1. A particle moves three times, counterclockwise, along the circle

$$x^2 + (y-1)^2 = 9.$$

If the particle starts at the point $(3,1)$, then the parametric equations of the path of the particle are

(a) $x = 3\cos t$, $y = 1 + 3\sin t$, $0 \le t \le 6\pi$

(b) $x = 3\cos t$, $y = 1 - 3\sin t$, $0 \le t \le 6\pi$

(c) $x = 3\sin t$, $y = 1 + 3\cos t$, $0 \le t \le 6\pi$

(d) $x = 3\cos(3t)$, $y = 1 - 3\sin(3t)$, $0 \le t \le 2\pi$

(e) $x = 3\sin(3t)$, $y = 1 + 3\cos(3t)$, $0 \le t \le 2\pi$

$x = 3\cos t$, $y = 1 + 3\sin t$
$0 \le t \le 2\pi$
At $t = 0$, $(x,y) = (3,1)$
As t increases from 0 to 6π, particle moves 3 times counterclock.

$\frac{10.1}{33(b)}$

2. The curve given parametrically by

$$x = \tan\theta,\ y = 2\sec\theta,\ -\pi/2 < \theta < \pi/2$$

is

(a) a hyperbola

(b) an ellipse

(c) a parabola with horizontal axis of symmetry

(d) a parabola with vertical axis of symmetry

(e) a straight line

$$x^2 = \frac{y^2}{4} - 1$$

$\frac{10.1}{\text{Similar } 18}$

3. The curve given by the parametric equations

$$x = 2\cos^3\theta,$$

$$y = 2\sin^3\theta, \qquad 0 \le \theta < \frac{7\pi}{4}$$

has

$\frac{dx}{d\theta} = -6\cos^2\theta\sin\theta$

$\frac{dy}{d\theta} = 6\sin^2\theta\cos\theta$

$\frac{dy}{dx} = -\tan\theta = 0$

$\Rightarrow \theta = 0, \pi$

(a) only one horizontal and only one vertical tangent lines

(b) only one horizontal tangent line and no vertical tangent line

(c) no horizontal tangent line and only one vertical tangent line

(d) two horizontal and one vertical tangent lines

(e) one horizontal and two vertical tangent lines

$\frac{dy}{dx} = \pm\infty \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\left(\frac{dy}{dx} \to \pm\infty \text{ as } \theta \to \frac{\pi}{2}^+, \frac{\pi}{2}^-, \frac{3\pi}{2}^+ \text{ or } \frac{3\pi}{2}^-\right)$

10.2 Similar 28(b)

$\boxed{\begin{array}{l}\theta = 0, \pi,\ y = 0 \\ \theta = \frac{\pi}{2}, \frac{3\pi}{2}, x = 0\end{array}}$

At $\theta = 0$ as well as $\theta = \pi \Rightarrow$ one horizontal line ($y=0$)

At $\theta = \pi/2, \frac{3\pi}{2} \Rightarrow$ one vertical tangent line ($x=0$)

4. The curve given by the parametric equations

$$x = t^3 - 3t \text{ and } y = t^2$$

is

$\frac{dy}{dx} = \frac{2t}{3(t^2-1)} = 0$

(a) concave up when $-1 < t < 1$

(b) concave down when $t < 0$

(c) concave up when $t > 0$

(d) concave down when $-\frac{1}{2} < t < \frac{1}{2}$

(e) concave up when $t > 1$

$\frac{dy}{dx} = 0 \Rightarrow t = 0, \quad \frac{dx}{dt} = 0 \Rightarrow t = \pm 1$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{6(t^2-1) - 12t^2}{9(t^2-1)^2}}{3(t^2-1)}$

$= \frac{-2(t^2+1)}{9(t^2-1)^3}$

10.2 Example P650

$\frac{d^2y}{dx^2} > 0$, when $-1 < t < 1$ CU

< 0, when $t > 1$ or $t < 1$ CD

5. The area of the region enclosed by the curve that is given by the parametric equations $x = 2\cos\theta$, $y = \frac{1}{2}\sin\theta$, $0 \le \theta \le 2\pi$, is

$$A = 4\int_0^2 y\,dx$$

$x = 0 \Rightarrow \theta = \pi/2$
$x = 2 \Rightarrow \theta = 0$

(a) π
(b) 3π
(c) 0
(d) 2π
(e) 4π

$$= 4\int_{\pi/2}^{0} \frac{1}{2}\sin\theta\,(-2\sin\theta)\,d\theta$$

$$= 4\int_0^{\pi/2} \sin^2\theta\,d\theta = \int_0^{\pi/2} (2 - 2\cos 2\theta)\,d\theta$$

$$= \left(2\theta - \sin 2\theta\right)\Big|_0^{\pi/2}$$

$$= (\pi - 0) - (0 - 0) = \pi$$

$\frac{10.2}{Q.\ 31}$

6. The surface area of the solid generated by rotating the curve given by the parametric equations

$$x = 6t,\ y = 3t^2,\ 0 \le t \le 1,$$

about the y-axis is

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \sqrt{36 + 36t^2}\,dt$$
$$= 6\sqrt{1+t^2}\,dt$$

