The Key by Codes is in the Last page

~ #11 §10.3

1. The **slope** of the tangent line to the parametric curve

$$x = t^3 - 5t + 1,\ y = t^2 + 3t$$

at the point corresponding to $t = 2$ is equal to

(a) 1 ______ (correct)

(b) $\frac{7}{10}$

(c) $\frac{5}{2}$

(d) $\frac{1}{3}$

(e) $\frac{2}{9}$

• $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t+3}{3t^2-5}$

• slope $= \left.\frac{dy}{dx}\right|_{t=2} = \frac{4+3}{12-5} = \frac{7}{7} = 1$

~ #50 §10.3

2. The **arc length** of the parametric curve

$$x = t^2,\ y = \frac{1}{3}t^3 + 1,\ 0 \le t \le 1$$

is equal to

(a) $\frac{1}{3}(5\sqrt{5} - 8)$ ______ (correct)

(b) $\frac{2}{3}(5\sqrt{5} - 4)$

(c) $\frac{3}{4}(\sqrt{5} - 2)$

(d) $\frac{1}{3}(5\sqrt{5} + 6)$

(e) $\frac{2}{3}(4\sqrt{5} - 8)$

$\frac{dx}{dt} = 2t$; $\frac{dy}{dt} = t^2$

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{4t^2 + t^4} = t\sqrt{4+t^2}$ $t \ge 0$

$L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \int_0^1 t\sqrt{4+t^2}\, dt$

$= \frac{1}{2} \cdot \frac{2}{3} \left.(4+t^2)^{3/2}\right]_0^1$

$= \frac{1}{3}(5^{3/2} - 4^{3/2})$

$= \frac{1}{3}(5\sqrt{5} - 8)$

3. The **area** of one petal (one loop) of the rose curve $r = 2\sin(5\,\theta)$ is equal to

~#12
§10.5

$\circ\ r = 0 \Rightarrow 2\sin(5\theta) = 0$
$\Rightarrow \sin(5\theta) = 0 \Rightarrow 5\theta = 0, \pi, 2\pi, \dots$
$\Rightarrow \theta = \underbrace{0, \frac{\pi}{5}}, \frac{2\pi}{5}, \dots$

(a) $\dfrac{\pi}{5}$ ____________ (correct)

(b) $\dfrac{2\pi}{5}$

(c) $\dfrac{\pi}{10}$

(d) $\dfrac{3\pi}{10}$

(e) $\dfrac{3\pi}{5}$

$$A = \int_0^{\pi/5} \frac{1}{2} r^2\, d\theta$$
$$= \int_0^{\pi/5} \frac{1}{2}\cdot 4\sin^2(5\theta)\, d\theta$$
$$= 2\int_0^{\pi/5} \frac{1-\cos(10\theta)}{2}\, d\theta$$
$$= \theta - \frac{1}{10}\sin(10\theta)\Big]_0^{\pi/5}$$
$$= \frac{\pi}{5}$$

4. The **volume** of the parallelepiped having adjacent edges

~#36
§11.4

$$\vec{u} =< 1, 3, 1 >,\ \vec{v} =< 0, 2, 2 >,\ \vec{w} =< 1, -2, 4 >$$

is equal to

(a) 16 ____________ (correct)

(b) 18

(c) 20

(d) 14

(e) 12

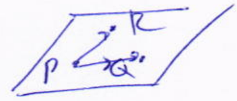
$$\begin{array}{cccc} & + & - & + \\ \vec{u}\cdot(\vec{v}\times\vec{w}) = & \begin{vmatrix} 1 & 3 & 1 \\ 0 & 2 & 2 \\ 1 & -2 & 4 \end{vmatrix} \end{array}$$

$= 1(8+4) - 3(0-2) + 1(0-2)$

$= 12 + 6 - 2$

$= 16$

~#47
§11.5



5. If $ax + by + cz = 10$ is an equation of the plane that passes through the points $P\,(1, 2, 3)$, $(3, 2, 1)$, and $(-1, -2, 2)$, then $a + b + c =$

Q R

$\vec{PQ} = \langle 2, 0, -2 \rangle$; $\vec{PR} = \langle -2, -4, -1 \rangle$

(a) 5 ______________________________ (correct)

