#19
§11.5

1. A set of **parametric equations** for the line passing through the point $(2, 3, 4)$ and is perpendicular to the plane given by $3x + 2y - z = 6$ is given by

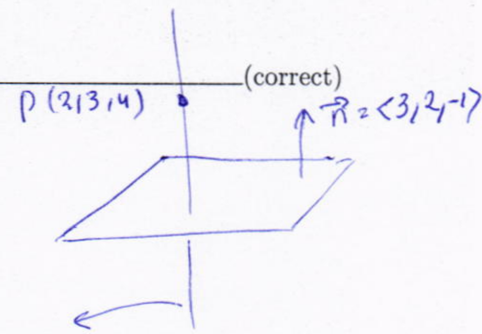
(a) $x = 2 + 3t,\ y = 3 + 2t,\ z = 4 - t$ ________________ (correct)

(b) $x = 2 - t,\ y = 3 + 2t,\ z = 4 + 3t$

(c) $x = 3 + 2t,\ y = 2 + 3t,\ z = -1 + 4t$

(d) $x = 1 + t,\ y = -1 + t,\ z = 2 + t$

(e) $x = 2 + 2t,\ y = 3 + 3t,\ z = 4 + 4t$



$x = 2 + 3t$
$y = 3 + 2t$, $t \in \mathbb{R}$
$z = 4 - t$

~ #45
§11.5

2. If $x + by + cz = d$ is an equation for the plane that passes through the points $P(0, 0, 0)$, $Q(1, 0, 1)$, and $R(-1, 1, 0)$, then $b + c + d =$

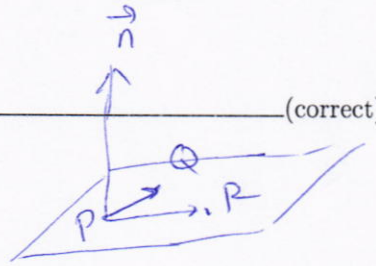
(a) 0 ________________ (correct)

(b) 1

(c) −1

(d) 2

(e) −2



$\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix}$

$= \langle -1, -1, 1 \rangle$

P(0,0,0)

eq: $-1(x-0) - 1(y-0) + 1(z-0) = 0$

$-x - y + z = 0$

$\Rightarrow x + y - z = 0$

$b = 1,\ c = -1,\ d = 0$

$\Rightarrow b + c + d = 0$

~ #83

§11.5

3. If (a, b, c) is the **point of intersection** between the plane $x + 3y + z = 6$ and the line $x = t$, $y = 1 - t$, $z = 2 + 3t$, then $a + b + c =$

(a) 6 ______________________________ (correct)

(b) 5

(c) 8

(d) -4

(e) -7

$t + 3(1-t) + (2+3t) = 6$

$t + 3 \cancel{-3t} + 2 \cancel{+3t} = 6$

$t + 5 = 6 \Rightarrow t = 1$

$\Rightarrow (a, b, c) = (1, 1-1, 2+3(1))$

$= (1, 0, 5)$

$\Rightarrow a + b + c = 1 + 0 + 5 = 6$

~ Example 2

§11.6

4. The **graph** of the equation $4x^2 - 3y^2 - 12z^2 = 0$ is

$4x^2 = 3y^2 + 12z^2$

(a) an elliptic cone ______________________________ (correct)

(b) a hyperboloid of two sheets

(c) a hyperboloid of one sheet

(d) an ellipsoid

(e) an elliptic paraboloid

an elliptic cone

~ #31
§13.1

5. The **domain** of the function $f(x, y) = \ln(y - x + 2)$ is

$y - x + 2 > 0$

(a) $\{(x, y) : y > x - 2\}$ ______ (correct)

$\Rightarrow y > x - 2$

(b) $\{(x, y) : y \geq x - 2\}$

(c) $\{(x, y) : y = x - 2\}$

(d) $\{(x, y) : y > x - 1\}$

(e) $\{(x, y) : y < x - 2\}$

#57
§13.1

6. An equation of the **level curve** of the function $f(x, y) = \dfrac{x}{x^2 + y^2}$ that passes through the point $(1, 1)$ is

$\dfrac{x}{x^2+y^2} = C$

(a) $x^2 - 2x + y^2 = 0$ ______ (correct)

(b) $2x^2 - x + 2y^2 = 0$

(c) $x^2 - 3x + y^2 = -1$

(d) $2x^2 - 3x + 2y^2 = 1$

(e) $x^2 + x + y^2 = 3$

$\dfrac{1}{1+1} = C \Rightarrow C = \dfrac{1}{2}$

$\Rightarrow \dfrac{x}{x^2+y^2} = \dfrac{1}{2}$

$\Rightarrow 2x = x^2 + y^2$

$\Rightarrow x^2 - 2x + y^2 = 0$

~#60
§ 13.2

7. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\ln(2x^2+2y^2+1)}{x^2+y^2} = \lim_{r\to 0^+} \frac{\ln(2r^2+1)}{r^2}$

$\overset{LR}{=} \displaystyle\lim_{r\to 0^+} \frac{\frac{1}{2r^2+1}\cdot 4r}{2r}$

