1. The exact length of the curve $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \le t \le 1$ is

(a) $4\sqrt{2} - 2$
(b) $6\sqrt{2} - 1$
(c) $\sqrt{2} + 1$
(d) $\sqrt{2} + 3$
(e) $\sqrt{2} - 3$

$$L = \int_0^1 \sqrt{(x')^2 + (y')^2}\, dt$$

$$= \int_0^1 \sqrt{(6t)^2 + (6t^2)^2}\, dt$$

$$= 6 \int_0^t t\sqrt{1+t^2}\, dt$$

$1+t^2 = u$
$t\,dt = \frac{du}{2}$

$$= 3 \cdot \frac{2}{3} \left[(1+t^2)^{3/2}\right]_0^1$$

$$= 2\left[2^{3/2} - 1\right] = 2\left[2\sqrt{2} - 1\right]$$

$$= 4\sqrt{2} - 2$$

10.2
Exercise 41

2. The graph of $r \cot\theta = 6 \sec\theta$ is

(a) a parabola
(b) an ellipse
(c) a circle
(d) a hyperbola
(e) a straight line

$$r \cot\theta = 6 \sec\theta$$

$$r \cdot \frac{x}{y} = 6 \cdot \frac{r}{x}$$

$$y = \frac{x^2}{6}$$

10.3
Exercises 29-46

3. If $\vec{a} =< -2, 3, 1 >$ and $\vec{b} =< 1, 2, 2 >$, then $\text{comp}_{\vec{a}}\vec{b} =$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{-2+6+2}{\sqrt{4+9+1}} = \frac{6}{\sqrt{14}}$$

(a) $\dfrac{6}{\sqrt{14}}$

(b) $\dfrac{10}{\sqrt{14}}$

(c) $\dfrac{8}{\sqrt{14}}$

(d) $\dfrac{4}{\sqrt{14}}$

(e) $\dfrac{3}{\sqrt{14}}$

12.3

Example 6

4. The value of k for which the vectors

$$\vec{a} =< 1, 2, 3 >, \vec{b} =< 0, 1, 1 > \text{ and } \vec{c} =< k, 0, -1 >$$

lie in the same plane is

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ k & 0 & -1 \end{vmatrix} = 0$$

$$1(-1-0) - 2(0-k) + 3(0-k) = 0$$

$$-1 + 2k - 3k = 0$$

$$-1 - k = 0$$

$$k = -1$$

(a) -1

(b) -2

(c) 0

(d) 1

(e) 2

12.4

Exercises 33-34

5. The distance between the parallel planes $10x + 2y + 2z = 10$ and $5x + y + z = 2$ is

(a) $\dfrac{1}{\sqrt{3}}$

(b) $\dfrac{1}{\sqrt{2}}$

(c) $\sqrt{3}$

(d) $\sqrt{2}$

(e) 1

$(1,0,0)$

$$D = \frac{|5\cdot 1 + 1\cdot 0 + 1\cdot 0 - 2|}{\sqrt{5^2+1^2+1^2}} = \frac{3}{\sqrt{27}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

12.5
Example 9

6. The quadratic surface $x^2 - y^2 + z^2 - 4x + 2y - 4z + 6 = 0$ represents

(a) a hyperboloid of one sheet

(b) a hyperboloid of two sheets

(c) a cone

(d) a paraboloid

(e) an ellipsoid

$$(x-2)^2 - 4 - (y-1)^2 + 1 + (z-2)^2 - 4 + 6 = 0$$

$$(x-2)^2 - (y-1)^2 + (z-2)^2 = 1$$

12.6
Similar Exer. 37

7. The maximum rate of change of $f(x, y) = xe^y$ at the point $P(2, 0)$ is

(a) $\sqrt{5}$
(b) $\sqrt{3}$
(c) $\sqrt{2}$
(d) 1
(e) 0

$\nabla f = \langle e^y, xe^y \rangle$

$\nabla f(2,0) = \langle 1, 2 \rangle$

$\|\nabla f\| = \sqrt{1+4} = \sqrt{5}$

14.6
Exer. 21-26

8. If $w = xy$, $x = \cos t$ and $y = \sin t$, then $\dfrac{dw}{dt} =$

(a) $\cos 2t$
(b) $\sin 2t$
(c) $2\cos 2t$
(d) $2\sin 2t$
(e) 1

$\dfrac{\partial w}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial w}{\partial y}\dfrac{dy}{dt}$

