~ #15
§ 11.5

1. A set of **parametric equations** for the line passing through the points $P(2, 0, 1)$ and $Q(1, -1, 2)$ is given by

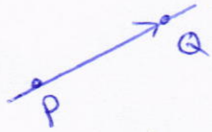
(a) $x = 2 - t$, $y = -t$, $z = 1 + t$ ______________________________(correct)

(b) $x = 2 - t$, $y = -2t$, $z = 1 + t$

(c) $x = 2 + t$, $y = -t$, $z = 1 - t$

(d) $x = 2 - 2t$, $y = 1 - t$, $z = 1 + t$

(e) $x = -t$, $y = 2 - t$, $z = t$



- $P(2,0,1)$
- $\overrightarrow{PQ} = \langle 1-2, -1-0, 2-1 \rangle$
  $= \langle -1, -1, 1 \rangle$
  $=$ a vector parallel to the line

para. eq:

$x = 2 - t$
$y = 0 - t$, $t \in \mathbb{R}$
$z = 1 + t$

Example 2
§11.6

2. The **graph** of the equation

$$4x^2 - 3y^2 + 12z^2 + 12 = 0 \Rightarrow -4x^2 + 3y^2 - 12z^2 = 12$$

is $\Rightarrow -\frac{x^2}{3} + \frac{y^2}{4} - z^2 = 1$

2 negative signs $\Rightarrow$ a hyperboloid of two sheets

(a) a hyperboloid of two sheets ______________________________(correct)

(b) a hyperboloid of one sheet

(c) an elliptic cone

(d) an ellipsoid

(e) an elliptic paraboloid

\# 90
§11.5

3. The **distance** between the point $(1, 3, -1)$ and the plane $3x - 4y + 5z = 6$ is equal to

(a) $2\sqrt{2}$ ____(correct)

(b) $4\sqrt{2}$

(c) $6\sqrt{2}$

(d) $\frac{\sqrt{2}}{3}$

(e) $\frac{\sqrt{2}}{4}$

$$d = \frac{|3(1) - 4(3) + 5(-1) - 6|}{\sqrt{3^2 + (-4)^2 + 5^2}}$$

$$= \frac{|3 - 12 - 5 - 6|}{\sqrt{9 + 16 + 25}} = \frac{|3 - 23|}{\sqrt{50}} = \frac{20}{5\sqrt{2}}$$

$$= \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 2\sqrt{2}$$

~
\#52
§11.5

4. If $ax + by + cz = 4$ is an equation for the plane that passes through the point $\overset{Q}{(2, 2, 1)}$ and contains the line $x = y - 4 = z$, then $abc =$

→ a point on the line: $P(0, 4, 0)$
a vector parallel to the line: $\langle 1, 1, 1 \rangle = \vec{v}$

(a) -12 ____(correct)

(b) -4

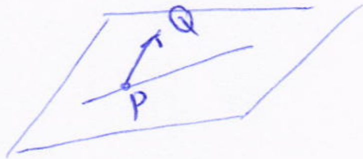
(c) 3

(d) 10

(e) -15

$\vec{PQ} = \langle 2, -2, 1 \rangle$

$$\vec{PQ} \times \vec{v} = \begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$



$$= \langle -3, -1, 4 \rangle$$

with $Q(2,2,1)$, equation of the plane is

$$-3(x-2) - (y-2) + 4(z-1) = 0$$

$$\Rightarrow -3x + 6 - y + 2 + 4z - 4 = 0$$

$$\Rightarrow -3x - y + 4z = -4$$

$$\Rightarrow 3x + y - 4z = 4$$

$$a = 3, \quad b = 1, \quad z = -4$$

$$\Rightarrow abc = -12$$

\# 36
§ 13.1

5. The domain D and the range R of the function

$$f(x, y) = 6 - 2x - 3y$$

are

the graph is a general plane (neither horizontal nor vertical)

(a) $D = \{(x, y) : -\infty < x < \infty,\ -\infty < y < \infty\},\ R = (-\infty, \infty)$ __________(correct)

