#33
§10.2

1. The curve represented by the parametric equations

$$x = e^{-t},\ y = e^{3t},\ t \geq 0$$

is represented by the rectangular equation

(a) $y = \dfrac{1}{x^3}$, $0 < x \leq 1$ ________ (correct)

(b) $y = \dfrac{1}{x^3}$, $x \geq 1$

(c) $y = \dfrac{1}{\sqrt[3]{x}}$, $x \geq 1$

(d) $y = x^3$, $0 < x \leq 1$

(e) $y = \dfrac{1}{3x}$, $x \geq 1$

• $y = (e^{-t})^{-3} = x^{-3} = \dfrac{1}{x^3}$

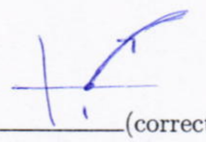
• $x = e^{-t}$

$t \geq \infty \Rightarrow 0 < x \leq 1$

~ #5 → 22
§10.2

2. Which set of the following parametric equations represents a parabola or part of a parabola ?

(a) $x = t + 1,\ y = \sqrt{t},\ t \geq 0$ ________ (correct) $t = y^2 \Rightarrow x = y^2 + 1$



(b) $x = 2t - 1,\ y = 2t + 1,\ t \in (-\infty, \infty)$ $\rightarrow y = x + 2$, line

(c) $x = 1 + \cos t,\ y = \sin t,\ t \in [0, 2\pi]$ $(x-1)^2 + y^2 = 1$, a Circle

(d) $x = \dfrac{1}{t},\ y = t - 1,\ t \in (0, \infty)$ $\longrightarrow y = \dfrac{1}{x} - 1$

(e) $x = \sec t,\ y = \cos t,\ t \in \left[0, \dfrac{\pi}{2}\right)$ $\longrightarrow y = \dfrac{1}{x}$

#21 (C)
§10.3

3. An **equation of the tangent line** to the parametric curve

$$x = t^2 - 4,\ y = t^2 - 2t$$

at the point $(x, y) = (-3, 3)$ is given by

$x = -3 \Rightarrow t^2 - 4 = -3 \Rightarrow t^2 = 1 \Rightarrow t = \pm 1$

• $t = 1 \Rightarrow y = 1 - 2 = -1 \neq 3$

• $t = -1 \Rightarrow y = 1 + 2 = 3$ ✓

(a) $y = 2x + 9$ ______ (correct)
(b) $y = 3$
(c) $y = x + 6$
(d) $y = -2x - 3$
(e) $x = -3$

So $t = -1$.

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-2}{2t}$

slope $= \left.\frac{dy}{dx}\right|_{t=-1} = \frac{-2-2}{-2} = \frac{-4}{-2} = 2$

Eq: $y - 3 = 2(x+3)$

$\Rightarrow y = 2x + 6 + 3$

$\Rightarrow y = 2x + 9$

~#44
§10.3

4. The curve given by the parametric equations

$$x = 1 + t^2,\ y = 1 + t^2 + t^3$$

is **concave upward** when

• $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t + 3t^2}{2t} = 1 + \frac{3}{2}t = y'$

(a) $t > 0$ ______ (correct)
(b) $t < 0$
(c) $-\infty < t < \infty$
(d) $-1 < t < 1$
(e) $t > -2$

• $\frac{d^2y}{dx^2} = \frac{dy'}{dx} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}} = \frac{\frac{3}{2}}{2t} = \frac{3}{4t}$

$-$ 0 $+$ → t

∩ 0 ∪

Concave upward when $t > 0$

~ #71

§10.3

5. The **surface area** of the surface generated by revolving the parametric curve

$$x = \cos^2 t,\ y = 2\cos t,\ 0 \le t \le \frac{\pi}{2}$$

about the x-axis is given by

$\int_0^{\pi/2} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\int_0^{\pi/2} 2\pi \cdot 2\cos t \sqrt{(-2\cos t \sin t)^2 + (-2\sin t)^2}\, dt$