$= y(-\sin t) + x(\cos t)$

$= -\sin^2 t + \cos^2 t$

$= \cos 2t$

14.5
Like Exer 1-6

9. If $f(x, y) = x^y$, then $f_{xy}(x, y)$ at $x = 1$ and $y = 2$ is

(a) 1
(b) 3
(c) 4
(d) 5
(e) 2

$f_y = x^y \ln x$

$f_{yx} = x^{y-1} + y x^{y-1} \ln x$

$f_{yx}\big|_{(1,2)} = 1^{2-1} + 2 \cdot 1^{2-1} \cdot \ln 1$

$= 1 + 0$

$= 1$

14.3
Exer. 28

10. The absolute maximum of $f(x, y) = e^{-x^2-y^2}(2x^2 + y^2)$ on the disk $x^2 + y^2 \leq 4$ is

(a) $\frac{2}{e}$
(b) $\frac{2}{e^2}$
(c) $\frac{4}{e}$
(d) $\frac{4}{e^2}$
(e) $\frac{1}{e}$

Interior of $x^2+y^2 \leq 4$:

$f_x = -2x\, e^{-x^2-y^2}(2x^2+y^2) + e^{-x^2-y^2}(4x)$

$= 2x\, e^{-x^2-y^2}(2 - 2x^2 - y^2) = 0$

$f_y = 2y\, e^{-x^2-y^2}(1 - 2x^2 - y^2) = 0$

C.P: $(0,0), (0,\pm 1), (\pm 1, 0)$

$f = 0 \qquad \frac{1}{e} \qquad \frac{2}{e}$

Boundary of $x^2+y^2 \leq 4$:

$f(x,y) = e^{-4}(x^2+4), \quad -2 \leq x \leq 2$

C.P. in $[-2,2]$ is $x = 0$.

$f(0, \pm 2) = \frac{4}{e^4}, \quad f(\pm 2, 0) = \frac{8}{e^4}$

Absolute maximum of f on the disk is $\frac{2}{e}$

14.7
Exer. 56, Page 984

11. The function $f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$ has local minimum value at $(x,y) =$

(a) $(1,1)$
(b) $(0,0)$
(c) $(-1,1)$
(d) $(-1,-1)$
(e) $(-1,0)$

$f_x = y - \frac{1}{x^2} = 0 \Rightarrow y = \frac{1}{x^2}$

$f_y = x - \frac{1}{y^2} = 0 \Rightarrow x = \frac{1}{y^2}$ $\Rightarrow x = \frac{1}{\frac{1}{x^4}} = x^4$

$\Rightarrow x(x^3-1) = 0 \Rightarrow x = 0$ or $x = 1$

f is not defined.

Only one c.P. is $(1,1)$

$f_{xx} = \frac{2}{x^3}$, $f_{xy} = 1$, $f_{yy} = \frac{2}{y^3}$

$D(1,1) = \left[f_{xx} f_{yy} - (f_{xy})^2\right]_D = 2.2 - 1 > 0$

and $f_{xx}(1,1) = 2 > 0$

$\therefore \quad f(1,1) = 1+1+1 = 3$

14.7
Like exer. 14

12. The sum of the extreme values of the function $f(x,y) = xe^y$, subject to constraint $x^2 + y^2 = 2$ is

(a) 0
(b) 1
(c) e
(d) $-e$
(e) $2e$

$\nabla f = \lambda \nabla g$

$\Rightarrow e^y = 2\lambda x$ — (1)

$xe^y = 2\lambda y$ — (2)

$x^2 + y^2 = 2$ — (3)

(1) $\Rightarrow x \neq 0$.

If $y = 0$, then (2) gives $x = 0$, so $y \neq 0$

(1) & (2) $\Rightarrow \frac{e^y}{2x} = \lambda = \frac{xe^y}{2y} \Rightarrow 2ye^y = 2x^2e^y$

$\Rightarrow y = x^2$. — (4)

14.8
Exer. 6

(3) & (4) $\Rightarrow x^2 + x^4 = 2$

$\Rightarrow (x^2+2)(x^2-1) = 0 \Rightarrow x = \pm 1$

Pts are $(1,1)$, $(-1,1)$

$f = e$ $\quad f = -e$

Sum $= e - e = 0$.

