1. Symmetric equations for the line of intersection of the planes $x + 2y + 3z = 1$ and $x - y + z = 1$ are given by

(a) $\dfrac{x-1}{5} = \dfrac{y}{2} = \dfrac{z}{-3}$

(b) $\dfrac{x+1}{5} = \dfrac{y}{2} = \dfrac{z}{-3}$

(c) $\dfrac{x}{5} = \dfrac{y-1}{2} = \dfrac{z}{3}$

(d) $\dfrac{x}{5} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$

(e) $\dfrac{x-1}{5} = \dfrac{y-1}{2} = \dfrac{z+1}{3}$

$x + 2y + 3z = 1$
$x - y + z = 1$
Let $z = 0$. Then $x = 1$, $y = 0$.
$(1, 0, 0)$ is a point in the intersecting line

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{vmatrix}$$

$= \langle 5, 2, -3 \rangle$ (parallel to the line)

The symmetric equations are

$$\frac{x-1}{5} = \frac{y}{2} = \frac{z}{-3}$$

12.5
Exercise 12

2. The point of intersection of the line whose parametric equations are $x = 2 - 2t$, $y = 3t$, $z = 1 + t$ and the plane $x + 2y - z = 7$ is (a, b, c), then $a + b + c =$

(a) 7

(b) 6

(c) 5

(d) 4

(e) 0

Substituting x, y, z in the equation of the plane to get

$2 - 2t + 6t - 1 - t = 7$

$3t = 6 \Rightarrow t = 2$

$(a, b, c) = (2-4, 6, 1+2)$

$= (-2, 6, 3)$

$a + b + c = 7$

12.5
Excercise 45

3. The equation of the plane through the point $(3, -2, 8)$ and parallel to the plane $z = x + y$ is given by

(a) $x + y - z + 7 = 0$

(b) $x - y + z + 13 = 0$

(c) $2x + y - z - 4 = 0$

(d) $x + 2y - z + 9 = 0$

(e) $x - y - z + 3 = 0$

$\vec{n} = \langle 1, 1, -1 \rangle$

Equation of the plane is

$1(x-3) + 1(y+2) - 1(z-8) = 0$

$x + y - z - 3 + 2 + 8 = 0$

$x + y - z + 7 = 0$

12.5

Excercise 18

4. Which of the following statements is true about the surface

$$x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$$

(a) circular cone with vertex $(2, -1, 1)$ and axis parallel to y-axis

(b) circular cone with vertex $(-1, 0, 2)$ and axis parallel to y-axis

(c) circular cone with vertex $(0, 0, 0)$ and axis parallel to y-axis

(d) circular cone with vertex $(-1, -1, -1)$ and axis parallel to y-axis

(e) circular cone with vertex $(1, 3, 0)$ and axis parallel to y-axis

$(x-2)^2 - 4 - (y+1)^2 + 1 + (z-1)^2 - 1 + 4 = 0$

$(x-2)^2 + (z-1)^2 = (y+1)^2$

12.6

Similar Exercise 16

5. Consider the following statements about the surface

$$x^2 = 4 - 4y^2 + z^2$$

I. Axis of symmetry is the y-axis.
II. The surface is called hyperboloid of one sheet
III. The surface is called hyperboloid of two sheets
IV. Axis of symmetry is z-axis

Which of the following is true?

$$x^2 + 4y^2 - z^2 = 4$$

$$\frac{x^2}{2^2} + \frac{y^2}{1^2} - \frac{z^2}{2^2} = 1$$

Hyperboloid of one sheet and axis of symmetry is Z-axis.