$\int_0^{\pi/2} 4\pi \cos t \sqrt{4\cos^2 t \sin^2 t + 4\sin^2 t}\, dt$

(a) $\int_0^{\frac{\pi}{2}} 4\pi \sin(2t) \cdot \sqrt{1+\cos^2 t}\, dt$ ________________(correct)

$\int_0^{\pi/2} 4\pi \cos t \cdot \sqrt{4\sin^2 t(\cos^2 t + 1)}\, dt$

(b) $\int_0^{\frac{\pi}{2}} 8\pi \cos t \cdot \sqrt{1+\cos^2 t}\, dt$

$\int_0^{\pi/2} 4\pi \underbrace{\cos t \cdot 2\sin t}\sqrt{\cos^2 t + 1}\, dt$

(c) $\int_0^{\frac{\pi}{2}} 2\pi \cos(2t) \cdot \sqrt{1+\cos^2 t}\, dt$

$\int_0^{\pi/2} 4\pi \sin(2t) \cdot \sqrt{1+\cos^2 t}\, dt$

(d) $\int_0^{\frac{\pi}{2}} 8\pi \sin t \cdot \sqrt{1+\cos^2 t}\, dt$

(e) $\int_0^{\frac{\pi}{2}} 2\pi \cos^2 t \cdot \sqrt{1+\cos^2 t}\, dt$

~ #36

§10.3

6. Which one of the following statements is **TRUE** about the graph given by the parametric equations

$$x = t^2 - t,\ y = 2t^3 - 6t + 5$$

$\frac{dx}{dt} = 2t - 1$; $\frac{dy}{dt} = 6t^2 - 6 = 6(t-1)(t+1)$

• $\frac{dx}{dt} = 0 \Rightarrow t = \frac{1}{2}$

$\left.\frac{dy}{dt}\right|_{t=\frac{1}{2}} = 6(-\frac{1}{2})(\frac{3}{2}) \neq 0$

$\Rightarrow$ V.T. when $t = \frac{1}{2}$

(a) It has two horizontal tangents and one vertical tangent ________________(correct)

(b) It has one horizontal tangent and one vertical tangent

(c) It has no horizontal tangent and two vertical tangents

(d) It has one horizontal tangent and no vertical tangent

(e) It has one horizontal tangent and two vertical tangents

• $\frac{dy}{dt} = 0 \Rightarrow t = \pm 1$

$\left.\frac{dx}{dt}\right|_{t=\pm 1} = 2(\pm 1) - 1 \neq 0$

$\Rightarrow$ 2 H.T.

#7
§10.4

7. If a point has polar coordinates $(r, \theta) = \left(-4, -\frac{3\pi}{4}\right)$, then the corresponding rectangular coordinates for the point are

(a) $(x, y) = (2\sqrt{2},\, 2\sqrt{2})$ ______________________________ (correct)
(b) $(x, y) = (2\sqrt{2},\, -2\sqrt{2})$
(c) $(x, y) = (-2\sqrt{2},\, 2\sqrt{2})$
(d) $(x, y) = (-2\sqrt{2},\, -2\sqrt{2})$
(e) $(x, y) = (\sqrt{2},\, \sqrt{2})$

$x = r\cos\theta = -4\cos(-\frac{3\pi}{4})$
$= -4 \cdot -\frac{\sqrt{2}}{2} = 2\sqrt{2}$
$y = r\sin\theta = -4\sin(-\frac{3\pi}{4})$
$= -4 \cdot -\frac{\sqrt{2}}{2} = 2\sqrt{2}$

$(x, y) = (2\sqrt{2}, 2\sqrt{2})$

~ #88
§10.4

8. The graph of the polar equation $r = \dfrac{6}{\sin\theta - 3\cos\theta}$ is

(a) a line ______________________________ (correct)
(b) an ellipse
(c) a circle
(d) a parabola
(e) a hyperbola

$\Rightarrow r\sin\theta - 3r\cos\theta = 6$
$\Rightarrow y - 3x = 6$
$\Rightarrow y = 3x + 6$, a line