(b) $D = \{(x, y) : -\infty < x < \infty,\ -\infty < y < \infty\},\ R = (-\infty, 6)$

(c) $D = \{(x, y) : -\infty < x < \infty,\ -\infty < y < \infty\},\ R = (6, \infty)$

(d) $D = \{(x, y) : x \geq 0,\ y \geq 0\},\ R = [0, \infty)$

(e) $D = \{(x, y) : x \geq 0,\ y \leq 0\},\ R = (-\infty, 0)$

\# 12, Review ch 13
p. 964

§ 13.1

6. The **level surface** of

$$f(x, y, z) = 4x^2 - y^2 + z^2$$

that passes through the point $(0, 1, 1)$ is

$f(x,y,z) = C$

$4x^2 - y^2 + z^2 = C$

sub. $(0,1,1)$

$0 - 1 + 1 = C \Rightarrow C = 0$

The equation of the level surface is

$4x^2 - y^2 + z^2 = 0$

$\Rightarrow y^2 = 4x^2 + z^2$

an elliptic cone

(a) an elliptic cone ____________________(correct)

(b) an elliptic paraboloid

(c) a hyperbolic paraboloid

(d) a hyperboloid of one sheet

(e) a hyperboloid of two sheets

~ #46
§ 13.2

7. $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x-y^2}{x^3+y}$

(a) does not exist ________ (correct)

(b) is equal to 2

(c) is equal to -1

(d) is equal to 0

(e) is equal to 1

, along the y-axis : $x=0$

$$\lim_{\substack{(x,y)\to(0,0)\\ x=0}} \frac{2x-y^2}{x^3+y} = \lim_{y\to 0} \frac{0-y^2}{0+y} = \lim_{y\to 0} -y = 0$$

, along the line $y=x$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=x}} \frac{2x-y^2}{x^3+y} = \lim_{x\to 0} \frac{2x-x^2}{x^3+x}$$

$$= \lim_{x\to 0} \frac{2-x}{x^2+1} = \frac{2-0}{0+1} = 2$$

Since the limit along the two paths are not equal, then the given limit does not exist.

~ #58
§ 13.2

8. $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(2x^2+2y^2)}{4x^2+4y^2} = \lim\limits_{r\to 0^+} \dfrac{\sin(2r^2)}{4r^2}$

$$\overset{L'R}{=} \lim_{r\to 0^+} \frac{\cos(2r^2)\cdot 4r}{8r}$$

(a) $\frac{1}{2}$ ________ (correct)

(b) 1

(c) 0

(d) 2

(e) $\frac{1}{4}$

$$= \lim_{r\to 0^+} \frac{\cos(2r^2)}{2}$$

$$= \frac{\cos(0)}{2} = \frac{1}{2}$$

#49(b)
§13.2

9. Which one of the following statements is **True** about the function

$$f(x,y) = \begin{cases} \dfrac{x^4 - y^4}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 1 & (x,y) = (0,0) \end{cases}$$

$\lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 - y^4}{x^2+y^2}$

(a) f is not continuous at $(0,0)$ ______ (correct)

(b) f is continuous at $(0,0)$

(c) $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist

(d) $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 1$

(e) $\lim\limits_{(x,y)\to(0,0)} f(x,y) = -1$

$= \lim\limits_{(x,y)\to(0,0)} (x^2 - y^2)$

$= 0 - 0$

$= 0 \neq 1 = f(0,0)$

so f is not continuous at $(0,0)$

~ #35
§13.3

10. If $f(x,y) = e^{-2y}\sin(\pi xy)$, then $f_y\left(1, \dfrac{1}{2}\right) =$

(a) $-\dfrac{2}{e}$ ______ (correct)

(b) $\dfrac{1}{e}$

(c) π

(d) $-\dfrac{\pi}{2}$

(e) $e\pi$

$f_y(x,y) = e^{-2y} \cdot \cos(\pi xy) \cdot \pi x + \sin(\pi xy) \cdot -2e^{-2y}$

