King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 201

Major Exam I

231

October 04, 2023

Net Time Allowed: 120 Minutes

MASTER VERSION

1. Let C be the portion of the parametric curve $x = 3\cos t$ and $y = 3\sin t$ from the point $(3, 0)$ to $(\frac{3}{2}, \frac{3\sqrt{3}}{2})$. The area of the surface obtained by rotating C about the x-axis is

   (a) 9π
   (b) $\frac{2}{3}$
   (c) 3π
   (d) $\frac{3}{2}$
   (e) $\frac{4}{5}\pi$

2. If $\vec{a}$ and $\vec{b}$ are unit vectors in space and the angle between them is $\frac{2\pi}{3}$, then $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{a})$ is equal to

   (a) $-\frac{3}{4}$
   (b) $\frac{2}{3}$
   (c) -3
   (d) 0
   (e) $\frac{4}{5}$

3. If the equation $x^2 + y^2 + z^2 + 2x - 4y + 6z + 5 = 0$ represents a sphere with center (a, b, c) and radius r, then $a + b + c + r =$

(a) 1
(b) -2
(c) -3
(d) 0
(e) 2

4. A set of parametric equations for the rectangular equation $y = 2x - 5$ that represents the point $(3, 1)$ when $t = 0$ is

(a) $x = 3 - t, \quad y = 1 - 2t$
(b) $x = 3 + 2t, \quad y = 1 + 2t$
(c) $x = 3 - t, \quad y = 2 + t$
(d) $x = 3 + t, \quad y = 1 - t$
(e) $x = 3 - 2t, \quad y = 1 + t$

5. The parametric curve

$$x = t^2, \qquad y = t^3 - 3t$$

is concave upward on the interval

(a) $t \in (0, \infty)$
(b) $t \in (-\infty, \infty)$
(c) $t \in (-\infty, 1)$
(d) $t \in (-\infty, 0)$
(e) $t \in (-1, \infty)$

6. The slope of the tangent line to the curve

$$x = 2t - 1, \qquad y = t + t^2$$

at the point $(1, 2)$ is

(a) $\frac{3}{2}$
(b) 1
(c) $\frac{1}{3}$
(d) 0
(e) $-\frac{1}{2}$

7. If $\vec{u} = \langle 2, -4 \rangle$, $\vec{v} = \langle 2, -3 \rangle$ and a, b are scalars such that $a\vec{u} + b\vec{v} = \langle -4, 1 \rangle$, then $2a + b =$

(a) 3

(b) -2

(c) 2

(d) 0

(e) 1

8. The sum of all possible values of m such that the points $(0, 2, 1)$, $(m-1, 0, m)$, $(5, -m, 6)$ are collinear is equal to

(a) -1

(b) -2

(c) 2

(d) 0

(e) 3

9. The area of the triangle with vertices $A(0, 2, 2)$, $B(2, 0, -1)$, $C(3, 4, 0)$ is

(a) $\frac{15}{2}$
(b) $\frac{5}{4}$
(c) 5
(d) $\frac{13}{3}$
(e) 1

10. The slope of the tangent line to the cardioid $r = 1 + \sin\theta$ when $\theta = \frac{\pi}{3}$ is equal to

(a) -1
(b) -2
(c) 2
(d) 3
(e) $-\frac{1}{3}$

11. The area enclosed by one loop of the four-leaved rose $r = \cos 2\theta$ is equal to

(a) $\dfrac{\pi}{8}$
(b) 8π
(c) $\dfrac{2\pi}{3}$
(d) $\dfrac{5\pi}{8}$
(e) π

12. The exact length of the polar curve

$$r = 2\cos\theta, \qquad 0 \le \theta \le \pi$$

is equal to

(a) 2π
(b) 5π
(c) $\dfrac{3\pi}{2}$
(d) $\dfrac{5\pi}{7}$
(e) 3π

13. If the vectors $\vec{a} = \langle 2, 2, -1 \rangle$ and $\vec{b} = \langle 5, -4, m \rangle$ are orthogonal. Then $m =$

(a) 2
(b) -1
(c) 0
(d) 3
(e) -3

14. Consider the vectors $\vec{u} = \langle -3, 1, 2 \rangle$ and $\vec{v} = \langle 1, 2, -3 \rangle$. If the vector projetion of $\vec{u}$ onto $\vec{v}$ is $\langle a, b, c \rangle$, then $a + b + c =$

(a) 0
(b) -1
(c) 2
(d) 3
(e) 1

15. The area of the region that lies inside the polar curve $r = 2 + \sin\theta$ and outside the circle $r = 3\sin\theta$ is

(a) $\dfrac{9\pi}{4}$
(b) $\dfrac{4\pi}{3}$
(c) $\dfrac{5\pi}{2}$
(d) π
(e) 3π

16. The Cartesian equation of the parametric curve

$$x = \ln t \qquad y = \sqrt{t}, \qquad t \geq 1$$

is given by

(a) $y = e^{x/2}, \quad x \geq 0$
(b) $y = e^{x}, \quad x \geq 0$
(c) $y = e^{2x}, \quad x \geq 1$
(d) $y = e^{x}, \quad x \geq 1$
(e) $y = e^{2x}, \quad x \geq 0$

17. Consider the points $A(2, 1, -1)$, $B(3, 0, 2)$, $C(4, -2, -1)$ and $D(3, m, 0)$. If the volume of the parallelepiped determined by the vectors $\vec{AB}$, $\vec{AC}$, and $\vec{AD}$ is 4, then the **sum** of all possible values of m is

(a) $-\frac{2}{3}$
(b) $\frac{3}{2}$
(c) $-\frac{4}{3}$
(d) $\frac{3}{5}$
(e) $\frac{3}{7}$

18. The parametric curve

$$x = t^2 + 4t, \qquad y = 6t^2$$

has a vertical tangent line at the point

(a) $(-4, 24)$
(b) $(0, 0)$
(c) $(5, 6)$
(d) $(-3, 6)$
(e) $(2, -24)$

19. A vector $\vec{v}$ of length 3 that has the direction opposite to the vector $\vec{a} = \langle 1, 2, -3 \rangle$ is

(a) $\frac{1}{\sqrt{14}}\langle -3, -6, 9 \rangle$

(b) $\frac{1}{\sqrt{11}}\langle 3, 6, -9 \rangle$

(c) $\frac{1}{\sqrt{14}}\langle -1, -2, 9 \rangle$

(d) $\frac{1}{\sqrt{11}}\langle 1, 2, -3 \rangle$

(e) $\frac{1}{\sqrt{14}}\langle 2, 4, -6 \rangle$

20. If $\|\vec{u}\| = \sqrt{3}$, $\|\vec{v}\| = 2$ and $\vec{u} \cdot \vec{v} = \sqrt{6}$. The angle θ between the two vectors $\vec{u}$ and $\vec{v}$ is

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{3}$

(c) $\frac{3\pi}{2}$

(d) $\frac{\pi}{5}$

(e) $\frac{5\pi}{4}$