(a) II and IV
(b) I and II
(c) III and IV
(d) II and III
(e) I and III

12.6
Example 6

6. The domain of the function

$$f(x, y, z) = \ln(x - y) + yz\cos x$$

is

$\ln(x-y)$ is defined for $x-y>0$

(a) $\{(x, y, z) \in R^3 : x > y\}$
(b) $\{(x, y, z) \in R^3 : x < z\}$
(c) $\{(x, y, z) \in R^3 : y > z\}$
(d) $\{(x, y, z) \in R^3 : y < z\}$
(e) $\{(x, y, z) \in R^3 : y^2 + z^2 = 1\}$

14.1
Similar Example P.898

7. The range of $f(x, y) = \ln(16 - x^2 - y^2)$ is

Domain = $\{(x,y) : 16 - x^2 - y^2 > 0\}$

(a) $(-\infty, 4\ln 2]$
(b) $(-\infty, \ln 2]$
(c) $(-\infty, \ln 5]$
(d) $(-\infty, 2\ln 2]$
(e) $(-\infty, \infty)$

$-\infty < z = f(x,y) \leq \ln 16 = 4\ln 2$

14.1
Similar Excercise 15

8. The level curves of $f(x, y) = \sqrt[3]{x^2 + y^2}$ are

(a) a family of circles
(b) a family of ellipses
(c) a family of lines
(d) a family of parabolas
(e) a family of hyperbolas

Let $f(x,y) = C$. Then

$C = \sqrt[3]{x^2+y^2}$

$C^{3/2} = x^2 + y^2$

$C \geq 0$ $\Rightarrow$ circles with centered origin

14.1
Excercise 51

Page 6 of 10 MASTER

9. The limit

$$\lim_{(x,y)\to(0,0)} \frac{8x^2 \sin y}{3x^3 + 5y^3}$$

(a) does not exist
(b) is equal to 0
(c) is equal to 1
(d) is equal to -1
(e) is equal to ∞

Along line $x = 0$:

$$\lim_{y\to 0} \frac{0}{5y^3} = 0$$

Along line $y = x$:

$$\lim_{x\to 0} \frac{8x^2 \sin x}{3x^3 + 5x^3}$$

$$= \lim_{x\to 0} \frac{8x^2 \sin x}{8x^3}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} = 1$$

Limit DNE

14.2
Execercise 11

10. The limit

$$\lim_{(x,y)\to(3,3)} \frac{x + 2y - 9}{\sqrt{x + 2y} - 3}$$

(a) is equal to 6
(b) is equal to 9
(c) does not exist
(d) is equal to 3
(e) is equal to 1

$$\lim_{(x,y)\to(3,3)} \frac{x+2y-9}{\sqrt{x+2y}-3} \times \frac{\sqrt{x+2y}+3}{\sqrt{x+2y}+3}$$

$$= \lim_{(x,y)\to(3,3)} \sqrt{x+2y} + 3$$

$$= \sqrt{3+6} + 3$$

$$= 3 + 3$$

$$= 6$$

14.2
Similar Exccercise 17

11.

$$\text{If } f(x,y) = \begin{cases} \dfrac{2-2\cos(x+y)}{(x+y)^2} & \text{if}(x,y) \neq (0,0) \\ k & \text{if}(x,y) = (0,0) \end{cases},$$

then $f(x,y)$ is

(a) continuous at $(0,0)$ only if $k=1$

(b) continuous at $(0,0)$ only if $k=-1$

(c) continuous at $(0,0)$ only if $k=0$

(d) not continuous at $(0,0)$ for all values of k

(e) continuous at $(0,0)$ for any value of the real number k

Let $u = x+y$, Then

$$\lim_{(x,y)\to(0,0)} \frac{2-2\cos(x+y)}{(x+y)^2}$$

$$= \lim_{u\to 0} \frac{2-2\cos u}{u^2} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{u\to 0} \frac{2\sin u}{2u} = 1$$

$\Rightarrow$ k should be equal to 1 for the Function $f(x,y)$ to be Continuous at $(0,0)$.