9. The **slope** of the tangent line to the polar graph $r = 3 - 3\cos\theta$ at the point corresponding to $\theta = \frac{\pi}{4}$ is

$f'(\theta) = 3\sin\theta$

~# 65

§10.4

$$\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

(a) $\frac{\sqrt{2}}{2-\sqrt{2}}$ (correct)

(b) -1

(c) $-\frac{1}{\sqrt{2}}$

(d) $\frac{\sqrt{2}}{1-\sqrt{2}}$

(e) $\frac{-\sqrt{2}}{2+\sqrt{2}}$

$$= \frac{3\sin\theta\cdot\sin\theta + (3-3\cos\theta)\cos\theta}{3\sin\theta\cdot\cos\theta - (3-3\cos\theta)\cdot\sin\theta}$$

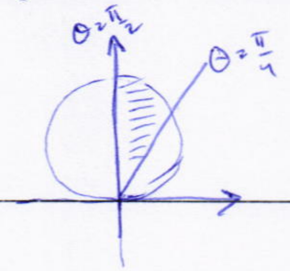
$$\text{slope} = \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{3\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} + \left(3-\frac{3\sqrt{2}}{2}\right)\frac{\sqrt{2}}{2}}{3\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} - \left(3-\frac{3\sqrt{2}}{2}\right)\frac{\sqrt{2}}{2}}$$

$$= \frac{\frac{3}{2} + 3\frac{\sqrt{2}}{2} - \frac{3}{2}}{\frac{3}{2} - \frac{3\sqrt{2}}{2} + \frac{3}{2}} = \frac{\frac{3\sqrt{2}}{2}}{3 - \frac{3\sqrt{2}}{2}}$$

$$= \frac{3\sqrt{2}}{6-3\sqrt{2}} = \frac{\sqrt{2}}{2-\sqrt{2}}$$

$\sim$ #3
§10.5

11. The **area** of the region inside the graph of the polar equation $r = 2\sin\theta$ for $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$ is equal to



(a) $\frac{\pi}{4} + \frac{1}{2}$ ______ (correct)

(b) $\frac{\pi}{2} + \frac{1}{2}$

(c) $\frac{\pi}{2} - \frac{1}{2}$

(d) $\frac{\pi}{4} + 1$

(e) $\frac{3\pi}{4} - \frac{1}{2}$

$$A = \int_{\pi/4}^{\pi/2} \frac{1}{2} r^2 \, d\theta$$

$$= \int_{\pi/4}^{\pi/2} 2\sin^2\theta \, d\theta = \int_{\pi/4}^{\pi/2} [1 - \cos(2\theta)] \, d\theta$$

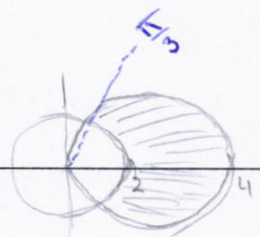
$$= \theta - \frac{1}{2}\sin(2\theta)\Big]_{\pi/4}^{\pi/2}$$

$$= \left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$= \frac{\pi}{4} + \frac{1}{2}$$

$\sim$ #43
§10.5

12. The **area** of the region inside the polar curve $r = 4\cos\theta$ and outside the polar curve $r = 2$ is equal to



pts of intersection:
$4\cos\theta = 2 \Rightarrow \cos\theta = \frac{1}{2}$
$\Rightarrow \theta = \frac{\pi}{3} \in 1^{st}$ quadrant

(a) $\frac{4\pi + 6\sqrt{3}}{3}$ ______ (correct)

