Key by Codes is in last page

$\sim$ #21 §10.2

1. The curve represented by the parametric equations

$$x = \sec^2 t,\ y = \cos t,\ \ 0 \le t < \frac{\pi}{2}$$

is represented by the rectangular equation

$x = \sec^2 t = \frac{1}{\cos^2 t} = \frac{1}{y^2}$

(a) $y = \dfrac{1}{\sqrt{x}}$, $x \ge 1$ ________ (correct)

$\Rightarrow y^2 = \frac{1}{x}$

(b) $y = \dfrac{1}{\sqrt{x}}$, $x > 0$

$\Rightarrow y = \frac{1}{\sqrt{x}}$, on $\le t < \frac{\pi}{2}$, $x = \sec^2 t > 0$, $y = \cos t > 0$

(c) $y = -\dfrac{1}{\sqrt{x}}$, $x \ge 1$

. $x = \sec^2 t \ge 1$ on $0 \le t \le \frac{\pi}{2}$

(d) $y = \dfrac{1}{x^2}$, $x > 0$

$\Rightarrow y = \frac{1}{\sqrt{x}}$, $x \ge 1$

(e) $y = \dfrac{1}{x^2}$, $x \ge 1$

#37(b) §10.2

2. The graph of the parametric equations

$$x = \cos t,\ y = 2\cos t + 1$$

is

$y = 2\cos t + 1$

$\Rightarrow y = 2x + 1$, $-1 \le x \le 1$

a line segment

(a) a line segment ________ (correct)

(b) a circle

(c) an ellipse

(d) a hyperbola

(e) a parabola

#29
§10.3

3. The **slope** of the tangent line to the parametric curve

$$x = t^2 - t,\ y = t^3 - 3t - 1$$

at the point corresponding to $t = 2$ is equal to

(a) 3 ____ (correct)

(b) -2

(c) $-\frac{1}{3}$

(d) 6

(e) 4

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2-3}{2t-1}$$

$$\text{slope} = \left.\frac{dy}{dx}\right|_{t=2} = \frac{3.4-3}{2.2-1} = \frac{9}{3} = 3$$

~ #51
§10.3

4. The **arc length** of the parametric curve

$$x = e^t \cos t,\ y = e^t \sin t,\ \ 0 \le t \le 1$$

is equal to

- $\frac{dx}{dt} = -e^t \sin t + \cos t \cdot e^t = e^t(\cos t - \sin t)$

$\left(\frac{dx}{dt}\right)^2 = e^{2t}(\cos^2 t - 2\sin t \cos t + \sin^2 t)$
$= e^{2t}(1 - \sin(2t))$

(a) $\sqrt{2}\,(e-1)$ ____ (correct)

(b) $\sqrt{2}\,(e^2-1)$

(c) $e+1$

(d) $2(e-1)$

(e) $\sqrt{2}(e+1)$

- $\frac{dy}{dt} = e^t \cos t + \sin t \cdot e^t = e^t(\cos t + \sin t)$

$\left(\frac{dy}{dt}\right)^2 = e^{2t}(\cos^2 t + 2\sin t \cos t + \cos^2 t)$
$= e^{2t}(1 + \sin(2t))$

- $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 2e^{2t}$

- $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{2e^{2t}} = \sqrt{2}\cdot e^t$

$$L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^1 \sqrt{2}\cdot e^t\, dt = \sqrt{2}\cdot e^t\Big]_0^1 = \sqrt{2}\ (e-1)$$

~#34
§10.4

5. Converting the rectangular equation

$$(x^2 + y^2)^2 - 4x^2 + 4y^2 = 0$$

$r^4 - 4r^2\cos^2\theta + 4r^2\sin^2\theta = 0$

$r^2(r^2 - 4\cos^2\theta + 4\sin^2\theta) = 0$

to polar form, we get

$\Rightarrow r^2 = 0$ or $r^2 - 4\cos^2\theta + 4\sin^2\theta = 0$

$\Rightarrow r = 0$ or $r^2 = 4\cos^2\theta - 4\sin^2\theta$

(a) $r^2 = 4\cos(2\theta)$ ____________________(correct)