14.2

Similar Excercises 7, 8, 38, 37, 43

12. If $f(x,y) = \tan^{-1}(x+y^2)$, then $f_y(1,2) =$

(a) $\dfrac{2}{13}$

(b) $\dfrac{4}{13}$

(c) $\dfrac{5}{13}$

(d) $\dfrac{3}{13}$

(e) $\dfrac{1}{13}$

$$f_y(x,y) = \frac{1}{1+(x+y^2)^2} \cdot 2y$$

$$f_y(1,2) = \frac{4}{1+(1+4)^2}$$

$$= \frac{4}{1+25}$$

$$= \frac{4}{26}$$

$$= \frac{2}{13}.$$

14.3

Similar Exercise 26

13. If $f(x,y) = \cos(x^2y)$, then $f_x(x,y) - f_y(x,y) =$

$f_x(x,y) = -\sin(x^2y) \cdot 2xy$
$f_y(x,y) = -\sin(x^2y) \cdot x^2$

(a) $(x^2 - 2xy)\sin(x^2y)$
(b) $x^2\sin(x^2y)$
(c) $x^2\cos(x^2y)$
(d) $2xy\cos(x^2y)$
(e) $(x^2 + 2xy)\sin(xy)$

$f_x(x,y) - f_y(x,y)$
$= (x^2 - 2xy)\sin(x^2y)$

14.3
Excercise 37

14. The base radius and height of a right circular cone are measured as 10 cm and 25 cm, respectively with a possible error in measurement as much as 0.01 cm in each. The maximum error in the calculated volume of the cone is

(**Hint:** volume of a cone is $V = \frac{\pi r^2 h}{3}$)

$dV = \frac{\partial V}{\partial r} dr + \frac{\partial V}{\partial h} dh$

(a) 2π
(b) 3π
(c) π
(d) 4π
(e) 5π

$= \frac{2\pi r h}{3} dr + \frac{\pi r^2}{3} dh$

Error is at most $|\Delta r| \leq 0.01$, $|\Delta h| \leq 0.01$
We take $dr = 0.01$ and $dh = 0.01$

$dV = \frac{2 \cdot \pi \cdot 10 \cdot 25}{3}(0.01) + \frac{\pi \cdot 100}{3}(0.01)$

14.4
Example 5

$= \left(\frac{500\pi}{3} + \frac{100\pi}{3}\right)(0.01)$

$= \frac{600\pi}{3}(0.01)$

$= (200 \times 0.01)\pi$

$= 2\pi$

15. Consider the function $f(x,y) = y\ln\left(\frac{x}{y}\right)$. The linearization $L(x,y)$ of $f(x,y)$ at the point $(e,1)$ is:

(a) $L(x,y) = \dfrac{x}{e}$

(b) $L(x,y) = \dfrac{y}{e} + e$

(c) $L(x,y) = \dfrac{x}{e} + e$

(d) $L(x,y) = \dfrac{x}{e} - 1$

(e) $L(x,y) = \dfrac{y}{e} - 1$

$f(e,1) = 1 \ln(e) = 1$

$f_x(x,y) = y \dfrac{1}{\frac{x}{y}} \cdot \dfrac{1}{y} = \dfrac{y}{x}$

$f_x(e,1) = \dfrac{1}{e}$

$f_y(x,y) = \ln\left(\frac{x}{y}\right) + y \dfrac{1}{\frac{x}{y}}\left(-\dfrac{x}{y^2}\right)$

$= \ln\left(\frac{x}{y}\right) - 1$

$f_y(e,1) = \ln e - 1 = 1 - 1 = 0$

14.4

Similar Excercise 11

Hence,

$L(x,y) = 1 + (x-e)\cdot\frac{1}{e} + (y-1)\cdot 0$

$= 1 + \frac{x}{e} - 1 = \frac{x}{e}$

16. If $e^z = xyz$, then $\dfrac{\partial z}{\partial x} =$

(a) $\dfrac{yz}{e^z - xy}$

(b) $\dfrac{xy}{e^z - xy}$

(c) $\dfrac{x}{e^z - xy}$

(d) $\dfrac{xz}{e^z - xy}$

(e) $\dfrac{y}{e^z - xy}$

$e^z \dfrac{\partial z}{\partial x} = y\left(x\dfrac{\partial z}{\partial x} + z\cdot 1\right)$

