King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 201

Major Exam II

212

March 23, 2022

Net Time Allowed: 90 Minutes

MASTER VERSION

Key

1. The distance from the point $A(1,1,1)$ to the plane that contains the point $B(1,-2,1)$ and is perpendicular to the vector from the origin to B, is

(a) $\sqrt{6}$
(b) $\sqrt{7}$
(c) $\sqrt{5}$
(d) 2
(e) 1

$OB = \langle 1,-2,1 \rangle$

$\Rightarrow$ eq^n of the plane:

$(x-1) - 2(y+2) + (z-1) = 0$

$\Rightarrow \boxed{x - 2y + z - 6 = 0}$

distance $= \dfrac{|\cancel{1} - \cancel{2} + \cancel{1} - 6|}{\sqrt{1+4+1}} = \dfrac{6}{\sqrt{6}}$

$= \sqrt{6}$

2. The equation of the plane that contains the two lines

$L_1 : \; x = 1+t, \quad y = 1-t, \quad z = 2t$ $\qquad V_1 = \langle 1,-1,2 \rangle$

$L_2 : \; x = 2-s, \quad y = s, \quad z = 2$ $\qquad V_2 = \langle -1,1,0 \rangle$

is

$\vec{n} = \vec{V_1} \times \vec{V_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ -1 & 1 & 0 \end{vmatrix} = \langle -2,-2,0 \rangle$

$P = (1,1,0)$

(a) $x + y - 2 = 0$
(b) $3x - y + z = 1$
(c) $x - y + 3 = 0$
(d) $x - 5y - z = 4$
(e) $2x - 2y + 7z = 3$

$\Rightarrow$ eq^n of the plane is:

$-2(x-1) - 2(y-1) = 0$

$\Rightarrow -2x + 2 - 2y + 2 = 0$

$\Rightarrow \boxed{x + y - 2 = 0}$

3. The surface S defined by

$$x^2 + 2z^2 - 6x - y = -10$$

represents

$(x^2 - 6x + 9) + 2z^2 = y - 10 + 9$

$\Rightarrow \boxed{(x-3)^2 + 2z^2 = (y-1)}$

elliptic paraboloid

(a) an elliptic paraboloid with axis parallel to the y-axis

(b) a cone with axis parallel to the z-axis

(c) a hyperboloid of one-sheet with axis parallel to the y-axis

(d) an ellipsoid centered at $(-1, 2, 1)$

(e) a hyperbolic paraboloid with axis parallel to the x-axis

4. Consider the following statements about the surface

$$S: \; z = \sqrt{x^2 + 2y^2 - 4y + 2x + 3}$$

(I) It represents a hyperboloid of one-sheet

(II) It has a vertex at $(-1, 1, 0)$

(III) Its axis is parallel to the z-axis

Which of the above statements are true about S?

$z^2 = (x^2 + 2x + 1) + 2(y^2 - 2y + 1) + \cancel{3} - \cancel{3}$

$\boxed{z^2 = (x+1)^2 + 2(y-1)^2}$

(a) II and III

(b) III only

(c) I, II and III

(d) II only

(e) I and II

· upper half of a cone with z-axis as axis of symmetry and vertex $(-1, 1, 0)$

5. The domain of the function

$$f(x,y) = \frac{1}{\sqrt{9-x^2-y^2}}$$

is

$9 - x^2 - y^2 > 0$

$\Rightarrow \quad x^2 + y^2 < 9$

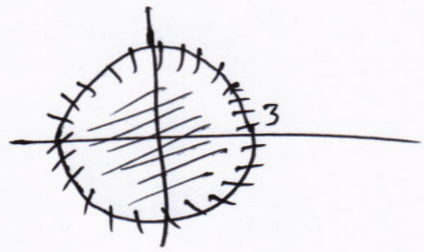
(a) the region inside the circle $x^2+y^2=9$