$\Rightarrow f_y(1, \tfrac{1}{2}) = e^{-1} \underbrace{\cos(\tfrac{\pi}{2})}_{=0} \cdot \pi + \underbrace{\sin(\tfrac{\pi}{2})}_{=1} \cdot -2e^{-1}$

$= -\dfrac{2}{e}$

~#94

§13.3 11. If $f(x,y,z) = \dfrac{z^2}{x+2y}$, then $f_{xyz}(x,y,z) =$

$f_x(x,y,z) = z^2 \cdot \dfrac{-1}{(x+2y)^2} = -z^2(x+2y)^{-2}$

(a) $\dfrac{8z}{(x+2y)^3}$ ________________ (correct)

$f_{xy}(x,y,z) = -z^2 \cdot -2(x+2y)^{-3} \cdot 2$

(b) $-\dfrac{4z}{(x+2y)^3}$

$f_{xyz}(x,y,z) = -2z \cdot -4(x+2y)^{-3}$

(c) $\dfrac{2z}{(x+2y)^2}$

$= \dfrac{8z}{(x+2y)^3}$

(d) $\dfrac{z}{(x+2y)^3}$

(e) $\dfrac{2z+x}{(x+2y)^2}$

#15

§13.4

12. Using **differentials**, the quantity

$$(2.01)^2(9.02) - 2^2 \cdot 9$$

is approximately equal to

Let $f(x,y) = x^2 y$

(x,y) changes from $(2,9)$ to $(2.01, 9.02)$

so $\Delta x = 2.01 - 2 = 0.01$ & $\Delta y = 9.02 - 9 = 0.02$

$\Delta z \approx dz = f_x\,dx + f_y\,dy$

$= (2xy)\,dx + (x^2)\,dy$

(a) 0.44 ________________ (correct)

(b) 0.24

(c) 0.48

(d) 0.39

(e) 0.36

$dx = \Delta x = 0.01$

$dy = \Delta y = 0.02$

$x = 2,\ y = 9$

$= 36(0.01) + 4(0.02)$

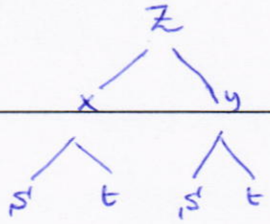
$= 0.36 + 0.08$

$= 0.44$

13. Let $z = e^{xy}$, $x = \ln\left(\frac{s}{t}\right)$, $y = s^2 - t^2$. The value of $\frac{\partial z}{\partial t}$ when $s = 2$ and $t = 1$ is equal to

$= \ln s - \ln t$ $\Rightarrow x = \ln 2$, $y = 3$

~#18, 21
§13.5



(a) $-24 - 16\ln 2$ ______ (correct)

(b) $-8 - 8\ln 2$

(c) $16 - 16\ln 2$

(d) $16 - 8\ln 2$

(e) $12 + 16\ln 2$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= e^{xy}\cdot y\cdot\frac{-1}{t} + e^{xy}\cdot x\cdot(-2t)$$

$$\left.\frac{\partial z}{\partial t}\right|_{\substack{s=2\\t=1}} = e^{3\ln 2}\cdot 3\cdot(-1) + e^{3\ln 2}\cdot\ln 2\cdot(-2)$$

$$= 8\cdot 3\cdot(-1) + 8\cdot\ln 2\cdot(-2)$$

$$= -24 - 16\ln 2$$

#34
§13.5

14. If $x\ln y + y^2 z + z^2 = 8$, then $\frac{\partial z}{\partial y} =$

Let $F(x,y,z) = x\ln y + y^2 z + z^2 - 8$

$$= -\frac{F_y}{F_z}$$

(a) $-\frac{x + 2y^2 z}{y^3 + 2yz}$ ______ (correct)

(b) $-\frac{x + 2y^2 z}{y^2 + 2z}$

(c) $-\frac{\ln y}{y^2 + 2z}$

(d) $\frac{x - 2y^2 z}{y^3 + 2yz}$

(e) $\frac{x - 2yz}{y^3 - 2yz}$

$$= -\frac{\frac{x}{y} + 2yz}{y^2 + 2z}\cdot\frac{y}{y}$$

$$= -\frac{x + 2y^2 z}{y^3 + 2yz}$$

#7
§13.6

15. The **directional derivative** of $f(x,y) = 3x - 4xy + 9y$ at $(1,2)$ in the direction of $\vec{v} = 3\vec{i} + 4\vec{j}$ is equal to