(b) $r^2 = 2\cos(2\theta)$

(c) $r^2 = 4(\cos\theta - \sin\theta)$

(d) $r = 2\sin(2\theta)$

(e) $r^2 = 4\sin(2\theta)$

origin rejected

$r^2 = 4(\cos^2\theta - \sin^2\theta)$

$\Rightarrow r^2 = 4\cos(2\theta)$

#71
§10.4

6. The polar curve $r = 2\csc\theta + 3$ has a **horizontal tangent line** when

- $x = r\cos\theta = (2\csc\theta + 3)\cos\theta = 2\cot\theta + 3\cos\theta$
- $y = r\sin\theta = (2\csc\theta + 3)\sin\theta = 2 + 3\sin\theta$

(a) $\theta = \frac{\pi}{2}$ ____________________(correct)

(b) $\theta = \frac{\pi}{3}$

(c) $\theta = \frac{\pi}{4}$

(d) $\theta = \frac{\pi}{6}$

(e) $\theta = \frac{\pi}{8}$

- $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{3\cos\theta}{-2\csc^2\theta - 3\sin\theta}$

Checking at the given values $\theta = \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{8}$

we find that

$\left.\dfrac{dy}{dx}\right|_{\boxed{\theta = \frac{\pi}{2}}} = 0$ only

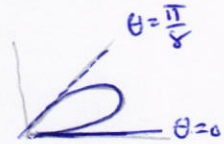
$\Rightarrow$ Horizontal tangent when $\theta = \frac{\pi}{2}$

#11
§10.5

7. The **area** of two petals (two loops) of the rose curve

$$r = \sin(8\,\theta)$$

is equal to



$\cdot\ r = 0 \Rightarrow \sin(8\theta) = 0$
$\Rightarrow 8\theta = 0, \pm\pi, \pm 2\pi, \ldots$
$\Rightarrow \theta = 0, \pm\frac{\pi}{8}, \pm\frac{\pi}{4}, \ldots$
$\Rightarrow \theta = 0, \frac{\pi}{8} \in QI$

(a) $\dfrac{\pi}{16}$ ______ (correct)

(b) $\dfrac{\pi}{8}$

(c) $\dfrac{3\pi}{8}$

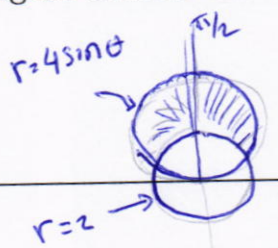
(d) $\dfrac{5\pi}{16}$

(e) $\dfrac{\pi}{4}$

$$A = 2 \cdot \int_0^{\pi/8} \tfrac{1}{2} r^2 \, d\theta$$
$$= 2 \cdot \tfrac{1}{2} \int_0^{\pi/2} \sin^2(8\theta)\, d\theta$$
$$= \int_0^{\pi/8} \tfrac{1}{2}\left[1 - \cos(16\theta)\right] d\theta$$
$$= \tfrac{1}{2}\left[\theta - \tfrac{1}{16}\sin(16\theta)\right]_0^{\pi/8}$$
$$= \tfrac{1}{2}\left[\left(\tfrac{\pi}{8} - 0\right) - (0 - 0)\right]$$
$$= \tfrac{1}{2} \cdot \tfrac{\pi}{8} = \tfrac{\pi}{16}$$

~ #41
§10.5

8. The **area** of the region outside the circle $r = 2$ and inside the circle $r = 4\sin\theta$ is equal to



$\cdot$ pts of intersection
$r = r \Rightarrow 2 = 4\sin\theta \Rightarrow \sin\theta = \frac{1}{2}$
$\Rightarrow \theta = \frac{\pi}{6} \in QI$

(a) $\dfrac{4\pi}{3} + 2\sqrt{3}$ ______ (correct)

(b) $\dfrac{2\pi}{3} + 2\sqrt{3}$

(c) $\dfrac{2\pi}{3} + \sqrt{3}$

(d) $\dfrac{5\pi}{3} - 2\sqrt{3}$

(e) $\dfrac{4\pi}{3} + \sqrt{3}$

By symmetry about the y-axis:

$$A = 2 \cdot \int_{\pi/6}^{\pi/2} \tfrac{1}{2}\left[(4\sin\theta)^2 - (2)^2\right] d\theta$$
$$= \int_{\pi/6}^{\pi/2} \left[16\left(\frac{1 - \cos(2\theta)}{2}\right) - 4\right] d\theta$$
$$= \int_{\pi/6}^{\pi/2} \left[4 - 8\cos(2\theta)\right] d\theta$$
$$= 4\theta - 4\sin(2\theta)\Big]_{\pi/6}^{\pi/2}$$
$$= (2\pi - 0) - \left(\tfrac{4\pi}{6} - 4 \cdot \tfrac{\sqrt{3}}{2}\right)$$
$$= 2\pi - \tfrac{2\pi}{3} + 2\sqrt{3} \quad = \tfrac{4\pi}{3} + 2\sqrt{3}$$