(b) 10

(c) -4

(d) -3

(e) 8

$$\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 2 & 0 & -2 \\ -2 & -4 & -1 \end{vmatrix} = \langle -8, 6, -8 \rangle$$

$P(1,2,3)$

$-8(x-1) + 6(y-2) - 8(z-3) = 0$

$\Rightarrow -4(x-1) + 3(y-2) - 4(z-3) = 0$

$\Rightarrow -4x + 4 + 3y - 6 - 4z + 12 = 0$

$\Rightarrow -4x + 3y - 4z = -10$

$\Rightarrow \boxed{4x - 3y + 4z = 10}$

$a + b + c = 4 - 3 + 4 = 5$

~Example 4
§13.2

6. $\displaystyle \lim_{(x,y)\to(0,0)} \frac{2x^2 - y^2}{x^2 + 2y^2} =$

along $x = 0$; $\displaystyle \lim_{\substack{(x,y)\to(0,0) \\ x=0}} \frac{0 - y^2}{0 + 2y^2} = \lim_{y\to 0} -\frac{1}{2} = -\frac{1}{2}$

(a) does not exist ______________________________ (correct)

along $y = 0$; $\displaystyle \lim_{\substack{(x,y)\to(0,0) \\ y=0}} \frac{2x^2 - 0}{x^2 + 0} = \lim_{x\to 0} 2 = 2$

(b) 2

(c) $-\frac{1}{2}$

(d) 1

(e) 0

The two limits are not equal, and so the given limit does not exist.

~ #62

§13.2 7. The function $f(x) = \dfrac{\sqrt{y-2x}}{x^2+y^2-4}$ is **continuous** on

$y - 2x \geq 0$ & $x^2+y^2-4 \neq 0$

$\Rightarrow$ $y \geq 2x$ and $x^2+y^2 \neq 4$

(a) $\{(x,y) : x^2+y^2 \neq 4,\ y \geq 2x\}$ ——————————(correct)

(b) $\{(x,y) : x^2+y^2 \neq 4,\ y > 2x\}$

(c) $\{(x,y) : x^2+y^2 > 4,\ y \geq 2x\}$

(d) $\{(x,y) : -2 \leq x \leq 2,\ -2 \leq y \leq 2\}$

(e) $\{(x,y) : x^2+y^2 \geq 4,\ y > 2x\}$

#32, Review Exc., ch 13, page 964

8. If $f(x,y) = \dfrac{x}{x+y}$, then $f_{yy}(x,y) - f_{xx}(x,y) =$

• $f_x = \dfrac{(x+y)\cdot 1 - x\cdot 1}{(x+y)^2} = \dfrac{y}{(x+y)^2}$

(a) $\dfrac{2}{(x+y)^2}$ ——————————(correct)

• $f_{xx} = \dfrac{(x+y)^2\cdot 0 - y\cdot 2(x+y)}{(x+y)^4} = \dfrac{-2y}{(x+y)^3}$

(b) $\dfrac{2x}{(x+y)^3}$

(c) $\dfrac{2(x-y)}{(x+y)^3}$

• $f_y = \dfrac{(x+y)\cdot 0 - x\cdot 1}{(x+y)^2} = \dfrac{-x}{(x+y)^2}$

(d) $\dfrac{-2}{(x+y)^2}$

• $f_{yy} = \dfrac{(x+y)^2\cdot 0 - (-x)\cdot 2(x+y)}{(x+y)^4} = \dfrac{2x}{(x+y)^3}$

(e) $\dfrac{1}{(x+y)^3}$

• $f_{yy}(x,y) - f_{xx}(x,y) = \dfrac{2x}{(x+y)^3} - \dfrac{-2y}{(x+y)^3}$

$= \dfrac{2(x+y)}{(x+y)^3}$

$= \dfrac{2}{(x+y)^2}$

#13

§13.4

9. Let $z = ye^x$. If (x, y) changes from $(2, 1)$ to $(2.1,\ 1.05)$, then using **differentials** the change in z is approximately equal to
[Use the approximation : $e^2 \approx 7.40$]

