1. The equation of the tangent line to the curve represented by the parametric equations $x = t^4 + 2$, $y = t^3 + t$ at the point $(3, -2)$ is

(a) $y = 1 - x$ ________________________________ (correct)

(b) $y = 5 + x$

(c) $y = 7 + 2x$

(d) $2y + 3x = 0$

(e) $2y = -x + 4$

2. The arc length of the curve represented by the parametric equations

$$x = e^{-t} \cos t,\ y = e^{-t} \sin t,\ 0 \le t \le \frac{\pi}{2}$$

is

(a) $\sqrt{2}(1 - e^{-\frac{\pi}{2}})$ ________________________________ (correct)

(b) $2(1 + e^{-\frac{\pi}{2}})$

(c) $3(1 + e^{\frac{\pi}{2}})$

(d) $\sqrt{3}(1 + e^{\frac{\pi}{2}})$

(e) $2 - 2e^{-\frac{\pi}{2}}$

3. The area of the surface formed by revolving the curve represented by the equations

$$x = 2\cos t,\ y = 2\sin t,\ 0 \le t \le \frac{\pi}{3}$$

about the x-axis is

(a) 4π ________ (correct)

(b) 5π

(c) 6π

(d) 2π

(e) π

4. The slope of the tangent line to the polar curve $r = 3\sin\theta$ at the point $\left(\frac{3}{2}, \frac{\pi}{6}\right)$ is equal to

(a) $\sqrt{3}$ ________ (correct)

(b) $-\sqrt{3}$

(c) $\frac{1}{\sqrt{3}}$

(d) $\frac{-2}{\sqrt{3}}$

(e) $\frac{1}{2}$

5. The polar coordinates (r, θ) corresponding to the rectangular coordinates $(5, -5\sqrt{3})$ with $r < 0$ and $0 \leq \theta < 2\pi$ is

(a) $\left(-10, \frac{2\pi}{3}\right)$ ________________ (correct)

(b) $\left(-10, \frac{\pi}{3}\right)$

(c) $\left(-5, \frac{2\pi}{3}\right)$

(d) $\left(-5, \frac{\pi}{3}\right)$

(e) $\left(-10, \frac{5\pi}{3}\right)$

6. The area of the region that lies inside $r = 4\sin\theta$ and outside $r = 2$ is equal to

(a) $4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (4\sin^2\theta - 1)\, d\theta$ ________________ (correct)

(b) $8\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (4\sin^2\theta - 1)\, d\theta$

(c) $4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 - 4\sin^2\theta)\, d\theta$

(d) $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4\sin^2\theta - 1)\, d\theta$

(e) $8\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (1 - 4\sin^2\theta)\, d\theta$

7. The arc length of the polar curve $r = 2\cos\theta$, $0 \leq \theta \leq \frac{\pi}{2}$ is equal to

(a) π ______________________________ (correct)
(b) $\frac{3\pi}{2}$
(c) 2π
(d) $\frac{2\pi}{3}$
(e) 3π

8. A unit vector that is perpendicular to the graph of the function $f(x) = -x^2 + 5$ at the point $(1, 4)$ is

(a) $\left\langle -\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle$ ______________________________ (correct)
(b) $\left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$
(c) $\left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$
(d) $\left\langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \right\rangle$
(e) $\left\langle \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right\rangle$

9. The equation of the sphere that has a diameter with end points $(5,0,-1)$ and $(1,-4,1)$ is

(a) $(x-3)^2+(y+2)^2+z^2=9$ ________________ (correct)
(b) $(x+3)^2+(y+2)^2+z^2=9$
(c) $(x-3)+y^2+z^2=9$
(d) $(x-3)^2+(y-2)^2+z^2=9$
(e) $(x-3)+y^2+(z-1)^2=9$

10. If $\vec{w}$ is a vector with length 6 and has the same direction as $\vec{v}=<2,0,\sqrt{12}>$, then $\vec{w}=$

(a) $<3,0,3\sqrt{3}>$ ________________ (correct)
(b) $<12,0,12\sqrt{12}>$
(c) $<\sqrt{6},0,\sqrt{30}>$
(d) $<0,3,3\sqrt{3}>$
(e) $<3\sqrt{3},0,3>$

11. Let $\vec{u}$ and $\vec{v}$ be two vectors such that $\vec{u} \cdot \vec{v} = -6$ and $\vec{v} =< 1, 2, k >$.
If $\text{proj}_{\vec{v}}\vec{u} = -\vec{v}$ and $k > 0$, then $k =$

(a) 1 ________________ (correct)