(b) the circle $x^2+y^2=9$

(c) the region outside the circle $x^2+y^2=9$

(d) the region inside and on the circle $x^2+y^2=9$

(e) the region outside and on the circle $x^2+y^2=9$



boundary not included

6. $$\lim_{(x,y)\to(1,1)} \frac{y-x}{(1-y)+3\ln x} =$$

(a) does not exist

(b) 0

(c) -1

(d) 1

(e) ∞

Along the line $x=1$:

$$\lim_{(1,y)\to(1,1)} \frac{y-1}{1-y} = -1 \quad -(1)$$

Along the line $y=1$:

$$\lim_{(x,1)\to(1,1)} \frac{1-x}{3\ln x} = \lim_{x\to 1} \frac{-1}{\frac{3}{x}}$$

$$= \frac{-1}{3} \quad -(2)$$

$(1) \neq (2)$

7. Let

$$f(x,y,z) = x^2yz^2 + \arctan(\frac{x^2}{\sqrt{z}})$$

Then $f_{xzy}(1,1,1) =$

(a) 4

(b) 0

(c) π

(d) 4π

(e) 1

$$f_y = x^2 z^2$$

$$f_{yx} = 2xz^2$$

$$f_{yxz} = 4xz = f_{xzy}$$

$$\Rightarrow f_{xzy}(1,1,1) = \underline{\underline{4}}$$

8. Let

$$x \sin z - z^2 y = \frac{1}{3}$$

Which one of the following is true?

(a) $z^2 \cdot \frac{\partial z}{\partial x} + \sin z \cdot \frac{\partial z}{\partial y} = 0$

(b) $\sin z \cdot \frac{\partial z}{\partial x} + z^2 \cdot \frac{\partial z}{\partial y} = 0$

(c) $\sin z \cdot \frac{\partial z}{\partial x} + z^2 \cdot \frac{\partial z}{\partial y} = \frac{1}{3}$

(d) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

(e) $\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = \frac{1}{3}$

Derivative w.r.t. x:

[redacted]

$$\frac{\partial z}{\partial x} = \frac{-\sin z}{x \cos z - 2yz}$$

$$\frac{\partial z}{\partial y} = \frac{z^2}{x \cos z - 2yz}$$

$$\Rightarrow \boxed{z^2 \frac{\partial z}{\partial x} + \sin z \frac{\partial z}{\partial y} = 0}$$

9. If the linearization of the function $f(x,y) = 1 - xy\cos(\pi y)$, at the point $(1,1)$ is given by $L(x,y) = ax + by + c$, then $a - b + c =$

(a) 0

(b) 1

(c) 2

(d) -1

(e) -2

$$f(1,1) = 1 - \cos(\pi) = \underline{\underline{2}}$$

$$f_x = -y\cos(\pi y)\Big|_{(1,1)} = \underline{\underline{1}}$$

$$f_y = \left(-x\cos(\pi y) + \pi xy\sin(\pi y)\right)\Big|_{(1,1)} = \underline{\underline{1}}$$