(a) $24\,\pi\,[2\sqrt{2} - 1]$
(b) $12\,\pi\,[\sqrt{2} - 1]$
(c) $9\,\pi\,[1 - 2\sqrt{2}]$
(d) $6\,\pi\,[\sqrt{2} + 1]$
(e) $3\,\pi\,[2 - \sqrt{2}]$

$$S = \int_0^1 2\pi x\,ds$$

$$= \int_0^1 2\pi \cdot 6t \cdot 6\sqrt{1+t^2}\,dt$$

$$= 36\pi \int_0^1 2t\sqrt{1+t^2}\,dt$$

$1 + t^2 = u$
$2t\,dt = du$

$\frac{10.2}{Q.\ 65}$

$$= 36\pi \int u^{1/2}\,du$$

$$= 36\pi \left[\frac{2}{3}(1+t^2)^{3/2}\right]_0^1$$

$$= 24\pi\left[2^{3/2} - 1\right]$$

$$= 24\pi\left[2\sqrt{2} - 1\right]$$

7. The polar equation $r = \tan\theta \sec\theta$ represents a

(a) Parabola
(b) Circle
(c) Line
(d) Ellipse
(e) Hyperbola

$r = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$

$r\cos^2\theta = \sin\theta$

$r^2\cos^2\theta = r\sin\theta$

$x^2 = y$

$\Rightarrow$ Parabola

$\frac{10.3}{Q.18}$

8. The graph of the polar curve $r = 3\cos\theta$, has a vertical tangent line when $(r, \theta) =$

(a) $(0, \pi/2)$
(b) $(0, \pi)$
(c) $(0, \pi/4)$
(d) $(3, \pi/2)$
(e) $(0, 2\pi)$

$x = r\cos\theta$

$= 3\cos^2\theta$

$\frac{dx}{d\theta} = -6\cos\theta\sin\theta = 0$

$\Rightarrow \sin 2\theta = 0$

$\Rightarrow \theta = 0, \frac{\pi}{2}$

At $\theta = 0$, $r = 3$

At $\theta = \pi/2$, $r = 0$

$(r, \theta) = (0, \frac{\pi}{2})$

$\frac{10.3}{Q.61}$

9. The slope of tangent line to the ~~graph~~ polar curve $r = \dfrac{1}{\theta}$ at $\theta = \pi$ is given by

(a) $-\pi$
(b) π
(c) 0
(d) -2
(e) 2

$x = \frac{1}{\theta}\cos\theta,\ y = \frac{1}{\theta}\sin\theta,\ f(\theta) = 1/\theta,\ f'(\theta) = -\frac{1}{\theta^2}$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-\frac{1}{\theta^2}\sin\theta + \frac{1}{\theta}\cos\theta}{-\frac{1}{\theta^2}\cos\theta - \frac{1}{\theta}\sin\theta}$$

$$= \frac{-\sin\theta + \theta\cos\theta}{-\cos\theta - \theta\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{x=\pi} = \frac{0-\pi}{1-0} = -\pi$$

$\dfrac{10.3}{Q.57}$

10. The area of the region enclosed by the polar curve $r = 2 - \cos\theta$ is

(a) $\dfrac{9\pi}{2}$
(b) $\dfrac{7\pi}{2}$
(c) $\dfrac{5\pi}{2}$
(d) $\dfrac{3\pi}{2}$
(e) $\dfrac{11\pi}{2}$

$$A = 2\int_0^\pi \frac{1}{2}(2-\cos\theta)^2\, d\theta$$

$$= \int_0^\pi (4 + \cos^2\theta - 4\cos\theta)\, d\theta$$

$$= \int_0^\pi \left(4 + \frac{1+\cos 2\theta}{2} - 4\cos\theta\right) d\theta$$

$$= \left[4\theta + \frac{1}{2}\theta + \frac{\sin 2\theta}{4} - 4\sin\theta\right]_0^\pi$$

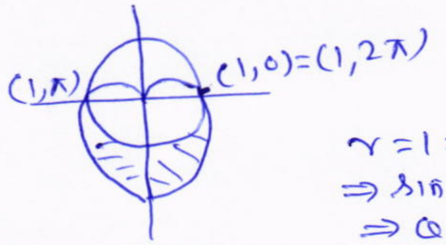
$$= 4\pi + \frac{\pi}{2} + 0 - 0 - 0 - 0 - 0 \cdots$$

$$= \frac{9\pi}{2}$$

$\dfrac{Q.12}{\text{Sec } 10.4}$

11. The area of the region that lies inside the curve $r = 1 - \sin\theta$ and outside the curve $r = 1$ is

(a) $\frac{\pi}{4} + 2$
(b) $\pi + \frac{1}{2}$
(c) $\pi - 2$
(d) $\frac{\pi}{2} + 2$
(e) $\pi + 2$