$\nabla f(x,y) = \langle f_x, f_y \rangle = \langle 3-4y, -4x+9 \rangle$
$\nabla f(1,2) = \langle -5, 5 \rangle$

(a) 1 ________________ (correct)

(b) -1

(c) -8

(d) 3

(e) $\dfrac{13}{5}$

$\|\vec{v}\| = \sqrt{9+16} = 5$

$\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|} = \langle \frac{3}{5}, \frac{4}{5} \rangle$

$D_{\vec{v}} f(1,2) = D_{\vec{u}} f(1,2) = \nabla f(1,2) \cdot \vec{u}$
$= \langle -5, 5 \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle = -5(\frac{3}{5}) + 5(\frac{4}{5})$
$= -3 + 4$
$= 1$

#35
§13.6

16. The maximum value of the directional derivative of

$$f(x,y,z) = \sqrt{x^2+y^2+z^2}$$

at the point $(1,4,2)$ is equal to

$\nabla f(x,y,z) = \langle f_x, f_y, f_z \rangle$

(a) 1 ________________ (correct)

(b) 2

(c) $\dfrac{1}{4}$

(d) $\dfrac{3}{2}$

(e) $\dfrac{1}{2}$

$= \langle \dfrac{x}{\sqrt{x^2+y^2+z^2}}, \dfrac{y}{\sqrt{x^2+y^2+z^2}}, \dfrac{z}{\sqrt{x^2+y^2+z^2}} \rangle$

$\nabla f(1,4,2) = \langle \dfrac{1}{\sqrt{21}}, \dfrac{4}{\sqrt{21}}, \dfrac{2}{\sqrt{21}} \rangle$

$\|\nabla f(1,4,2)\| = \sqrt{\dfrac{1}{21} + \dfrac{16}{21} + \dfrac{4}{21}} = 1$

max. value of directional derivative is
$\|\nabla f(1,4,2)\| = 1$

~ Example 8 § 13.6

17. The direction of **minimum increase** of

$$f(x,y,z) = e^{2y}\cos(x+z^3)$$

at the point $\left(\frac{\pi}{2}, \frac{1}{2}, 0\right)$ is given by

. $\nabla f(x,y,z) = \langle f_x, f_y, f_z \rangle$
$= \langle -e^{2y}\sin(x+z^3), 2e^{2y}\cos(x+z^3), -3e^{2y}z^2\sin(x+z^3) \rangle$

. $\nabla f(\frac{\pi}{2}, \frac{1}{2}, 0) = \langle -e, 0, 0 \rangle$

(a) $e\vec{i}$ ________ (correct)

(b) $e\vec{j}$

(c) $e\vec{i} - e\vec{j}$

(d) $-ei + 2e\vec{j} + ek$

(e) $-e\vec{i} + e\vec{k}$

. Direction of mn. increase is
$-\nabla f(\frac{\pi}{2}, \frac{1}{2}, 0) = \langle e, 0, 0 \rangle$
$= e\vec{i}$

#7 § 13.7

18. An equation of the **tangent plane** to the surface $z = x^2 + y^2 + 3$ at the point $(2,1,8)$ is

$= f(x,y)$

. $f_x(x,y) = 2x \Rightarrow f_x(2,1) = 4$

. $f_y(x,y) = 2y \Rightarrow f_y(2,1) = 2$

(a) $4x + 2y - z = 2$ ________ (correct)

(b) $3x + y - z = -1$

(c) $4x + y + z = 17$

(d) $2x + 2y - z = -2$

(e) $4x - 2y - z = -2$

. Eq of the tangent plane is

$f_x(2,1)(x-2) + f_y(2,1)(y-1) - (z-8) = 0$

$4(x-2) + 2(y-1) - z + 8 = 0$

$4x - 8 + 2y - 2 - z + 8 = 0$

$\Rightarrow 4x + 2y - z = 2$

#39
§13.7

19. If (a, b, c) is a point on the surface

$$z = x^2 - xy + y^2 - 2x - 2y \quad = f(x,y)$$

at which the tangent plane is **horizontal**, then $a^2 + b^2 + c^2 =$

Horizontal tangent plane $\Rightarrow$

(a) 24 ______________________________ (correct)