$$\Rightarrow L(x,y) = 2 + (x-1) + (y-1)$$

$$\Rightarrow \boxed{L(x,y) = x + y}$$

$a = 1$
$b = 1$
$c = 0$

10. The tangent plane to the surface $z = x^2 + 2y^2$, at the point $(1,1,3)$ contains the point

(a) $(0,0,-3)$

(b) $(0,0,1)$

(c) $(1,1,1)$

(d) $(1,1,0)$

(e) $(1,-1,2)$

$$f(x,y) = x^2 + 2y^2$$

$$f_x = 2x\Big|_{(1,1,3)} = \underline{\underline{2}}$$

$$f_y = 4y\Big|_{(1,1,3)} = \underline{\underline{4}}$$

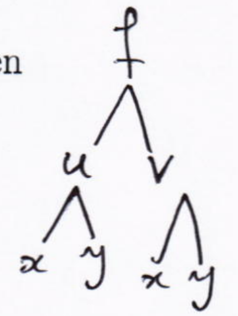
$$\Rightarrow z - 3 = 2(x-1) + 4(y-1)$$

$$\Rightarrow \boxed{2x + 4y - z = 3}$$

$$(x,y,z) = (0,0,-3)$$

lies on the plane

11. Let $f(x, y) = \ln(u^3 + v^4)$, where $u = 2x + y$, and $v = 3x - y$. Then



$f_y(1, 1) =$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y}$$

(a) $\frac{-5}{43}$

(b) $\frac{5}{44}$

(c) $\frac{-3}{41}$

(d) $\frac{-1}{42}$

(e) $\frac{3}{47}$

$$= \frac{3u^2}{u^3+v^4}\cdot 1 - \frac{4v^3}{u^3+v^4}\Bigg|_{(3,2)}$$

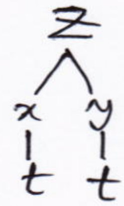
$(x,y) = (1,1)$

$(u,v) = (3,2)$

$$= \frac{3\cdot 3^2}{3^3+2^4} - \frac{4\,2^3}{3^3+2^4} = \frac{27-32}{27+16} = \frac{-5}{43}$$

12. Let $z = x^2y + 3xy^4$, where $x = \sin(2t)$ and $y = \cos(t)$, then the value of $\frac{dz}{dt}$ at $t = 0$ is

$t = 0 \Rightarrow (x,y) = (0,1)$



(a) 6

(b) -6

(c) 3

(d) -3

(e) 0

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$= 2(2xy + 3y^4)\cos(2t) - (x^2 + 12xy^3)\sin t$$

$$\Rightarrow \frac{dz}{dt}\Bigg|_{(0,1)} = 2(3) - 0 = \underline{\underline{6}}$$

13. An equation of the tangent plane to the ellipsoid

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$$

at the point $(-2, 1, -3)$ is

$\nabla = \langle \frac{x}{2}, 2y, \frac{2z}{9} \rangle$

$\nabla \big|_{(-2,1,-3)} = \langle -1, 2, -\frac{2}{3} \rangle$

(a) $3x - 6y + 2z = -18$

(b) $x + 6y + 4z = 1$

(c) $3x + 6y - z = -2$

(d) $3x + 6y - 7z = 11$

(e) $x - 6y - 3z = -13$

$\Rightarrow$ eq^n of the plane:

$-(x+2) + 2(y-1) - \frac{2}{3}(z+3) = 0$

$\Rightarrow -3x + 6y - 2z = 18$

$\Rightarrow \boxed{3x - 6y + 2z = -18}$

14. The maximum rate of change of $f(x, y, z) = e^{xz-y^2}$ at the point $(-2, 1, -1)$ is

$\nabla f = \langle z e^{xz-y^2}, -2y e^{xz-y^2}, x e^{xz-y^2} \rangle$

(a) $3e$

(b) $-3e$

(c) e

(d) 1

(e) 3

$\nabla f \big|_{(-2,1,-1)} = \langle -e, -2e, -2e \rangle$

$|\nabla f| = \sqrt{e^2 + 4e^2 + 4e^2} = \sqrt{9e^2}$

$= \underline{\underline{3e}}$

15. If the line

$$L: \; x = 2 + 3t, \quad y = -4t, \quad z = 5 + t$$

intersects the plane $2x - y + z = -2$, at the point (a, b, c), then $a + b + c =$

$$2(2+3t) + 4t + 5 + t = -2$$

(a) 7

(b) 8

(c) -5

(d) 4

(e) -6

$$\Rightarrow 11t = -11 \Rightarrow \boxed{t = -1}$$

$$\Rightarrow x = -1$$

$$y = 4$$

$$z = 4$$

$$(a, b, c) = (-1, 4, 4)$$

$$\Rightarrow -1 + 4 + 4 = \underline{7}$$