$r = 1 = 1 - \sin\theta$
$\Rightarrow \sin\theta = 0$
$\Rightarrow \theta = 0, \pi$

$$A = \int_{\pi}^{2\pi} \frac{1}{2}\left[(1-\sin\theta)^2 - 1\right]d\theta$$
$$= \frac{1}{2}\int_{\pi}^{2\pi} (\sin^2\theta - 2\sin\theta)\,d\theta$$
$$= \frac{1}{4}\int_{\pi}^{2\pi} (1 - \cos 2\theta - 4\sin\theta)\,d\theta$$
$$= \frac{1}{4}\left[\theta - \frac{\sin 2\theta}{2} + 4\cos\theta\right]_{\pi}^{2\pi}$$
$$= \frac{1}{4}\left[2\pi - 0 + 4 - (\pi - 0 - 4)\right] = \frac{1}{4}\left[\pi + 8\right]$$

$\frac{10.4}{Q.24}$

12. The exact length of the polar curve

$$r = 2\theta^2,\ \theta \in [0, \pi]$$

is

(a) $\frac{2}{3}((\pi^2 + 4)^{3/2} - 8)$
(b) $\frac{2}{3}(\pi^3 - 8)$
(c) $(\pi^2 + 4)^{3/2}$
(d) 8
(e) 16

$$L = \int_0^{\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$$
$$= \int_0^{\pi} \sqrt{4\theta^4 + 16\theta^2}\,d\theta$$
$$= \int_0^{\pi} 2\theta\sqrt{\theta^2 + 4}\,d\theta$$

Let $\theta^2 + 4 = u$.
$2\theta\,d\theta = du$

$$= \frac{2}{3}\left[(\theta^2+4)^{3/2}\right]_0^{\pi}$$
$$= \frac{2}{3}\left[(\pi^2+4)^{3/2} - 8\right]$$

$\frac{10.4}{\text{Similar } Q.47}$

$(2^2)^{3/2}$

13. If M is the distance from the point $P(3,-2,4)$ to the xz-plane and m is the distance from $P(3,-2,4)$ to the y-axis, then $M+m$ is equal to

$M = |-2| = 2$

$m = \sqrt{3^2+4^2} = 5$

$M+m = 7$

(a) 7
(b) 6
(c) 8
(d) 4
(e) 3

12.1
Similar Q.12

14. The unit vectors that are parallel to the tangent line to the curve $y = 2x^2 - x - 1$ at the point $(1, 0)$ are

(a) $\left\langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \right\rangle$ and $\left\langle \frac{-1}{\sqrt{10}}, \frac{-3}{\sqrt{10}} \right\rangle$
(b) $\left\langle \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right\rangle$ and $\left\langle \frac{-3}{\sqrt{10}}, \frac{-1}{\sqrt{10}} \right\rangle$
(c) $\langle 1,0\rangle$ and $\langle -1,0\rangle$
(d) $\langle 0,1\rangle$ and $\langle 0,-1\rangle$
(e) $\left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$ and $\left\langle \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right\rangle$

$y' = 4x - 1$

$y'|_{x=1} = 3$

$\pm\langle 1,3\rangle$ are parallel to the tangent line.

Unit vectors are

$\pm \frac{\langle 1,3\rangle}{\sqrt{10}}$

12.2
Similar Q.41

15. If $\vec{a} = \langle 3, 0, -1 \rangle$, then a vector $\vec{b} = \langle 0, u, v \rangle$ such that $\text{comp}_{\vec{a}} \vec{b} = 2$ is

(a) $\langle 0, 10, -2\sqrt{10} \rangle$
(b) $\langle 0, 1, -\sqrt{10} \rangle$
(c) $\langle 0, -1, 2\sqrt{10} \rangle$
(d) $\langle 10, 10, 3\sqrt{10} \rangle$
(e) $\langle 0, 0, 3\sqrt{10} \rangle$

$$\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|} = 2$$

$$\langle 3,0,-1 \rangle \cdot \langle 0,u,v \rangle = 2\sqrt{10}$$

$$\Rightarrow v = -2\sqrt{10}$$

$\frac{12.3}{\text{Similar Q47}}$

16. If the angle between the vectors $\langle 2, 1, -1 \rangle$ and $\langle 1, x, 0 \rangle$ is $\frac{\pi}{4}$, then $x =$

