1. The area under one arch of the cycloid given by the parametric equations: $x = 2(\theta - \sin\theta)$ and $y = 2(1 - \cos\theta)$ is equal to

   (a) $12\pi$ (correct)

   (b) $6\pi$

   (c) $10\pi$

   (d) $9\pi$

   (e) $8\pi$

2. The polar coordinates $(r, \theta)$ of the point $(-6, 0)$ where $r < 0$ and $0 \leq \theta \leq 2\pi$ is

   (a) $(-6, 0)$ (correct)

   (b) $(-6, \pi)$

   (c) $(-5, 0)$

   (d) $(-5, 2\pi)$

   (e) $(6, \pi)$

3. The unit vector in the direction of $2\vec{u} - \vec{v}$, where $\vec{u} = \langle 1, 0, -1 \rangle$ and $\vec{v} = \langle 2, -1, 0 \rangle$ is:

(a) $\left\langle 0, \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}} \right\rangle$ (correct)

(b) $\left\langle \frac{1}{\sqrt{5}}, 0, -\frac{2}{\sqrt{5}} \right\rangle$

(c) $\left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right\rangle$

(d) $\left\langle -\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 0 \right\rangle$

(e) $\left\langle \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle$

4. If $\vec{a} = \langle 2, -3, 1 \rangle$, $\vec{b} = \langle \sqrt{2}, -2\sqrt{2}, -\sqrt{2} \rangle$, $\vec{c} = \langle -1, 1, -2 \rangle$ and $\vec{a} \times (\vec{b} \times \vec{c}) + (\vec{a} \cdot \vec{b})\vec{c} = \langle x, y, z \rangle$, then $x + y + z =$

(a) $14\sqrt{2}$ (correct)

(b) $7\sqrt{2}$

(c) $-14\sqrt{2}$

(d) $-7\sqrt{2}$

(e) 0

5. The line through the points $(-3, 1, 0)$ and $(-1, 5, 6)$ intersects the plane $x+y+z = 4$ at

   (a) $(-2, 3, 3)$ _____ (correct)
   (b) $(1, 2, 1)$
   (c) $(-1, 4, 1)$
   (d) $(0, 3, 1)$
   (e) $(4, 1, -1)$

6. $\lim_{(x,y)\to(0,0)} \dfrac{x^6 y}{x^6 + y^6} =$

   (a) 0 _____ (correct)
   (b) DNE
   (c) $\infty$
   (d) 1
   (e) 2

7. The tangent plane to the surface $z = 2x - y^2$ at the point $(2, 1, 3)$ contains the point

(a) $(-1, -2, -3)$ _____ (correct)
(b) $(0, 0, 3)$
(c) $(0, -1, 0)$
(d) $(1, 0, 6)$
(e) $(1, 0, 0)$

8. The rate of change of $f(x, y, z) = x^2yz - xyz^3$ at the point $(2, -1, 1)$ in the direction of $\vec{u} = \left\langle 0, \frac{4}{5}, -\frac{3}{5} \right\rangle$ is

(a) $\frac{2}{5}$ _____ (correct)
(b) $\frac{1}{5}$
(c) $\frac{4}{5}$
(d) $\frac{1}{4}$
(e) $\frac{3}{4}$

9. The shortest distance between the point $(1, 0, -1)$ and the surface $y^2 = 9 + xz$ is

(a) $\sqrt{7}$ _____(correct)
(b) $\sqrt{17}$
(c) $\sqrt{10}$
(d) 3
(e) 4

10. The function $f(x, y) = xy(1 - x - y)$ has

(a) one local maximum and three saddle points _____(correct)
(b) one local minimum and three saddle points
(c) one local maximum, one local minimum and two saddle points
(d) one local minimum and two saddle points
(e) one local maximum, one local minimum and three saddle points

11. If $M$ is the maximum value and $m$ is the minimum value of the function $f(x,y) = 2x + y^2$ on the region $D = \{(x,y)|x^2 + y^2 \leq 1\}$, then $M + m =$

(a) 0 (correct)

(b) 1

(c) 2

(d) $-1$

(e) 3

12. If the maximum value and the minimum value of the function $f(x,y) = y^2 - x^2$ on the ellipse $x^2 + 4y^2 = 4$ are $M$ and $m$ respectively, then $M - m =$

(a) 5 (correct)

(b) 4

(c) 6

(d) 3

(e) 0

13. The maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 1$ is

(a) $3 + \sqrt{5}$ (correct)

(b) $3 - \sqrt{5}$

(c) $3 + \frac{\sqrt{5}}{5}$

(d) $3 - \frac{\sqrt{5}}{5}$

(e) 4

14. An estimate of the volume of the solid that lies below the surface $z = 1 + x^2 + 2y$ and above the rectangle $R = [1, 3] \times [0, 3]$ using a Riemann sum with $m = n = 2$ and sample points to be the lower left corner of each rectangle is

(a) 30 (correct)