(b) $\frac{4\pi + 5\sqrt{3}}{3}$

(c) $\pi + \frac{\sqrt{3}}{2}$

(d) $\frac{2\pi - \sqrt{3}}{6}$

(e) $\frac{3\pi + \sqrt{3}}{6}$

By symmetry about the polar axis

$$A = 2 \cdot \int_0^{\pi/3} \frac{1}{2}\left[(4\cos\theta)^2 - (2)^2\right] d\theta$$

$$= \int_0^{\pi/3} (16\cos^2\theta - 4) \, d\theta$$

$$= 4\int_0^{\pi/3} 4\,\frac{1 + \cos(2\theta)}{2} - 1 \, d\theta$$

$$= 4\int_0^{\pi/3} 1 + 2\cos(2\theta) \, d\theta$$

$$= 4\left[\theta + \sin(2\theta)\right]_0^{\pi/3}$$

$$= 4\left[\frac{\pi}{3} + \sin\left(\frac{2\pi}{3}\right)\right] = 4\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$$

$$= 4 \cdot \frac{2\pi + 3\sqrt{3}}{6} = 2 \cdot \frac{2\pi + 3\sqrt{3}}{3} = \frac{4\pi + 6\sqrt{3}}{3}$$

~ Example 4

§ 10.5

13. The **length** of the polar curve

$$r = 2 - 2\cos\theta,\ 0 \le \theta \le \frac{\pi}{3}$$

is equal to

$$L = \int_0^{\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

- $r^2 = 4 - 8\cos\theta + 4\cos^2\theta$
- $\left(\frac{dr}{d\theta}\right)^2 = (2\sin\theta)^2 = 4\sin^2\theta$
- $r^2 + \left(\frac{dr}{d\theta}\right)^2 = 4 - 8\cos\theta + 4 = 8 - 8\cos\theta = 8(1-\cos\theta)$

$$L = \sqrt{8}\int_0^{\pi/3} \sqrt{1-\cos\theta}\, d\theta$$

$$= \sqrt{8}\int_0^{\pi/3} \sqrt{2\sin^2\left(\frac{\theta}{2}\right)}\, d\theta$$

$$= \sqrt{8}\int_0^{\pi/3} \sqrt{2}\cdot\sin\left(\frac{\theta}{2}\right), \quad \sin\left(\frac{\theta}{2}\right) > 0 \text{ on } \left[0, \frac{\pi}{3}\right]$$

$$= 4 \cdot -2\cos\left(\frac{\theta}{2}\right)\Big]_0^{\pi/3}$$

$$= -8\left(\cos\frac{\pi}{6} - 1\right) = -8\left(\frac{\sqrt{3}}{2} - 1\right) = 8 - 4\sqrt{3}$$

(a) $8 - 4\sqrt{3}$ ________________________________ (correct)

(b) $2 - 2\sqrt{3}$

(c) $4 + 2\sqrt{3}$

(d) $6 - \sqrt{3}$

(e) $4 + \sqrt{3}$

#

§ 11.2

14. The center C and the radius r of the sphere

$$x^2 + y^2 + z^2 - 2x + 6y + 8z + 1 = 0$$

are

$$x^2 - 2x + 1 + y^2 + 6y + 9 + z^2 + 8z + 16 = -1 + 1 + 9 + 16$$

$$(x-1)^2 + (y+3)^2 + (z+4)^2 = 25$$

(a) $C(1, -3, -4)$, $r = 5$ ________________________________ (correct)

(b) $C(-1, 3, 4)$, $r = 5$

(c) $C(-2, 6, 8)$, $r = 4$

(d) $C(1, 3, -4)$, $r = 3$

(e) $C(-1, -3, 4)$, $r = 5$

$C(1, -3, -4)$

$r = 5$

15. The vector with initial point $P(1,1,-3)$ and terminal point $Q(3,4,-4)$ is **parallel** to the vector

~ #63

§11.2

(a) $\left\langle -\frac{2}{3}, -1, \frac{1}{3} \right\rangle$ ______ (correct)

(b) $\langle 3,4,-4 \rangle$

(c) $\langle 1,1,-3 \rangle$

(d) $\langle 2,-9,3 \rangle$

(e) $\left\langle 1, \frac{3}{2}, -1 \right\rangle$

$\vec{PQ} = \langle 3-1, 4-1, -4+3 \rangle$

$= \langle 2, 3, -1 \rangle$

$-3 \langle -\frac{2}{3}, -1, \frac{1}{3} \rangle = \vec{PQ}$

#41

§11.3

16. The **projection** of $\vec{u} = < 0,3,3 >$ onto $\vec{v} = < -1,1,1 >$ is

(a) $< -2,2,2 >$ ______ (correct)