(b) 28

(c) 32

(d) 36

(e) 20

$f_x(x,y) = 0$ & $f_y(x,y) = 0$ $\Rightarrow \begin{cases} 2x - y - 2 = 0 & \sim (1) \\ -x + 2y - 2 = 0 & - (2) \end{cases}$

$\Rightarrow \begin{cases} 2x - y - 2 = 0 \\ -2x + 4y - 4 = 0 \end{cases}$

Sum: $3y - 6 = 0 \Rightarrow \boxed{y = 2} \xrightarrow{(2)} -x + 4 - 2 = 0$
$\Rightarrow \boxed{x = +2}$

$\Rightarrow z = 4 - 4 + 4 - 4 - 4 = -4$

So $(a, b, c) = (2, 2, -4)$

& $a^2 + b^2 + c^2 = 4 + 4 + 16 = 24$

#49
§13.7

20. If (a, b, c) is a point on the ellipsoid $3x^2 + y^2 + 3z^2 = 1$ where the tangent plane is **parallel** to the plane $-6x + y + 3z = 0$, then $a + b + c =$

- $F(x,y,z) = 3x^2 + y^2 + 3z^2$
  $\nabla F(x,y,z) = \langle 6x, 2y, 6z \rangle$

(a) 0 ______________________________ (correct)

(b) 1

(c) $\dfrac{1}{2}$

(d) -1

(e) $-\dfrac{1}{4}$

- $-6x + y + 3z = 0 \Rightarrow \vec{n} = \langle -6, 1, 3 \rangle$
- parallel planes $\Rightarrow \langle 6x, 2y, 6z \rangle = k \langle -6, 1, 3 \rangle$ for some $k \in \mathbb{R}$

$\Rightarrow 6x = -6k, \quad 2y = k, \quad 6z = 3k$

$\Rightarrow \boxed{x = -k, \quad y = \tfrac{1}{2}k, \quad z = \tfrac{1}{2}k} \quad (*)$

Sub. in eq. of the ellipsoid:

$3x^2 + y^2 + 3z^2 = 1 \Rightarrow 3k^2 + \frac{1}{4}k^2 + \frac{3}{4}k^2 = 1 \Rightarrow 4k^2 = 1 \Rightarrow k^2 = \frac{1}{4}$

$\Rightarrow k = \pm \frac{1}{2}$

- $k = \frac{1}{2} \overset{(*)}{\Rightarrow} (x, y, z) = \left(-\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$
- $k = -\frac{1}{2} \overset{*}{\Rightarrow} (x, y, z) = \left(\frac{1}{2}, -\frac{1}{4}, -\frac{1}{4}\right)$

In both cases, $a + b + c = 0$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	D 1	B 4	E 2	C 5
2	A	B 5	B 2	B 1	A 3
3	A	D 4	A 5	A 5	E 1
4	A	C 3	D 3	D 4	E 4
5	A	C 2	B 1	D 3	B 2
6	A	E 6	C 9	B 8	D 7
7	A	E 8	A 6	D 9	D 6
8	A	B 9	D 7	C 10	A 9
9	A	D 7	D 10	E 7	E 10
10	A	C 10	B 8	D 6	E 8
11	A	A 15	D 12	A 12	D 12
12	A	B 14	C 14	E 13	A 11
13	A	E 11	A 13	B 15	E 15
14	A	D 12	D 15	B 14	D 13
15	A	D 13	D 11	C 11	C 14
16	A	D 18	C 18	A 19	A 17
17	A	C 19	C 20	B 18	E 18
18	A	B 20	B 17	A 20	A 19
19	A	B 17	E 19	A 16	E 16
20	A	B 16	D 16	E 17	D 20