(a) $1 \pm \sqrt{6}/2$
(b) $1 \pm \sqrt{6}$
(c) $2 \pm \sqrt{6}$
(d) $1 \pm \sqrt{3}$
(e) $-1 \pm \sqrt{3}/2$

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$$

$$\langle 2,1,-1 \rangle \cdot \langle 1,x,0 \rangle = \sqrt{6}\sqrt{1+x^2} \cos\frac{\pi}{4}$$

$$2 + x = \sqrt{6}\sqrt{1+x^2} \cdot \frac{1}{\sqrt{2}}$$

$$2 + x = \sqrt{3}\sqrt{1+x^2}$$

$$4 + x^2 + 4x = 3 + 3x^2$$

$$2x^2 - 4x - 1 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4(2)(-1)}}{4}$$

$$= 1 \pm \frac{\sqrt{24}}{4}$$

$$= 1 \pm \frac{\sqrt{6}}{2}$$

$\frac{12.3}{Q.26}$

17. The vector component of $\vec{v} = < 1, 1, 1 >$ orthogonal to $\vec{b} = < 2, 2, 0 >$ is

(a) $\hat{k}$
(b) $-\hat{j}$
(c) $\hat{j}$
(d) $\hat{i}$
(e) $-\hat{i}$

$$\text{Proj}_{\vec{b}}\vec{v} = \frac{\vec{v} \cdot \vec{b}}{\|b\|^2}\vec{b}$$
$$= \frac{4}{8}\langle 2,2,0\rangle = \langle 1,1,0\rangle$$

Vector component of $\vec{v}$ orthogonal to $\vec{b}$ is

$$\vec{v} - \text{Proj}_{\vec{b}}\vec{v} = \langle 1,1,1\rangle - \langle 1,1,0\rangle$$
$$= \langle 0,0,1\rangle$$

12.3
Similar Q 45

18. Suppose that two vectors $\vec{a}, \vec{b}$ satisfy

$$\vec{a} \cdot \vec{b} = \sqrt{15},\ \vec{a} \times \vec{b} = \langle -2, 0, -1\rangle.$$

Then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$
(d) $\frac{\pi}{4}$
(e) $\frac{2\pi}{5}$

Let θ be the angle between $\vec{a}$ and $\vec{b}$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta = \sqrt{15}$$
$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta = \sqrt{5}$$
$$\tan\theta = \frac{|\vec{a} \times \vec{b}|}{\vec{a} \cdot \vec{b}} = \frac{\sqrt{5}}{\sqrt{15}}$$
$$= \frac{1}{\sqrt{3}}$$
$$\theta = \frac{\pi}{6}$$

12.4
Similar Q 43

19. Consider for $a \geq 0$, the following 3 points

$$A(-2,0,0),\ B(-1,1,3),\ C(-2,a,2).$$

Assume that the area of the triangle $\triangle ABC = \sqrt{6}$. Then $a =$

$\vec{AB} = \langle 1,1,3 \rangle$, $\vec{AC} = \langle 0,a,2 \rangle$

$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 1 & 1 & 3 \\ 0 & a & 2 \end{vmatrix}$

$= \langle 2-3a, -2, a \rangle$

$|\vec{AB} \times \vec{AC}| = \sqrt{4+9a^2-12a+4+a^2}$

$= \sqrt{10a^2-12a+8}$

$|\vec{AB} \times \vec{AC}| = 2\sqrt{6}$

$10a^2-12a+8 = 24$

$5a^2-6a-8=0$

$5a^2-10a+4a-8=0$

$5a(a-2)+4(a-2)=0$

$\Rightarrow a = -\frac{4}{5},\ a = 2 > 0$

(a) 2

(b) 0

(c) $\frac{3}{5}$

(d) 1

(e) $\frac{1}{5}$

12.4
Ex. 29-32

20. Consider the **four** points $A(0,3,-4)$, $B(2,2,-3)$, $C(0,6,-6)$ and $D(-2,10,a)$. Suppose these **four** points lie in the same plane. Then $a =$

$\vec{AB} = \langle 2,-1,1 \rangle$, $\vec{AC} = \langle 0,3,-2 \rangle$

$\vec{AD} = \langle -2,7,a+4 \rangle$

A, B, C, D lie in the same plane.

$\Leftrightarrow$ $\vec{AB}, \vec{AC}, \vec{AD}$ are coplanar.

$\begin{vmatrix} 2 & -1 & 1 \\ 0 & 3 & -2 \\ -2 & 7 & a+4 \end{vmatrix} = 0$

$2(3a+12+14)+1(0-4)+1(0+6)=0$

$6a+52-4+6=0$

$6a+54=0$

$a=-9$

(a) -9

(b) -8

(c) 7

(d) -10

(e) 3

12.4
Similar Q.28