(b) $< 2,-2,2 >$

(c) $< -3,3,3 >$

(d) $< 0,1,4 >$

(e) $< -1,4,4 >$

$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v}$; $\vec{u} \cdot \vec{v} = 0 + 3 + 3 = 6$

$\|\vec{v}\|^2 = 1 + 1 + 1 = 3$

$= \frac{6}{3} \langle -1, 1, 1 \rangle$

$= 2 \langle -1, 1, 1 \rangle$

$= \langle -2, 2, 2 \rangle$

17. The **area of the parallelogram** that has the vectors $\vec{u} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{v} = \vec{j} + \vec{k}$ as adjacent sides is equal to

#20
§ 11.4

(a) $\sqrt{2}$ ______ (correct)
(b) 2
(c) $2\sqrt{2}$
(d) $\dfrac{\sqrt{2}}{2}$
(e) $3\sqrt{2}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$= 0i - j + k$

$= \langle 0, -1, 1 \rangle$

area $= \| \vec{u} \times \vec{v} \| = \sqrt{0+1+1} = \sqrt{2}$

~ #69
§ 11.1

18. Which one of the following vectors is **perpendicular** to the tangent line to the graph of $f(x) = x^4 - 2x$ at the point $(1, -1)$?

(a) $< 2, -1 >$ ______ (correct)
(b) $< 2, 1 >$
(c) $< -1, 1 >$
(d) $< -2, 4 >$
(e) $< 1, 2 >$

$f'(x) = 4x^3 - 2$

slope of tangent line is $f'(1) = 4 - 2 = 2$

slope of a line perpendicular to the tangent line is $-\frac{1}{2}$.

a vector perpendicular to the tangent line is $\langle 1, -\frac{1}{2} \rangle$

or $\langle 2, -1 \rangle$

19. If the vectors $\vec{u} = <1, a, b>$, $\vec{v} = <a, b, 1>$, and $\vec{w} = <b, 1, a>$ lie in the **same plane**, then

Def^n of "lie in The same plane"

(a) $a^3 + b^3 = 3ab - 1$ ________________ (correct)

(b) $a^3 - b^3 = 2ab$

(c) $a = b$

(d) $a^2 + b^2 = 3ab + 1$

(e) $a^3 + b^3 = ab - 2$

The vectors lie in The Same plane if and only if

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = 0$$

$$\begin{vmatrix} 1 & a & b \\ a & b & 1 \\ b & 1 & a \end{vmatrix} = 0$$

$$1(ab-1) - a(a^2-b) + b(a-b^2) = 0$$

$$\Longleftrightarrow \quad ab - 1 - a^3 + ab + ab - b^3 = 0$$

$$\Longleftrightarrow \quad 3ab - 1 = a^3 + b^3$$

~ #89

§11.2 20. If $R(a, b, c)$ is the point that lies **two-thirds** of the way from $P(1, 0, 1)$ to $Q(0, 2, 0)$, then $a + b + c =$

(a) 2 ________________ (correct)

(b) $\frac{4}{3}$

(c) $-\frac{2}{3}$

(d) -3

(e) 4

$\vec{PR} \parallel \vec{PQ}$ and $\|\vec{PR}\| = \frac{2}{3}\|\vec{PQ}\|$

$$\Longleftrightarrow \quad \vec{PR} = \frac{2}{3}\vec{PQ}$$

$$<a-1, b-0, c-1> = \frac{2}{3}<0-1, 2-0, 0-1>$$

$$\Rightarrow \begin{cases} a - 1 = -\frac{2}{3} \\ b - 0 = \frac{4}{3} \\ c - 1 = -\frac{2}{3} \end{cases} \Rightarrow \begin{cases} a = \frac{1}{3} \\ b = \frac{4}{3} \\ c = \frac{1}{3} \end{cases}$$

$$a + b + c = \frac{1}{3} + \frac{4}{3} + \frac{1}{3} = \frac{6}{3} = 2$$