(b) $\sqrt{2}$

(c) $\sqrt{3}$

(d) $\frac{1}{2}$

(e) $\frac{1}{3}$

12. The angle between the vectors $\vec{u} =< -2, 0, 1 >$ and $\vec{v} =< 4, 0, -2 >$ is equal to

(a) π ________________ (correct)

(b) 2π

(c) 3π

(d) $\frac{\pi}{2}$

(e) $\frac{\pi}{3}$

13. Let $\vec{u} = < 1, 2, -1 >$ and $\vec{v} = < -1, 1, k >$. If $\vec{u}$ is orthogonal to $2\vec{u} - \vec{v}$, then $k =$

(a) -11 ________________________________ (correct)
(b) -8
(c) 11
(d) 8
(e) -9

14. The area of the parallelogram with vectors $< 1, 2, 0 >$ and $< -1, -3, 4 >$ as adjacent sides is equal to

(a) 9 ________________________________ (correct)
(b) 8
(c) 7
(d) 10
(e) 6

15. If $\vec{u} =< a, 1, 0 >$, $\vec{v} =< 0, a, 1 >$, $\vec{w} =< 1, 0, a >$ are such that $\vec{u} \cdot (\vec{v} \times \vec{w}) = 9$, then $a =$

(a) 2 ____ (correct)

(b) 1

(c) 3

(d) 4

(e) 0

16. Suppose that the three vectors $\vec{u} = 3\vec{i} + c\vec{j} + \vec{k}$, $\vec{v} = 2\vec{j} - 2\vec{k}$, and $\vec{w} =< 3, 1, 1 >$ are coplanar then $c =$

(a) 1 ____ (correct)

(b) 2

(c) -1

(d) 0

(e) -2

17. Which of the following has the same graph as the graph of the parametric equations

$x = 2t,\ y = 4t - 3$?

(a) $x = \theta^3,\ y = 2\theta^3 - 3$ ______ (correct)
(b) $x = 2t^2,\ y = 4t^2 - 3$
(c) $x = 2\cos\theta,\ y = 4\cos\theta - 3$
(d) $x = e^t,\ y = 2e^t - 3$
(e) $x = \dfrac{2}{\theta},\ y = \dfrac{4}{\theta} - 3$

18. The parametric equations

$x = 3\csc\theta - 1,\ y = 4\cot\theta - 1$

represent

(a) A hyperbola ______ (correct)
(b) An ellipse
(c) A parabola
(d) A pair of intersecting lines
(e) A line

19. The area of the surface formed by revolving the right part of the cardioid $r = 1 + \sin(\theta)$, $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ about the line $\theta = \frac{\pi}{2}$ is

(a) $\frac{32\pi}{5}$ ____(correct)

(b) $\frac{16\pi}{5}$

(c) $\frac{8\pi}{5}$

(d) $\frac{4\pi}{5}$

(e) $\frac{2\pi}{5}$

20. Consider the following statements about vectors:
(I) If $\vec{u}$ and $\vec{v}$ have the same magnitude and direction, then $\vec{u}$ and $\vec{v}$ are equivalent.
(II) If $\vec{u}$ and $\vec{v}$ are non-zero vectors sharing the same direction, then $\vec{v} = \| \vec{v} \| \vec{u}$.
(III) If $a = b$, then $\| a\vec{i} + b\vec{j} \| = \sqrt{2}a$.
(IV) If $\vec{u}$ and $\vec{v}$ have the same magnitude but opposite directions, then $\vec{u} + \vec{v} = \vec{0}$.
Then

(a) I and IV are true ____(correct)

(b) I and II are true

(c) I and III are true

(d) II and IV are true

(e) Only I is true

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	C 2	C 16	A 12	A 7
2	A	A 17	A 20	B 15	E 5
3	A	E 8	D 2	E 2	D 12
4	A	B 19	E 10	D 8	E 1
5	A	B 9	D 3	D 9	C 16
6	A	E 18	D 19	B 4	A 3
7	A	B 15	A 14	D 14	C 10
8	A	E 12	C 4	E 18	B 19
9	A	E 6	C 8	B 6	E 11
10	A	C 5	D 18	A 3	A 8
11	A	E 14	E 15	A 17	A 18
12	A	E 11	B 13	A 10	B 13
13	A	E 10	A 17	A 19	A 4
14	A	B 16	C 6	D 7	B 9
15	A	A 20	E 5	B 5	D 14
16	A	D 7	C 9	D 1	D 6
17	A	D 13	A 12	A 11	E 2
18	A	D 3	A 7	B 16	C 15
19	A	E 1	D 1	C 13	D 17
20	A	D 4	B 11	B 20	E 20