13. The function $f(x, y) = x^4 - 2x^2 + y^3 - 3y$ has how many critical points?

$f_x = 4x^3 - 4x = 0 \Rightarrow x = 0, \pm 1$

$f_y = 3y^2 - 3 = 0 \Rightarrow y = \pm 1$

(a) 6

(b) 4

(c) 5

(d) 8

(e) 3

Critical points are

$(0, \pm 1), (1, \pm 1), (-1, \pm 1)$

14. The iterated integral $\int_1^2 \int_0^3 (6x^2y - 4x)\, dy\, dx$ is equal to

(a) 45

(b) 42

(c) 48

(d) -52

(e) 21

$= \int_1^2 \left(3x^2y^2 - 4xy\right)\Big|_0^3 dx$

$= \int_1^2 (27x^2 - 12x)\, dx$

$= 9x^3 - 6x^2 \Big|_1^2$

$= (72 - 24) - (9 - 6)$

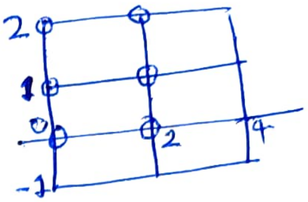
$= 48 - 3$

$= 45$

15.1

Exercise 15

15. Let $R = [0, 4] \times [-1, 2]$. Use a Riemann sum with $m = 2$ (number of subintervals in x-direction) and $n = 3$ (number of subintervals in y-direction) to estimate $\iint_R (2 - 2xy^2)\, dA$. Take the sample points to be upper left corners of the rectangles.



(a) -16

(b) -18

(c) -14

(d) -12

(e) -10

$$\iint_R (2-2xy^2)\,dA = \iint_R f(x,y)\,dA$$

$$= [f(0,0) + f(0,1) + f(0,2) + f(2,0) + f(2,1) + f(2,2)]\,\Delta A$$

$$= [2+2+2+2-2-14]\cdot 2$$

$$= -16$$

15.1
Exercise 2

16. The value of $\iint_D e^{-y^2}\, dA$, where $D = \{(x, y) : 0 \le y \le 2,\ 0 \le x \le y\}$ is equal to

(a) $\frac{1}{2}(1 - e^{-4})$

(b) $\frac{1}{2}(1 - e^{-6})$

(c) $\frac{1}{2}(1 - e^{-2})$

(d) $\frac{1}{2}(1 - e)$

(e) $\frac{1}{2}(1 - e^{-3})$

$$\int_{y=0}^{2}\int_{x=0}^{y} e^{-y^2}\,dx\,dy$$

$$= \int_0^2 \left[x e^{-y^2}\right]_{x=0}^{y} dy$$

$$= \int_0^2 y e^{-y^2}\,dy$$

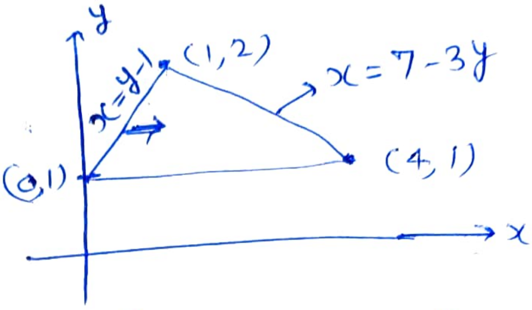
$$= -\frac{e^{-y^2}}{2}\Big|_0^2$$

$$= \frac{1}{2} - \frac{1}{2}e^{-4}$$

$$= \frac{1}{2}(1 - e^{-4})$$

15.2
Similar Exer. 9

17. If D is the triangular region with vertices $(0,1)$, $(1,2)$, $(4,1)$, then $\displaystyle\iint_D y^2 dA =$



(a) $\frac{11}{3}$

(b) $\frac{9}{3}$

(c) $\frac{13}{3}$

(d) $\frac{7}{3}$

(e) $\frac{17}{3}$

$$\int_{y=1}^{2}\int_{y-1}^{7-3y} y^2\,dx\,dy = \int_1^2 (7-3y-y+1)y^2\,dy$$

$$= \int_1^2 (8y^2-4y^3)\,dy = \frac{8}{3}y^3 - y^4\Big|_1^2$$

$$= \left(\frac{64}{3}-16\right)-\left(\frac{8}{3}-1\right) = \frac{16}{3}-\frac{5}{3}=\frac{11}{3}$$