$dx = \Delta x = 2.1 - 2 = 0.1$
$dy = \Delta y = 1.05 - 1 = 0.05$

(a) 1.11 ________________________________ (correct)
(b) 1.01
(c) 1.21
(d) 1.33
(e) 1.25

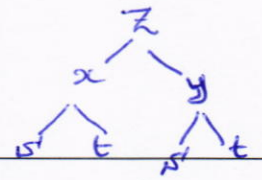
$dz = f_x\, dx + f_y\, dy$
$= ye^x\, dx + e^x\, dy$
$\Rightarrow dz = 1 \cdot e^2 (0.1) + e^2 (0.05)$
$= (0.15)\, e^2$

$\Delta z \approx dz$
$= (0.15)(7.40)$
$= 1.11$

~ #18

§13.5

10. Let $z = x^2 + y^2,\ x = s\cos t,\ y = \dfrac{\sin t}{s}$. The value of $\dfrac{\partial z}{\partial s}$ when $s = 2$ and $t = \dfrac{\pi}{4}$ is equal to



$x = 2\cos\frac{\pi}{4} = \frac{2}{\sqrt{2}} = \sqrt{2}$
$y = \frac{\sin(\pi/4)}{2} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

(a) $\dfrac{15}{8}$ ________________________________ (correct)
(b) $\dfrac{17}{4}$
(c) $\dfrac{5}{8}$
(d) $\dfrac{15}{4}$
(e) $\dfrac{3}{8}$

$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$
$= (2x) \cdot \cos t + (2y) \cdot \left(-\frac{\sin t}{s^2}\right)$

$\left.\frac{\partial z}{\partial s}\right|_{s=2,\ t=\frac{\pi}{4}} = 2\sqrt{2} \cdot \frac{1}{\sqrt{2}} + 2\,\frac{\sqrt{2}}{4} \cdot -\frac{1}{4\sqrt{2}}$

$= 2 - \frac{1}{8} = \frac{16-1}{8} = \frac{15}{8}$

#25
§13.6 11. Let $P(1,2)$ and $Q(2,3)$. The **directional derivative** of $g(x,y) = x^2 + y^2 + 1$ at P in the direction of the vector $\overrightarrow{PQ}$ is equal to

$\overrightarrow{PQ} = \langle 2-1, 3-2 \rangle = \langle 1, 1 \rangle$

(a) $3\sqrt{2}$ ——— (correct) $\frac{\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|} = \frac{1}{\sqrt{2}}\langle 1, 1 \rangle = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

(b) 4

(c) $4\sqrt{2}$ $\nabla g(x,y) = \langle 2x, 2y \rangle$

(d) 5 $\nabla g(x,y)\Big|_P = \langle 2, 4 \rangle$

(e) $5\sqrt{2}$

$D_{\overrightarrow{PQ}}\, g(1,2) = \nabla g(1,2) \cdot \frac{\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|}$

$= \langle 2, 4 \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$= \frac{2}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 3\sqrt{2}$

#19
§13.7 12. If $ax + by + cz = 7$ is an equation of the **tangent plane** to the ellipsoid $x^2 + 2y^2 + z^2 = 7$ at the point $(1, -1, 2)$, then $a + b + c =$

$\nabla F(x,y,z) = \langle 2x, 4y, 2z \rangle$, $F = x^2 + 2y^2 + z^2 - 7$

(a) 1 ——— (correct)

(b) 5 $\vec{n} = \nabla F(1,-1,2) = \langle 2, -4, 4 \rangle$

(c) 3 $\rightarrow 2(x-1) - 4(y+1) + 4(z-2) = 0$

(d) -2 $\Rightarrow (x-1) - 2(y+1) + 2(z-2) = 0$

(e) -4 $\Rightarrow x - 1 - 2y - 2 + 2z - 4 = 0$

$\Rightarrow x - 2y + 2z = 7$

• $a + b + c = 1 - 2 + 2 = 1$

#17

§13.8

13. If (a,b) is the only critical point of $f(x,y)=x^2+xy+\frac{1}{2}y^2-2x+y$, then

$f_x=0$, $f_y=0$ $\Rightarrow \begin{cases} 2x+y-2=0 & \sim (1) \\ x+y+1=0 & \sim (2) \end{cases}$