(a) 2 ______ (correct)

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) 0

(e) 1

$= \displaystyle\lim_{r\to 0^+} \frac{4r}{2r^2+1}\cdot\frac{1}{2r}$

$= \displaystyle\lim_{r\to 0^+} \frac{2}{2r^2+1} = \frac{2}{0+1} = 2$

#16
Review of Ch 13

8. Which one of the following statements is **TRUE**:

$$f(x,y) = \frac{x^2y}{x^4+y^2}$$

$\displaystyle\lim_{x\to 0} \frac{2x^3}{x^4+4x^2} = \lim_{x\to 0} \frac{2x}{x^2+4} = \frac{0}{0+4} = 0$

(a) along the line $y=2x$, $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = 0$ ______ (correct)

(b) along the line $y=-x$, $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = -1$ $\displaystyle = \lim_{x\to 0} \frac{-x^3}{x^4+x^2} = \lim_{x\to 0} \frac{-x}{x^2+1} = \frac{0}{0+1} = 0$

(c) along the parabola $y=x^2$, $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = 2$

$\displaystyle\lim_{x\to 0} \frac{x^4}{x^4+x^4} = \lim_{x\to 0} \frac{1}{2} = \frac{1}{2}$

(d) $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = \frac{1}{2}$

(e) $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = 0$

$\displaystyle\lim_{(x,y)\to 0} f(x,y)$ DNE

~Example 2

§ 13.3

9. The **slope** of the surface $f(x,y) = xe^{x^2y}$ at the point $(1, 2, e^2)$ in the x-**direction** is equal to

$f_x(x,y) = x \cdot e^{x^2y}(2xy) + e^{x^2y} \cdot 1$

(a) $5e^2$ ______________________________ (correct)

(b) $4e^2$

(c) $3e^2$

(d) $2e^2$

(e) e^2

slope $= f_x(1,2) = 1 \cdot e^2(4) + e^2 \cdot 1$
$= 4e^2 + e^2$
$= 5e^2$

#103

§ 13.3

10. If $z = e^{-t}\cos\left(\dfrac{x}{2}\right)$, then $\dfrac{\partial z}{\partial t} - 4\dfrac{\partial^2 z}{\partial x^2} =$

(a) 0 ______________________________ (correct)

(b) $2e^{-t}\cos\left(\dfrac{x}{2}\right)$

(c) $-2e^{-t}\cos\left(\dfrac{x}{2}\right)$

(d) $2e^{-t}\sin\left(\dfrac{x}{2}\right)$

(e) $-2e^{-t}\sin\left(\dfrac{x}{2}\right)$

$\dfrac{\partial z}{\partial t} = -e^{-t}\cos\left(\dfrac{x}{2}\right)$

$\dfrac{\partial z}{\partial x} = -e^{-t}\sin\left(\dfrac{x}{2}\right) \cdot \dfrac{1}{2}$

$\dfrac{\partial^2 z}{\partial x^2} = -e^{-t}\cos\left(\dfrac{x}{2}\right) \cdot \dfrac{1}{4}$

$\dfrac{\partial z}{\partial t} - 4\dfrac{\partial^2 z}{\partial x^2} = -e^{-t}\cos\left(\dfrac{x}{2}\right) - 4\left(-e^{-t}\cos\left(\dfrac{x}{2}\right)\cdot\dfrac{1}{4}\right)$
$= -e^{-t}\cos\left(\dfrac{x}{2}\right) + e^{-t}\cos\left(\dfrac{x}{2}\right)$
$= 0$

~ Example 3

§13.4 11. Let $z = \sqrt{29 - x^2 - y^2}$. If (x, y) moves from the point $(3, 4)$ to the point $(3.01, 3.97)$, then the change in z is approximately equal to

$dx = \Delta x = 3.01 - 3 = 0.01$

$dy = \Delta y = 3.97 - 4 = -0.03$

(a) $\frac{9}{200}$ ______________________________ (correct)

(b) $\frac{1}{20}$

(c) $\frac{1}{25}$

(d) $\frac{7}{200}$

(e) $\frac{3}{25}$

$$\Delta z \approx dz = \frac{-x}{\sqrt{29-x^2-y^2}}\,dx + \frac{-y}{\sqrt{29-x^2-y^2}}\,dy \qquad x=3,\ y=4$$

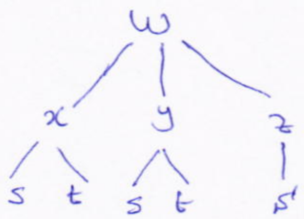
$$= \frac{-3}{2}(0.01) + \frac{-4}{2}(-0.03)$$

$$= \frac{-3}{200} + \frac{12}{200}$$

$$= \frac{9}{200}$$

~ Example 5

§13.5



12. Let $w = xy^2 + yz^2 + zx^2,\ x = s + t^2,\ y = \frac{s}{t},\ z = s^2$.

The value of $\frac{\partial w}{\partial t}$ when $s = 1$, $t = 1$ is equal to

$\Rightarrow x = 2,\ y = 1,\ z = 1$

(a) 5 ______________________________ (correct)