15.2
Like Exer. 22

18. $\displaystyle\int_0^1\int_{x^2}^1 \sqrt{y}\sin y\,dy\,dx =$

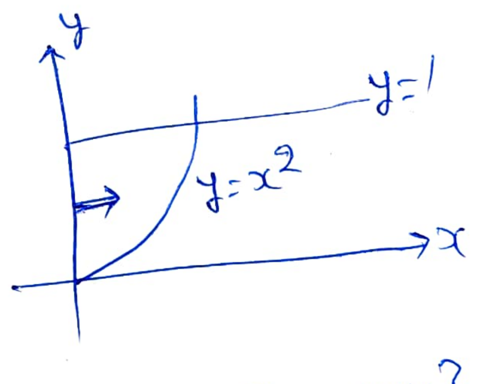
(a) $\displaystyle\int_0^1\int_0^{\sqrt{y}} \sqrt{y}\sin y\,dx\,dy$

(b) $\displaystyle\int_0^1\int_y^{\sqrt{y}} \sqrt{y}\sin y\,dx\,dy$

(c) $\displaystyle\int_0^1\int_0^{y} \sqrt{y}\sin y\,dx\,dy$

(d) $\displaystyle\int_0^1\int_0^{y^2} \sqrt{y}\sin y\,dx\,dy$

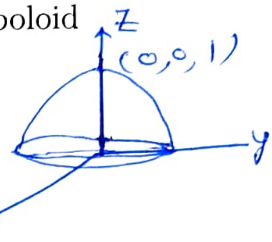
(e) $\displaystyle\int_0^1\int_0^{2y} \sqrt{y}\sin y\,dx\,dy$



$R = \{(x,y):\ 0 \le x \le \sqrt{y},\ 0 \le y \le 1\}$

15.2
Exer. 52

19. The volume of the solid bounded by the plane $z = 0$ and the paraboloid $z = 1 - x^2 - y^2$ is
(Hint: Use polar coordinates)



(a) $\frac{\pi}{2}$
(b) π
(c) 2π
(d) $\frac{\pi}{4}$
(e) 0

$$V = \iint_R \int_{Z=0}^{1-x^2-y^2} dZ\, dA$$
$$= \iint_R (1-x^2-y^2)\, dA$$
$$= \int_{\theta=0}^{2\pi} \int_0^1 (1-r^2) \cdot r\, dr\, d\theta$$
$$= 2\pi \left[\frac{r^2}{2} - \frac{r^4}{4} \right]_0^1$$
$$= 2\pi \left[\frac{1}{2} - \frac{1}{4} \right] = \frac{\pi}{2}$$

15.3
Example 2

20. The value of the integral

$$I = \int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (2x + y)\, dx\, dy$$

is

$$= \int_0^{\pi} \int_0^1 (2r\cos\theta + r\sin\theta) \cdot r\, dr\, d\theta$$
$$= \int_0^1 r^2\, dr \int_0^{\pi} (2\cos\theta + \sin\theta)\, d\theta$$
$$= \frac{1}{3} \left(2\sin\theta - \cos\theta \right)_0^{\pi}$$
$$= \frac{1}{3} \left[(0+1) - (0-1) \right]$$
$$= \frac{2}{3}$$

(a) $\frac{2}{3}$
(b) $\frac{4}{3}$
(c) $\frac{5}{3}$
(d) $\frac{8}{3}$
(e) $\frac{1}{3}$

15.3
Exer. 30

21. The value of

$$\iiint_E z\,dV$$

where E is the solid bounded by the coordinate planes and the plane $x+y+z=1$ is

$$I = \int_{x=0}^{1}\int_{y=0}^{1-x}\int_{z=0}^{1-x-y} z\,dz\,dy\,dx$$

$$= \int_0^1\int_0^{1-x} \frac{(1-x-y)^2}{2}\,dy\,dx$$

$$= \frac{1}{2}\int_0^1 \left[-\frac{(1-x-y)^3}{3}\right]_{y=0}^{1-x} dx$$

$$= \frac{1}{6}\int_0^1 (1-x)^3\,dx$$

$$= \frac{1}{6}\left[-\frac{(1-x)^4}{4}\right]_0^1 = \frac{1}{24}$$

(a) $\frac{1}{24}$
(b) $\frac{1}{12}$
(c) $\frac{1}{6}$
(d) $\frac{1}{48}$
(e) $\frac{1}{2}$