(a) f has a relative minimum at (a,b) ______ (correct)

(b) f has a relative maximum at (a,b)

(c) f has a saddle point at (a,b)

(d) $a+b=1$

(e) $f(a,b)=5$

$\Rightarrow x-3=0 \Rightarrow x=3$

$\overset{(2)}{\Rightarrow} y=-4$

$\boxed{(3,-4)}$

- $f_{xx}=2$, $f_{yy}=1$, $f_{xy}=1$

$D(x,y)=f_{xx}f_{yy}-f_{xy}^2$

$=2\cdot 1-1=1$

$\left.\begin{array}{l} D(3,-4)=1>0 \\ f_{xx}(3,-4)=2>0 \end{array}\right\} \Rightarrow$ f has a local min at $(3,-4)$

#39

§13.8

14. The **absolute maximum** of

$$f(x,y)=x^2-4xy+5$$

on the region $R=\{(x,y): 1\le x\le 4,\ 0\le y\le 2\}$ is

(a) 21 ______ (correct)

(b) 15

(c) -11

(d) -6

(e) 25

~ #5

(x, y, z)

§13.9 15. The **minimum distance** from the point $(-1, -1, 0)$ to the surface $z = \sqrt{1 - 2x - 2y}$ is equal to
(Hint: To simplify the computations, minimize the square of the distance).

$d^2 = (x+1)^2 + (y+1)^2 + (z-0)^2$

$f(x,y) = (x+1)^2 + (y+1)^2 + 1 - 2x - 2y$

(a) $\sqrt{3}$ ____________________ (correct)

(b) $\sqrt{2}$

(c) $3\sqrt{2}$

(d) $2\sqrt{3}$

(e) $\frac{\sqrt{3}}{2}$

$f_x = 2(x+1) - 2 = 0$

$f_y = 2(y+1) - 2 = 0$ $\Rightarrow (x, y) = (0, 0)$

$\cdot\ f_{xx} = 2\ ;\ f_{yy} = 2,\ f_{xy} = 0$

$D(x,y) = f_{xx} \cdot f_{yy} - f_{xy}^2 = 4$

$\left.\begin{array}{l} D(0,0) = 4 > 0 \\ f_{xx}(0,0) = 2 > 0 \end{array}\right\} \Rightarrow$ min at $(0,0)$

$x = 0, y = 0 \Rightarrow z = \sqrt{1-0-0} = 1 \Rightarrow (x,y,z) = (0,0,1)$

$d^2 = (0+1)^2 + (0+1)^2 + (1-0)^2 = 1+1+1 = 3$

$\Rightarrow d = \sqrt{3}.$

~ Example 3

§13.10 16. The **minimum value** of

$$f(x, y, z) = x^2 + 2y^2 + 3z^2$$

subject to the constraint $x - 2y + 6z = 45$ is

By Lagrange Method

(a) 135 ____________________ (correct)

(b) 115

(c) 95

(d) 105

(e) 125

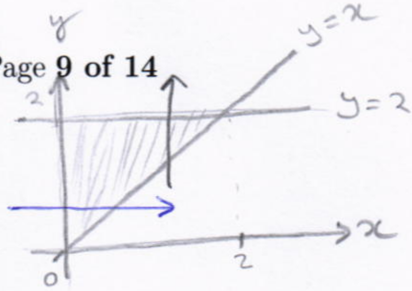
$\left\{\begin{array}{l} f_x = \lambda g_x \\ f_y = \lambda g_y \\ f_z = \lambda g_z \\ g = c \end{array}\right. \Rightarrow \left\{\begin{array}{l} 2x = \lambda \cdot 1 \\ 4y = \lambda \cdot (-2) \\ 6z = \lambda \cdot 6 \\ x - 2y + 6z = 45 = c \end{array}\right. \Rightarrow \begin{array}{l} x = \frac{\lambda}{2} \\ y = -\frac{\lambda}{2} \\ z = \lambda \end{array}$