(b) 4

(c) 12

(d) 8

(e) 0

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= (y^2 + 2xz)\cdot(2t) + (2xy + z^2)\cdot\left(-\frac{s}{t^2}\right)$$

$$\left.\frac{\partial w}{\partial t}\right|_{\substack{s=1\\t=1}} = (1+4)(2) + (4+1)(-1)$$

$$= 10 - 5$$

$$= 5$$

~ #31

§13.5

13. If z is defined implicity as a differentiable function of x and y by the equation

$$xyz = \sin(3x - 2y - z),$$

then $\dfrac{\partial z}{\partial x} =$

Let $F(x,y,z) = \sin(3x-2y-z) - xyz$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

(a) $\dfrac{3\cos(3x-2y-z) - yz}{\cos(3x-2y-z) + xy}$ ________________ (correct)

$$= -\frac{3\cos(3x-2y-z) - yz}{-\cos(3x-2y-z) - xy}$$

$$= \frac{3\cos(3x-2y-z) - yz}{\cos(3x-2y-z) + xy}$$

(b) $\dfrac{3\sin(3x-2y-z) - yz}{\sin(3x-2y-z) + xy}$

(c) $\dfrac{\cos(3x-2y-z) + yz}{3\cos(3x-2y-z) - xy}$

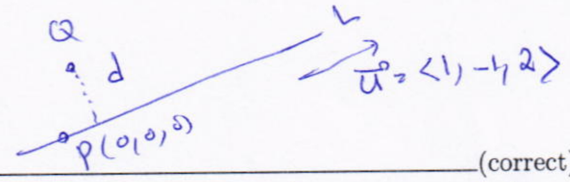
(d) $\dfrac{3yz - \cos(3x-2y-z)}{xy + \cos(3x-2y-z)}$

(e) $\dfrac{3\cos(3x-2y-z) + xy}{\cos(3x-2y-z) - yz}$

~ #95

§11.5

14. The **distance** between the point $Q(1,1,1)$ and the line $L: x = t,\ y = -t,\ z = 2t$ is equal to



(a) $\dfrac{\sqrt{21}}{3}$ ________________ (correct)

(b) $\dfrac{\sqrt{21}}{2}$

(c) $\dfrac{\sqrt{7}}{4}$

(d) $\dfrac{\sqrt{7}}{2}$

(e) $\dfrac{\sqrt{21}}{4}$

$$d = \frac{\|\vec{PQ} \times \vec{u}\|}{\|\vec{u}\|} = \frac{\sqrt{9+1+4}}{\sqrt{1+1+4}} = \frac{\sqrt{14}}{\sqrt{6}} = \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{21}}{3}$$

$$\vec{PQ} \times \vec{u} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = \langle 3, -1, -2 \rangle$$

~ #45

§11.6

15. The set of all points **equidistant** from the point $(0, 0, 3)$ and the plane $z = -1$ forms

(x, y, z) $z+1=0$

(a) an elliptic paraboloid ________________ (correct)

(b) a hyperbolic paraboloid

(c) an elliptic cone

(d) an ellipsoid

(e) a hyperboloid of one sheet

$$\sqrt{(x-0)^2+(y-0)^2+(z-3)^2} = \frac{|z+1|}{1}$$

$$\Rightarrow x^2+y^2+(z-3)^2 = |z+1|^2$$

$$\Rightarrow x^2+y^2+\cancel{z^2}-6z+9 = \cancel{z^2}+2z+1$$

$$\Rightarrow x^2+y^2+8 = 8z$$

or $\Rightarrow z = \frac{x^2}{8}+\frac{y^2}{8}+1$, an elliptic paraboloid

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	A $_{4}$	D $_{4}$	D $_{4}$	B $_{2}$
2	A	C $_{2}$	A $_{1}$	A $_{2}$	A $_{3}$
3	A	E $_{1}$	E $_{3}$	D $_{3}$	C $_{1}$
4	A	E $_{3}$	C $_{2}$	B $_{5}$	D $_{4}$
5	A	E $_{5}$	A $_{5}$	C $_{1}$	E $_{5}$
6	A	A $_{9}$	D $_{9}$	D $_{10}$	C $_{7}$
7	A	B $_{7}$	D $_{7}$	E $_{8}$	A $_{9}$
8	A	E $_{10}$	B $_{10}$	D $_{7}$	A $_{6}$
9	A	B $_{8}$	E $_{6}$	B $_{6}$	C $_{8}$
10	A	B $_{6}$	E $_{8}$	A $_{9}$	D $_{10}$
11	A	C $_{11}$	B $_{11}$	D $_{14}$	C $_{14}$
12	A	A $_{14}$	A $_{12}$	B $_{11}$	D $_{15}$
13	A	B $_{15}$	C $_{13}$	C $_{13}$	E $_{12}$
14	A	A $_{12}$	A $_{14}$	E $_{12}$	E $_{11}$
15	A	E $_{13}$	E $_{15}$	D $_{15}$	A $_{13}$