(b) 20

(c) 40

(d) 25

(e) 35

15. The volume of the solid that lies under the plane $4x + 6y - 2z + 15 = 0$ and above the rectangle

$$R = \{(x, y) | -1 \leq x \leq 2, -1 \leq y \leq 1\}$$

is

(a) 51 (correct)
(b) 45
(c) 52
(d) 49
(e) 56

16. $\int_{-1}^{1} \int_{0}^{1} x^6 \sin x \sqrt{1 + e^y}\, dy\, dx =$

(a) 0 (correct)
(b) 1
(c) $\sqrt{1 + e}$
(d) $\sqrt{1 + e} \sin 1$
(e) $-1$

17. $\int_0^1 \int_{x^2}^1 \sqrt{y} \sin(\pi y)\, dy\, dx =$

(a) $\frac{1}{\pi}$ _____ (correct)

(b) 0

(c) $\pi$

(d) $-\frac{1}{\pi}$

(e) $2\pi$

18. The volume of the solid bounded by

$$y^2 + z^2 = 4,\ x = 2y,\ x = 0, \text{ and } z = 0$$

in the first octant is

(a) $\frac{16}{3}$ _____ (correct)

(b) $\frac{8}{3}$

(c) $\frac{4}{3}$

(d) $\frac{3}{8}$

(e) $\frac{3}{2}$

19. The volume of the solid bounded by the plane $z = 0$ and the paraboloid $z = 1 - x^2 - y^2$ is

(a) $\frac{\pi}{2}$ _____ (correct)

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) $\pi$

(e) $2\pi$

20. The average value of the function $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$ on the region

$$R = \{(x, y) | 1 \leq x^2 + y^2 \leq 4\}$$

is

(a) $\frac{2}{3}$ _____ (correct)

(b) $\frac{3}{4}$

(c) $\frac{1}{2}$

(d) $\frac{3}{5}$

(e) $\frac{2}{5}$

21. $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}\, dy\, dx =$

(a) $\frac{\pi}{4}(1-e^{-4})$ (correct)

(b) $\frac{\pi}{4}(e^{-4}-1)$

(c) $\frac{\pi}{2}(1-e^{-4})$

(d) $\frac{\pi}{2}(e^{-4}-1)$

(e) $\frac{\pi}{2}(e^{4}-1)$

22. The volume of the solid enclosed by the cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 1$ equals

(a) $\frac{8}{15}$ (correct)

(b) $\frac{8}{25}$

(c) $\frac{7}{15}$

(d) $\frac{7}{25}$

(e) $\frac{13}{30}$

23. The volume of the solid enclosed by the paraboloids $y = x^2 + z^2$ and $y = 8 - x^2 - z^2$ is

(a) $16\,\pi$ (correct)

(b) $\dfrac{15\pi}{2}$

(c) $8\,\pi$

(d) $\dfrac{33\pi}{2}$

(e) 50

24. Let $E$ be the solid bounded by the parabolic cylinder $z = 1 - x^2$, the plane $y = 1 - x$ and the coordinate planes. The value of

$$\iiint_E x\,dV$$

is

(a) $\dfrac{7}{60}$ (correct)

(b) $\dfrac{7}{30}$

(c) $\dfrac{7}{15}$

(d) $\dfrac{7}{90}$

(e) $\dfrac{7}{20}$

25. Using cylindrical coordinates, we find that $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} x\,dz\,dy\,dx$ is equal to

(a) $\frac{64}{15}$ (correct)

(b) $\frac{32}{15}$

(c) $\frac{128}{15}$

(d) $\frac{256}{15}$

(e) 0

26. The volume of the solid that lies between the paraboloid $z = 16 - x^2 - y^2$ and the cone $z = 6\sqrt{x^2 + y^2}$ is equal to

(a) $24\,\pi$ (correct)

(b) $16\,\pi$

(c) $8\,\pi$

(d) $30\,\pi$

(e) $12\,\pi$

27. The integral $\iiint_E \sqrt{x^2+y^2+z^2}\,dV$ where $E$ is the region above $z=\sqrt{x^2+y^2}$ and between $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$, is equal to

(a) $\frac{15\pi}{2}\left(1-\frac{1}{\sqrt{2}}\right)$ _____(correct)

(b) $\frac{7\pi}{2\sqrt{2}}$

(c) $\frac{\pi}{2}(\sqrt{2}-1)$

(d) $\frac{15\pi}{4}\left(1+\frac{1}{\sqrt{2}}\right)$

(e) $\frac{\pi}{4}\left(1-\frac{1}{\sqrt{2}}\right)$

28. The surface whose equation is $\rho=\cos\phi$ is

(a) a sphere of radius $\frac{1}{2}$ _____(correct)

(b) a sphere of radius $\frac{1}{4}$

(c) a plane

(d) a cone centered at $\left(0,0,\frac{1}{2}\right)$

(e) a cone centered at $\left(\frac{1}{4},0,0\right)$