$\Rightarrow \frac{\lambda}{2} + \lambda + 6\lambda = 45 \Rightarrow \frac{15}{2}\lambda = 45 \Rightarrow \lambda = \frac{2 \cdot 45}{15} = 2 \cdot 3 = 6$

$\Rightarrow (x,y,z) = (3, -3, 6)$

$\Rightarrow f(3,-3,6) = 9 + 18 + 3(36) = 27 + 108 = 135$, the minimum value of f since f takes arbitrary large values on points of the given plane; e.g., $(45,0,0)$ is on the plane &

$f(45,0,0) = 45^2 = 2025 > 135.$



#61

§14.1 17. $\displaystyle\int_0^2 \int_x^2 x\sqrt{1+y^3}\,dy\,dx =$

(a) $\frac{26}{9}$ (correct)

(b) 3

(c) $\frac{26}{3}$

(d) $\frac{1}{3}(\sqrt{3}-1)$

(e) 9

$$= \int_0^2 \underbrace{\int_0^y x\sqrt{1+y^3}\,dx}_{\sqrt{1+y^3}\,\cdot\,\frac{1}{2}x^2\Big]_{x=0}^{x=y}}\,dy$$

$$= \int_0^2 \frac{1}{2}y^2\sqrt{1+y^3}\;dy$$

$$= \frac{1}{6}\left(1+y^3\right)^{3/2}\cdot\frac{2}{3}\Big]_0^2$$

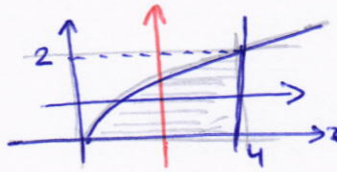
$$= \frac{1}{9}\left(9^{3/2}-1\right) = \frac{1}{9}(27-1) = \frac{26}{9}$$

~ Example 5

§14.1 18. $\displaystyle\int_0^2 \int_{y^2}^4 f(x,y)\,dx\,dy =$

$R: y^2 \le x \le 4,\ 0 \le y \le 2$

$x = y^2 \Rightarrow y = \sqrt{x}$



(a) $\displaystyle\int_0^4 \int_0^{\sqrt{x}} f(x,y)\,dy\,dx$ (correct)

$R: 0 \le y \le \sqrt{x},\ 0 \le x \le 4$

$$= \int_0^4 \int_0^{\sqrt{x}} f(x,y)\,dy\,dx$$

(b) $\displaystyle\int_0^4 \int_1^{\sqrt{x}} f(x,y)\,dy\,dx$

(c) $\displaystyle\int_0^2 \int_0^{\sqrt{x}} f(x,y)\,dy\,dx$

(d) $\displaystyle\int_0^2 \int_0^{x^2} f(x,y)\,dy\,dx$

(e) $\displaystyle\int_{y^2}^4 \int_0^2 f(x,y)\,dy\,dx$

#16
§14.1

19. $\int_0^1 \int_0^y (6x + 5y^3)\, dx\, dy$ $= \int_0^1 3x^2 + 5y^3 x \Big]_{x=0}^{x=y} dy$

$= \int_0^1 3y^2 + 5y^4 \, dy$

(a) 2 ______ (correct)

(b) 0

(c) 3

(d) 1

(e) 4

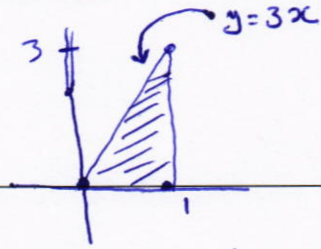
$= y^3 + y^5 \Big]_0^1$

$= 1+1$

$= 2$

~ #52, 54
§16.2

20. The **average value** of $f(x, y) = 2xy$ over the triangular region with vertices $(0, 0)$, $(1, 0)$, $(1, 3)$ is equal to



(a) $\frac{3}{2}$ ______ (correct)

(b) $\frac{6}{5}$

(c) $\frac{5}{2}$

(d) $\frac{1}{2}$

(e) 2

Area $= \frac{1}{2} \cdot 1 \cdot 3 = \frac{3}{2}$

$R: 0 \le x \le 1, \quad 0 \le y \le 3x$

$\text{Avg} = \frac{1}{\text{area}} \iint_R f \, dA$

$= \frac{2}{3} \int_0^1 \int_0^{3x} 2xy \, dy\, dx$

$= \frac{2}{3} \int_0^1 xy^2 \Big]_{y=0}^{y=3x} dx$

$= \frac{2}{3} \int_0^1 9x^3 \, dx = \frac{2}{3} \cdot 9 \cdot \frac{x^4}{4} \Big]_0^1$