$(e^z - xy)\dfrac{\partial z}{\partial x} = yz$

$\dfrac{\partial z}{\partial x} = \dfrac{yz}{e^z - xy}$

14.5

Excercise 33

17. If $z = \frac{x}{y}$, $x = se^t$, $y = 1 - se^{-t}$, then $\frac{\partial z}{\partial s}$ at $x = 3$, $y = -2$, and $t = 0$ is

(a) $\frac{1}{4}$

(b) $\frac{3}{4}$

(c) $\frac{5}{4}$

(d) 1

(e) 4

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$= \frac{1}{y} \cdot e^t + \left(-\frac{x}{y^2}\right)\left(-e^{-t}\right)$$

$$= \frac{e^t}{y} + \frac{x e^{-t}}{y^2}$$

$$\left.\frac{\partial z}{\partial s}\right|_{x=3,\, y=-2,\, t=0} = -\frac{1}{2} + \frac{3}{4}$$

$$= \frac{-2+3}{4} = \frac{1}{4}$$

14.5
Excercises 1-12
(Similar)

18. If $w = xy + z^2$, $x = 2s - t$, $y = 3t$ and $z = s$, then

$$\frac{\partial w}{\partial s} + \frac{\partial w}{\partial t} =$$

(a) $8s$

(b) $2t + 2s$

(c) $2t - s$

(d) $6t$

(e) $4s$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial s}$$

$$= (y)(2) + (x)(0) + (2z)(1)$$

$$= 2y + 2z$$

$$\frac{\partial w}{\partial t} = (y)(-1) + (x)(3) + (2z)(0)$$

$$= -y + 3x$$

$$\frac{\partial w}{\partial s} + \frac{\partial w}{\partial t} = 3x + y + 2z$$

$$= 3(2s - t) + 3t + 2s$$

$$= 8s$$

14.5
Excercises 21-26
(Similar)

19. The equation of the tangent plane at the point $(-3, 1, -2)$ to the ellipsoid

$$\frac{x^2}{9} + y^2 + z^2 = 6$$

is $ax + by + 6z + 18 = 0$. Then $a + b =$

(a) -2
(b) -3
(c) -1
(d) -4
(e) -5

$\nabla f = \langle \frac{2x}{9}, 2y, 2z \rangle$

$\nabla f(-3,1,2) = \langle -\frac{2}{3}, 2, -4 \rangle$

Equation of the tangent plane is

$-\frac{2}{3}(x+3) + 2(y-1) - 4(z+2) = 0$

$-\frac{2}{3}x - 2 + 2y - 2 - 4z - 8 = 0$

$-2x + 6y - 12z - 6 - 6 - 24 = 0$

$x - 3y + 6z + 18 = 0$

$a = 1,\ b = -3 \quad ,\ a + b = -2$

14.6 Exercise 41-46

20. The directional derivative of $f(x, y, z) = xe^y + ye^z + ze^x$ at $(0, 0, 0)$ in the direction of $\vec{v} = < -3, 1, 2 >$ is

(a) 0
(b) $-\frac{3}{\sqrt{14}}$
(c) $\frac{1}{\sqrt{14}}$
(d) $\frac{2}{\sqrt{14}}$
(e) 1

$\nabla f = \langle e^y + ze^x, xe^y + e^z, ye^z + e^x \rangle$

$\nabla f(0,0,0) = \langle 1, 1, 1 \rangle$

$\vec{v} = \langle -3, 1, 2 \rangle$

$\hat{v} = \frac{\langle -3, 1, 2 \rangle}{\sqrt{14}}$

$D_{\hat{v}} f = \nabla f \cdot \hat{v}$

$= \langle 1, 1, 1 \rangle \cdot \frac{\langle -3, 1, 2 \rangle}{\sqrt{14}}$

$= \frac{-3 + 1 + 2}{\sqrt{14}} = 0$

14.6 Exercise 17