$0 \le \theta \le \pi$

Example 5

§10.5

9. The **area of the surface** formed by revolving the circle $r = \cos\theta$ about the line $\theta = \frac{\pi}{2}$ is given by

$$S = \int_0^{\pi} 2\pi \cdot r\cos\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \qquad , \frac{dr}{d\theta} = -\sin\theta$$

$$= \int_0^{\pi} 2\pi \cdot \cos\theta \cdot \cos\theta \sqrt{\cos^2\theta + \sin^2\theta}\, d\theta$$

$$= \int_0^{\pi} 2\pi \cos^2\theta \sqrt{1}\, d\theta$$

$$= \int_0^{\pi} 2\pi \cos^2\theta\, d\theta$$

(a) $\int_0^{\pi} 2\pi \cdot \cos^2\theta\, d\theta$ ______ (correct)

(b) $\int_0^{2\pi} 2\pi \cdot \cos^2\theta\, d\theta$

(c) $\int_0^{\pi} \pi \cdot \cos^2\theta\, d\theta$

(d) $\int_0^{2\pi} 2\pi \cdot \sin\theta \cdot \cos\theta\, d\theta$

(e) $\int_0^{\pi} \pi \cdot \sin\theta \cdot \cos\theta\, d\theta$

~ #45

§10.2

10. If $C(a, b, c)$ is the **center** of the sphere

$$3x^2 + 3y^2 + 3z^2 - 18x + 6z + 24 = 0$$

then $a^2 + b^2 + c^2 =$

$x^2 + y^2 + z^2 - 6x + 2z + 8 = 0$

$x^2 - 6x + 9 + y^2 + z^2 + 2z + 1 = -8 + 9 + 1$

(a) 10 ______ (correct)

(b) 11

(c) 5

(d) 14

(e) 3

$(x-3)^2 + y^2 + (z+1)^2 = 2$

Center $(3, 0, -1)$, radius $= \sqrt{2}$

$a^2 + b^2 + c^2 = 9 + 0 + 1 = 10$

13
§11.4

11. A vector that is **orthogonal** to both vectors

$$\vec{u} = i + j + k,\ \vec{v} = 2i + j - k$$

is

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

(a) $-2i + 3j - k$ (correct)

(b) $3j - k$

(c) $-2i - 3j - k$

(d) $2i + 3j$

(e) $-2i - j + k$

$$= i(-1-1) - j(-1-2) + k(1-2)$$

$$= -2i + 3j - k$$

~ # 41
§11.3

12. The **projection** of $\vec{u} = < 1, -2, 3 >$ onto $\vec{v} = < -1, 1, 2 >$ is

(a) $\left\langle -\frac{1}{2}, \frac{1}{2}, 1 \right\rangle$ (correct)

(b) $\langle -2, 2, 4 \rangle$

(c) $\left\langle \frac{1}{2}, -1, \frac{3}{2} \right\rangle$

(d) $\langle 2, -2, -4 \rangle$

(e) $\langle -1, 1, 2 \rangle$

$$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v}$$

$\vec{u} \cdot \vec{v} = -1 - 2 + 6 = 3$

$\|\vec{v}\|^2 = 1 + 1 + 4 = 6$

$$= \frac{3}{6} \langle -1, 1, 2 \rangle$$

$$= \frac{1}{2} \langle -1, 1, 2 \rangle$$

$$= \left\langle -\frac{1}{2}, \frac{1}{2}, 1 \right\rangle$$

~ #35

§11.4 13. The **volume** of the parallelepiped having adjacent edges

$$\vec{u} = <3, 1, 1>,\ \vec{v} = <2, 1, 0>,\ \vec{w} = <1, -1, -2>$$

is equal to

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} 3 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & -2 \end{vmatrix}$$

(a) 5 ________ (correct)