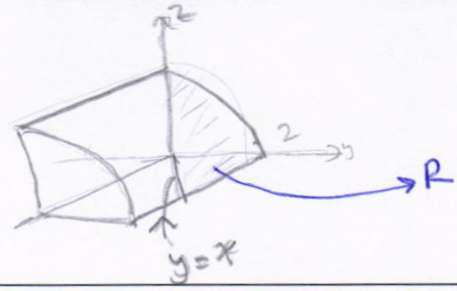
$= \frac{2}{3} \cdot 9 \cdot \frac{1}{4} = \frac{3}{2}$

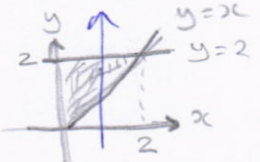
#26

§14.2 21. The **volume** of the solid lying in the **first octant** and bounded by the graphs of

$z = 4 - y^2$, $y = x$, $y = 2$

is equal to





$R: 0 \le x \le 2,\ x \le y \le 2$

(a) 4 ____________________ (correct)

(b) 6

(c) $\frac{17}{3}$

(d) $\frac{22}{3}$

(e) $\frac{32}{3}$

$V = \iint_R (4-y^2)\, dA$

$= \int_0^2 \int_x^2 (4-y^2)\, dy\, dx$

$= \int_0^2 \left. 4y - \frac{1}{3}y^3 \right]_{y=x}^{y=2} dx$

$= \int_0^2 8 - \frac{8}{3} - (4x - \frac{1}{3}x^3)\, dx$

$= \int_0^2 (\frac{16}{3} - 4x + \frac{1}{3}x^3)\, dx$

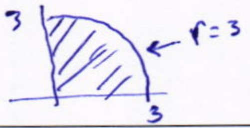
$= \left. \frac{16}{3}x - 2x^2 + \frac{1}{12}x^4 \right]_0^2 = \frac{32}{3} - 8 + \frac{16}{12} = \frac{32}{3} - 8 + \frac{4}{3}$

$= \frac{32 - 24 + 4}{3} = \frac{32-20}{3} = \frac{12}{3} = 4$

~ #29

§14.3

22. If $R = \{(x, y) : x^2 + y^2 \le 9,\ x \ge 0,\ y \ge 0\}$ then $\int\int_R (x + y)\, dA =$



$R: 0 \le r \le 3,\ 0 \le \theta \le \frac{\pi}{2}$

(a) 18 ____________________ (correct)

(b) 9

(c) 36

(d) 6π

(e) $\frac{2\pi}{3}$

$= \int_0^{\pi/2} \int_0^3 \underbrace{(r\cos\theta + r\sin\theta)\, r}_{(\cos\theta + \sin\theta) r^2} dr\, d\theta$

$(\cos\theta + \sin\theta) \left. \frac{r^3}{3} \right]_{r=0}^{r=3}$

$= \int_0^{\pi/2} 9 (\cos\theta + \sin\theta)\, d\theta$

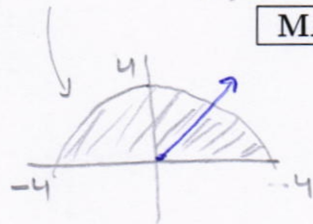
$= 9 \left. (\sin\theta - \cos\theta) \right]_0^{\pi/2}$

$= 9 \cdot [1 - (-1)]$

$= 9 \cdot 2$

$= 18$

$y = \sqrt{16-x^2} \Rightarrow x^2+y^2=16$
$\Rightarrow r=4$



$0 \le r \le 4$
$0 \le \theta \le \pi$

~ #25
§14.3

23. $\displaystyle\int_{-4}^{4}\int_{0}^{\sqrt{16-x^2}} \cos(x^2+y^2)\,dy\,dx =$

(a) $\displaystyle\int_0^{\pi}\int_0^4 r\cos(r^2)\,dr\,d\theta$ ________________ (correct)