15.6 Example 2

22. If

$$\iiint_E x\,dV = \int_0^a\int_0^b (cr^2 - r^4)\cos\theta\,dr\,d\theta,$$

where E is the solid bounded by the paraboloid $z = x^2+y^2$ and the plane $z=4$, then $a+b-c=$

$$\iint_D \int_{x^2+y^2}^{4} x\,dz\,dA$$

$$= \iint_D x(4-x^2-y^2)\,dA$$

$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{2} r(4-r^2)\cdot\cos\theta\cdot r\,dr\,d\theta$$

$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{2} (4r^2-r^4)\cos\theta\,dr\,d\theta$$

(a) $2(\pi-1)$
(b) 2π
(c) $4(\pi-1)$
(d) $\pi-2$
(e) $\pi-4$

15.6 Example 3

$a = 2\pi,\ b = 2,\ c = 4.$

$a+b-c = 2\pi+2-4 = 2\pi-2$
$= 2(\pi-1)$

23. The volume of the region enclosed by the cylinder $x^2 + (y-1)^2 = 1$ and the planes $z = 0$, $z = y$ is

$r^2\cos^2\theta + (r\sin\theta - 1)^2 = 1$

$r^2\cos^2\theta + r^2\sin^2\theta + 1 - 2r\sin\theta = 1$

$r^2 = 2r\sin\theta$

$\boxed{r = 2\sin\theta}$

$z = y = r\sin\theta.$

$E = \{(r,\theta,z);\ 0 \le \theta \le \pi,\ 0 \le r \le 2\sin\theta,\ 0 \le z \le r\sin\theta\}$

(a) $\displaystyle\int_0^{\pi}\int_0^{2\sin\theta}\int_0^{r\sin\theta} r\,dzdrd\theta$

(b) $\displaystyle\int_0^{2\pi}\int_0^{\sin\theta}\int_0^{\cos\theta} r\,dzdrd\theta$

(c) $\displaystyle\int_0^{\pi/2}\int_0^{\sin\theta}\int_0^{r\cos\theta} r\,dzdrd\theta$

(d) $\displaystyle\int_0^{\pi}\int_0^{\cos\theta}\int_0^{r\sin\theta} r\,dzdrd\theta$

(e) $\displaystyle\int_0^{\pi}\int_0^{2\cos\theta}\int_0^{r\sin\theta} r\,dzdrd\theta$

15.7
Like Exer 26(a)

24. Let E be the region bounded by the cone $z^2 = x^2 + y^2$ and the paraboloid $z = 2 - x^2 - y^2$. Then $\displaystyle\iiint_E z\,dV =$

$\displaystyle\iint_R \int_{z=r}^{2-r^2} z\,r\,dz\,dr\,d\theta$

$= \displaystyle\int_0^{2\pi}\int_0^1 \frac{r}{2}\left[(2-r^2)^2 - r^2\right] dr\,d\theta$

$= \pi\displaystyle\int_0^1 r(4 + r^4 - 4r^2 - r^2)\,dr$

$= \pi\displaystyle\int_0^1 (r^5 - 5r^3 + 4r)\,dr$

$= \pi\left[\frac{1}{6} - \frac{5}{4} + \frac{4}{2}\right]$

$= \dfrac{2 - 15 + 24}{12}\pi = \dfrac{11\pi}{12}$

(a) $\dfrac{11\pi}{12}$
(b) π
(c) $\dfrac{5\pi}{4}$
(d) $\dfrac{4\pi}{5}$
(e) 2π

15.7
Like Exer 25

25. $\displaystyle\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{\frac{x^2+y^2}{4}}^{\frac{\sqrt{x^2+y^2}}{2}} xz\,dz\,dx\,dy =$

$$\int_{-\pi/2}^{\pi/2}\int_{0}^{2}\int_{\frac{r^2}{4}}^{r/2} r\cos\theta \cdot z\, dz \cdot r\,dr\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\int_{0}^{2} \cos\theta\, r^2 \cdot \frac{z^2}{2}\Big|_{\frac{r^2}{4}}^{r/2} dr\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\int_{0}^{2} \cos\theta \cdot r^2\left(\frac{r^2}{8} - \frac{r^4}{32}\right)dr\,d\theta$$

