King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 201

Exam II

231

November 14, 2023

Net Time Allowed: 120 Minutes

MASTER VERSION

1. The area of the region that lies in the first quadrant as well as in the domain of the function $f(x, y) = \ln(6 - x - y)$ is equal to

   (a) 18 ____ (correct)

   (b) 30

   (c) 23

   (d) 12

   (e) 34

2. The level surfaces of $f(x, y, z) = \ln(4x^2 + 4y^2 - z^2)$ are

   (a) hyperboloids of one sheet ________________________________ (correct)

   (b) ellipsoids

   (c) elliptic cones

   (d) hyperbolic paraboloids

   (e) hyperboloids of two sheets

3. The range of

$$f(x, y) = 4\ln(3 - 2x^2 - y^2)$$

is

(a) $(-\infty, 4\ln 3]$ ________________ (correct)
(b) $(-1, 1)$
(c) $(-\ln 4, 0]$
(d) $(-\infty, \infty)$
(e) $[-1, \ln 4)$

4. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2} - 1}{x^2 + y^2} =$

(a) -1 ________________ (correct)
(b) 3
(c) -2
(d) 0
(e) does not exist

5. If the maximum increase of the function $f(x,y) = x^2 + y^2 - 4x - 2y$ at the point $(5, c)$ occurs in the direction of $\vec{i} + 2\vec{j}$, then $c =$

(a) 7 ____________________ (correct)

(b) 0

(c) 4

(d) 5

(e) 3

6. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2} =$

(a) does not exist ____________________ (correct)

(b) 1

(c) 0

(d) -1

(e) ∞

7. Let $z = \tan^{-1}(xy)$ where $x = s + \sin t$ and $y = \cos t$. Then $\dfrac{\partial z}{\partial s} + \dfrac{\partial z}{\partial t}$ at $(s,t) = (1,0)$ is equal to

(a) 1 ____________________ (correct)
(b) -4
(c) -2
(d) 8
(e) 0

8. Let

$$f(x,y) = \begin{cases} \dfrac{\sin(xy)}{xy}, & xy \neq 0 \\ c, & xy = 0 \end{cases}$$

The value of c that makes $f(x,y)$ continuous at $(0,0)$ is

(a) 1 ____________________ (correct)
(b) -1
(c) 2
(d) -4
(e) 0

9. The normal line to the surface

$$\ln\left(\frac{x}{y-z}\right) = x - 1$$

at the point $(1,4,3)$ passes through the point

(a) $(1,3,4)$ ________________ (correct)

(b) $(0,3,3)$

(c) $(1,2,1)$

(d) $(1,2,3)$

(e) $(2,3,3)$

10. If the radius and height of a right circular cone are measured as 10 cm and 25 cm, respectively, with a possible error in measurement of as much as 0.1 cm in each. The maximum error in the calculated volume of the cone is estimated as (**Hint:** The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$).

(a) $20\,\pi$ ________________ (correct)

(b) $2\,\pi$

(c) $10\,\pi$

(d) $4\,\pi$

(e) $12\,\pi$

11. Let $f(x, y, z) = xe^{xy} \sin^2 z$. Then $f_{xyz}(-1, 0, \pi/4) =$

(a) -2 ________________ (correct)
(b) 1
(c) 0
(d) 4
(e) -1

12. Let $W(s, t) = F(u(s, t), v(s, t))$, where F, u and v are differentiable functions, and

$$F_u(2, -3) = 1, \quad F_v(2, -3) = 3, \quad u(1, 0) = 2, \quad v(1, 0) = -3,$$

$$u_s(1, 0) = -2, \quad v_s(1, 0) = 5, \quad u_t(1, 0) = -6, \quad v_t(1, 0) = 4,$$

Then $W_s(1, 0) + W_t(1, 0) =$

(a) 19 ________________ (correct)
(b) 22
(c) 0
(d) 20
(e) 11

13. If $2yz + x\ln y = z^2$, then $\dfrac{\partial z}{\partial y}$ at $(2,1,2)$ equals

(a) 3 ________________________________ (correct)

(b) 5

(c) 0

(d) 4

(e) 1

14. The distance between the parallel planes

$$10x + 2y - 2z = 5 \quad \text{and} \quad 5x + y - z = 1$$

is equal to

(a) $\dfrac{\sqrt{3}}{6}$ ________________________________ (correct)

(b) $\sqrt{7}$

(c) $\dfrac{1}{\sqrt{3}}$

(d) $\sqrt{13}$

(e) $\dfrac{1}{\sqrt{2}}$

15. If the directional derivative of $f(x, y) = \ln(1 + xy)$ at $(a, 4)$ in the direction of $\vec{v} = \langle 1, -2 \rangle$ is 0, then $a =$

(a) 2 ________________ (correct)

(b) -1

(c) 0

(d) 4

(e) 1

16. If the line that passes through the points $P(1, 3, 4)$ and $Q(0, 5, 7)$ passes also through the point $R(2, 1, a)$, then $a =$

(a) 1 ________________ (correct)

(b) 3

(c) 2

(d) -2

(e) 5

17. If the equation of the plane containing the point $(1, 2, 3)$ and passing through the line of intersection of the planes

$$x + y + z = 1, \qquad 2x - y + 2z = 2$$

is given by $2x + ay + 2z = b$, then $a + b =$

(a) -1 ________________ (correct)
(b) 2
(c) 0
(d) 3
(e) -2

18. Consider the surface

$$x^2 - 3y^2 - 9z^2 = 0.$$

Which of the following is/are correct?
(I) The vertical trace in the xz-plane is the lines $x = \pm 3z$.
(II) The traces in the plane parallel to the yz-plane are ellipses.
(III) The surface represents a hyperboloid of two sheets.

(a) (I) and (II) only ________________ (correct)
(b) (I) only
(c) (II) only
(d) (II) and (III) only
(e) (III) only

19. If the line with symmetric equations

$$x - 2 = z - 1, \qquad y = 1$$

is perpendicular to the surface $3x^2 - 2xy + z^2 = 1$ at the point (x_0, y_0, z_0), then $x_0 + y_0 + z_0 =$

(a) 0 ____ (correct)

(b) 2

(c) 1

(d) 3

(e) 4

20. Let $w = \sqrt{x^2 + y^2 + z^2}$, where

$$x = \cos\theta, \quad y = \sin\theta, \quad z = \tan\theta.$$

Then $\dfrac{dw}{d\theta}$ at $\theta = \dfrac{\pi}{4}$ is equal to

(a) $\sqrt{2}$ ____ (correct)

(b) $\sqrt{3}$

(c) 1

(d) $3\sqrt{2}$

(e) $2\sqrt{3}$