$\displaystyle\int_0^{\pi}\int_0^4 \cos(r^2)\cdot r\,dr\,d\theta$

$\displaystyle= \int_0^{\pi}\int_0^4 r\cos(r^2)\,dr\,d\theta$

(b) $\displaystyle\int_0^{\pi}\int_0^4 \cos(r^2)\,dr\,d\theta$

(c) $\displaystyle\int_0^{\pi}\int_{-4}^4 r\cos(r^2)\,dr\,d\theta$

(d) $\displaystyle\int_0^{2\pi}\int_0^4 r\cos(r^2)\,dr\,d\theta$

(e) $\displaystyle\int_{-\pi}^{\pi}\int_0^4 r\cos(r^2)\,dr\,d\theta$

Example 4 (c)
§14.6

24. The **volume** of the solid region bounded below by the paraboloid $z = x^2+y^2$ and above by the sphere $x^2+y^2+z^2=6$ is given by

(a) $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}}\int_{x^2+y^2}^{\sqrt{6-x^2-y^2}} dz\,dy\,dx$ ________________ (correct)

intersection:

$\begin{cases} x^2+y^2+z^2=6 \\ x^2+y^2=z \end{cases}$

$\Rightarrow z^2+z=6 \Rightarrow z^2+z-6=0 \Rightarrow (z+3)(z-2)=0$

$\Rightarrow z=-3,\ z=2$

$\downarrow$

$x^2+y^2=2 \xrightarrow{R}$

(b) $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}}\int_{\sqrt{6-x^2-y^2}}^{x^2+y^2} dz\,dy\,dx$

(c) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{\sqrt{6-x^2-y^2}} dz\,dy\,dx$

(d) $\displaystyle\int_{0}^{\sqrt{2}}\int_{0}^{\sqrt{2-x^2}}\int_{0}^{\sqrt{6-x^2-y^2}} dz\,dy\,dx$

$\displaystyle V = \iiint_D 1\,dV$

$\displaystyle = \int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}}\int_{x^2+y^2}^{\sqrt{6-x^2-y^2}} dz\,dy\,dx$

(e) $\displaystyle\int_{0}^{\sqrt{2}}\int_{0}^{\sqrt{2-x^2}}\int_{x^2+y^2}^{\sqrt{6-x^2-y^2}} dz\,dy\,dx$

#8
§14.6

25. $\int_1^4 \int_1^{e^2} \int_0^{\frac{1}{xz}} (\ln z)\, dy\, dz\, dx = \int_1^4 \int_1^{e^2} (\ln z) y \Big]_{y=0}^{y=\frac{1}{xz}} dz\, dx$

$= \int_1^4 \int_1^{e^2} \frac{\ln z}{xz}\, dz\, dx$

(a) $2\ln 4$ (correct)

(b) $3\ln 4$

(c) $\ln 4$

(d) $\dfrac{\ln 4}{2}$

(e) $\dfrac{\ln 4}{3}$

$= \int_1^4 \frac{1}{x} \cdot \frac{1}{2} (\ln z)^2 \Big]_{z=1}^{z=e^2} dx$

$= \int_1^4 \frac{1}{x} \cdot \frac{1}{2} (4 - 0)\, dx$

$= 2 \int_1^4 \frac{1}{x}\, dx$

$= 2 \ln|x| \Big]_1^4 = 2(\ln 4 - 0)$

$= 2\ln 4$

#33
§11.7

26. The point with rectangular coordinates $(x, y, z) = (-2, 2\sqrt{3}, 4)$ is represented in spherical coordinates by

- $\rho = \sqrt{x^2+y^2+z^2} = \sqrt{4+12+16} = \sqrt{32} = 4\sqrt{2}$
- $\tan\theta = \frac{y}{x} = \frac{2\sqrt{3}}{-2} = -\sqrt{3} \Rightarrow \theta = \frac{2\pi}{3}$ since $(x,y) = (-2, 2\sqrt{3}) \in Q\,\text{II}$
- $z = \rho\cos\phi \Rightarrow \cos\phi = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \phi = \frac{\pi}{4}$ as $0 \le \phi \le \pi$