(b) 6

(c) 8

(d) 10

(e) 2

$$= 3 \begin{vmatrix} 1 & 0 \\ -1 & -2 \end{vmatrix} - (1) \begin{vmatrix} 2 & 0 \\ 1 & -2 \end{vmatrix} + (1) \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= 3(-2) - (-4) + (-2-1)$$

$$= -6 + 4 - 3$$

$$= -5$$

$$\text{Vol} = |\vec{u} \cdot (\vec{v} \times \vec{w})| = |-5| = 5$$

#85

§11.2 14. The vector of magnitude $\frac{3}{2}$ that has the same direction as the vector $< 2, -2, 1 >$ is

$$= \frac{3}{2} \cdot \frac{<2, -2, 1>}{\| <2, -2, 1> \|}$$

(a) $\left\langle 1, -1, \frac{1}{2} \right\rangle$ ________ (correct)

(b) $\langle 2, -1, 2 \rangle$

(c) $\langle 2, -2, 1 \rangle$

(d) $\left\langle -3, 3, \frac{3}{2} \right\rangle$

(e) $\left\langle -1, \frac{1}{2}, 1 \right\rangle$

$$= \frac{3}{2} \cdot \frac{<2, -2, 1>}{\sqrt{4+4+1}} = \frac{3}{2} \cdot \frac{<2, -2, 1>}{3}$$

$$= \frac{1}{2} <2, -2, 1>$$

$$= <1, -1, \frac{1}{2}>$$

15. If $R(a,b,c)$ is the point that lies **three-fifths** of the way from $P(0,1,2)$ to $Q(2,-1,-3)$, then $a+b+c=$

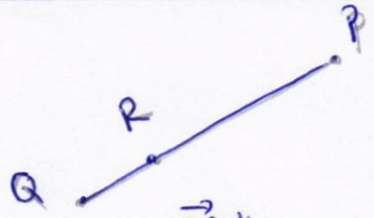
(a) 0 ____ (correct)

(b) $\frac{3}{5}$

(c) $-\frac{1}{5}$

(d) 1

(e) -3



$\vec{PR} \parallel \vec{PQ}$ and $\|\vec{PR}\| = \frac{3}{5}\|\vec{PQ}\|$

$\Leftrightarrow \quad \vec{PR} = \frac{3}{5}\vec{PQ}$

$\Rightarrow \langle a-0, b-1, c-2\rangle = \frac{3}{5}\langle 2-0, -1-1, -3-2\rangle$

$\Rightarrow \langle a, b-1, c-2\rangle = \frac{3}{5}\langle 2, -2, -5\rangle = \langle \frac{6}{5}, -\frac{6}{5}, -3\rangle$

$\Rightarrow \begin{cases} a = \frac{6}{5} \\ b-1 = -\frac{6}{5} \\ c-2 = -3 \end{cases} \Rightarrow \begin{cases} a = \frac{6}{5} \\ b = -\frac{1}{5} \\ c = -1 \end{cases}$

$\Rightarrow a+b+c = \frac{6}{5} - \frac{1}{5} - 1 = 1-1 = 0$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	C $_{1}$	E $_{1}$	E $_{2}$	E $_{2}$
2	A	A $_{4}$	E $_{3}$	D $_{1}$	E $_{3}$
3	A	D $_{3}$	E $_{2}$	A $_{4}$	B $_{4}$
4	A	B $_{2}$	B $_{4}$	A $_{3}$	E $_{1}$
5	A	C $_{7}$	A $_{8}$	D $_{7}$	B $_{9}$
6	A	D $_{9}$	A $_{5}$	C $_{6}$	B $_{6}$
7	A	D $_{5}$	C $_{9}$	A $_{8}$	E $_{8}$
8	A	E $_{8}$	E $_{7}$	D $_{5}$	D $_{5}$
9	A	E $_{6}$	A $_{6}$	A $_{9}$	A $_{7}$
10	A	E $_{10}$	A $_{12}$	D $_{10}$	D $_{15}$
11	A	C $_{14}$	C $_{10}$	C $_{11}$	C $_{11}$
12	A	B $_{15}$	B $_{13}$	E $_{13}$	E $_{12}$
13	A	B $_{13}$	A $_{14}$	C $_{12}$	E $_{10}$
14	A	E $_{12}$	A $_{11}$	E $_{14}$	A $_{13}$
15	A	B $_{11}$	B $_{15}$	C $_{15}$	D $_{14}$