$$= \sin\theta\Big|_{-\pi/2}^{\pi/2}\int_{0}^{2}\frac{4r^4-r^6}{32}\,dr$$

$$= \frac{2}{32}\int_{0}^{2}(4r^4-r^6)\,dr$$

(a) $\displaystyle\frac{1}{16}\int_{0}^{2}(4r^4-r^6)\,dr$

(b) $\displaystyle\frac{1}{16}\int_{0}^{2}(4r^3-r^5)\,dr$

(c) $\displaystyle\int_{0}^{1}(r^4-r^6)\,dr$

(d) $\displaystyle\int_{0}^{1}(r^3-r^5)\,dr$

(e) $\displaystyle\frac{1}{32}\int_{0}^{1}(4r^4-r^6)\,dr$

$\frac{15.7}{\text{Exer. 29}}$

26. Let E be the solid lying above the cone $z=\sqrt{x^2+y^2}$ and between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$. Then $\displaystyle\iiint_E \sqrt{x^2+y^2+z^2}\,dV =$

$z = \sqrt{x^2+y^2} \iff \phi = \pi/4$

$E = \{(\rho,\theta,\phi) : 1 \le \rho \le 2, 0 \le \theta \le 2\pi, 0 \le \phi \le \frac{\pi}{4}\}$

$$I = \int_{0}^{\pi/4}\int_{0}^{2\pi}\int_{1}^{2} \rho \cdot \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$$

$$= \frac{\rho^4}{4}\Big|_{1}^{2}\ \theta\Big|_{0}^{2\pi}\ (-\cos\phi)\Big|_{0}^{\pi/4}$$

$$= \left(\frac{16}{4}-\frac{1}{4}\right) \cdot 2\pi\left(1-\frac{\sqrt{2}}{2}\right)$$

$$= \frac{15}{4}\cdot 2\pi\left(1-\frac{\sqrt{2}}{2}\right)$$

$$= \frac{15}{2}\pi\left(1-\frac{\sqrt{2}}{2}\right)$$

(a) $\displaystyle\frac{15}{2}\pi\left(1-\frac{\sqrt{2}}{2}\right)$

(b) $\displaystyle\frac{15}{2}\pi\left(1+\frac{\sqrt{2}}{2}\right)$

(c) $\displaystyle\frac{15}{4}\pi\left(1-\frac{\sqrt{2}}{2}\right)$

(d) $\displaystyle\frac{15}{4}\pi\left(1+\frac{\sqrt{2}}{2}\right)$

(e) $\displaystyle\frac{15\pi}{2}\left(1-\sqrt{3}\right)$

$\frac{15.8}{\text{Exer. 26}}$

27. If B is the ball with center at the origin and radius 2,

then $\iiint_B (x^2 + y^2 + z^2)^2 \, dV =$

$$\int_{\phi=0}^{\pi} \int_{0}^{2\pi} \int_{\rho=0}^{2} \rho^4 \cdot \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$= \left.\frac{\rho^7}{7}\right|_0^2 \cdot 2\pi \cdot \left(-\cos\phi\right)\Big|_0^{\pi}$$

$$= \frac{128}{7} \cdot 2\pi \cdot 2 = \frac{512\pi}{7}$$

(a) $\dfrac{512\pi}{7}$

(b) $\dfrac{510\pi}{7}$

(c) $\dfrac{508\pi}{7}$

(d) $\dfrac{506\pi}{7}$

(e) $\dfrac{500\pi}{7}$

15.8
Exercise 21

28. The surface $\rho = \sin\theta \sin\phi$ represents

(a) a sphere with radius $\frac{1}{2}$ and center $\left(0, \frac{1}{2}, 0\right)$

(b) a sphere with radius $\frac{1}{4}$ and center $\left(0, \frac{1}{2}, 0\right)$

(c) a sphere with radius $\frac{1}{2}$ and center $(0, 1, 0)$

(d) a sphere with radius $\frac{1}{4}$ and center $(0, 1, 0)$

(e) a sphere with radius 1 and center $(0, 0, 0)$

15.8
Like Exer. 7

$$\rho = \sin\theta \sin\phi$$

$$\rho^2 = \rho \sin\theta \sin\phi$$

$$x^2 + y^2 + z^2 = y$$

$$x^2 + \left(y - \tfrac{1}{2}\right)^2 + z^2 = \left(\tfrac{1}{2}\right)^2 \rightarrow \text{sphere}$$