(a) $(\rho, \theta, \phi) = \left(4\sqrt{2}, \frac{2\pi}{3}, \frac{\pi}{4}\right)$ (correct)

(b) $(\rho, \theta, \phi) = \left(4\sqrt{2}, \frac{5\pi}{3}, \frac{\pi}{4}\right)$

(c) $(\rho, \theta, \phi) = \left(4\sqrt{2}, \frac{2\pi}{3}, \frac{3\pi}{4}\right)$

(d) $(\rho, \theta, \phi) = \left(2\sqrt{2}, \frac{2\pi}{3}, \frac{\pi}{4}\right)$

(e) $(\rho, \theta, \phi) = \left(4\sqrt{2}, \frac{\pi}{3}, \frac{3\pi}{4}\right)$

Example 4 , §11.7

27. The graph of the surface represented by the cylindrical equation

$$r^2\cos(2\theta) + z^2 + 1 = 0$$

$\Rightarrow r^2(\cos^2\theta - \sin^2\theta) + z^2 + 1 = 0$

$\Rightarrow r^2\cos^2\theta - r^2\sin^2\theta + z^2 + 1 = 0$

$\Rightarrow x^2 - y^2 + z^2 = -1$

$\Rightarrow -x^2 + y^2 - z^2 = 1$, a hype. of 2 sheets

is

(a) a hyperboloid of two sheets ______ (correct)

(b) a hyperboloid of one sheet

(c) an elliptic paraboloid

(d) a sphere

(e) an elliptic hyperboloid

~ #44 §14.7

28. $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx =$

$D: 0 \le x \le 2,\ 0 \le y \le \sqrt{4-x^2},\ 0 \le z \le \sqrt{4-x^2-y^2}$ $x^2+y^2+z^2=4$ $(\rho=2)$

= ~~1st octant~~ The solid inside the sphere in ~~side~~ the first octant

$0 \le \rho \le 2,\ 0 \le \phi \le \frac{\pi}{2},\ 0 \le \theta \le \frac{\pi}{2}$

(a) 2π ______ (correct)

(b) 6π

(c) 4π

(d) 8π

(e) π

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{2} \rho \cdot \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$\frac{1}{4}\rho^4\Big]_0^2$

$$= \int_0^{\pi/2}\underbrace{\int_0^{\pi/2} 4\sin\phi\, d\phi}_{-4\cos\phi\big]_0^{\pi/2}}\, d\theta \qquad = -4(0-1) = 4$$

$$= \int_0^{\pi/2} 4\, d\theta = 4\theta\Big]_0^{\pi/2} = 4\cdot\frac{\pi}{2} = 2\pi$$

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	D 6	C 1	E 7	B 7
2	A	B 4	B 4	E 2	B 4
3	A	E 3	C 3	C 1	E 3
4	A	C 7	E 2	A 6	A 1
5	A	C 5	E 7	E 5	E 6
6	A	A 2	D 6	E 3	B 5
7	A	A 1	B 5	D 4	B 2
8	A	E 13	A 14	D 9	B 12
9	A	B 11	E 9	E 11	C 8
10	A	E 10	E 11	E 8	E 9
11	A	A 8	E 8	B 10	D 14
12	A	B 14	B 13	B 13	E 13
13	A	B 12	C 10	C 14	A 10
14	A	B 9	A 12	E 12	A 11
15	A	C 19	C 17	A 20	C 19
16	A	D 21	B 20	A 19	E 21
17	A	A 20	B 19	A 21	B 15
18	A	E 17	B 15	A 18	A 20
19	A	A 15	D 18	C 17	E 18
20	A	B 18	A 16	A 16	C 16
21	A	C 16	E 21	A 15	B 17
22	A	A 26	A 26	E 25	E 28
23	A	E 24	E 23	E 28	B 26
24	A	C 28	D 24	E 26	A 22
25	A	D 22	C 27	D 22	A 23
26	A	A 25	C 25	E 27	D 27
27	A	A 27	E 28	D 24	A 25
28	A	C 23	E 22	A 23	D 24