Key by Coole in last page

~#20 §10.3

1. An equation for the **tangent line** to the parametric curve

$$x = 1 + 2\sin t, \quad y = t - \cos^2 t$$

$t=0 \Rightarrow (x,y) = (1,-1)$

at the point corresponding to $t = 0$ is

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-2\cos t(-\sin t)}{2\cos t}$

slope $= \frac{dy}{dx}\Big|_{t=0} = \frac{1}{2}$

(a) $y = \frac{1}{2}x - \frac{3}{2}$ (correct)

(b) $y = 2x - 3$

(c) $y = -\frac{1}{2}x - \frac{1}{2}$

(d) $y = -2x + 1$

(e) $y = \frac{1}{2}x + \frac{3}{2}$

Eq: $y+1 = \frac{1}{2}(x-1)$

$\Rightarrow y = \frac{1}{2}x - \frac{1}{2} - 1$

$\Rightarrow y = \frac{1}{2}x - \frac{3}{2}$

~#3 §10.5

2. The **area** of the region inside the circle $r = 3\sin\theta$ for $\frac{\pi}{6} \le \theta \le \frac{\pi}{3}$ is equal to

$A = \int_{\pi/6}^{\pi/3} \frac{1}{2} r^2 \, d\theta$

(a) $\frac{3\pi}{8}$ (correct)

(b) $\frac{\pi}{6}$

(c) $\frac{5\pi}{4}$

(d) $\frac{2\pi}{5}$

(e) $\frac{5\pi}{8}$

$= \frac{1}{2}\int_{\pi/6}^{\pi/3} 9\sin^2\theta \, d\theta$

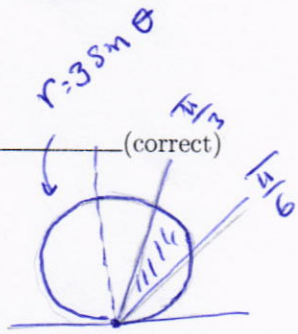
$= \frac{9}{4}\int_{\pi/6}^{\pi/3} [1-\cos(2\theta)] \, d\theta$

$= \frac{9}{4}\left[\theta - \frac{1}{2}\sin(2\theta)\right]_{\pi/6}^{\pi/3}$

$= \frac{9}{4}\left[\theta - \sin\theta\cos\theta\right]_{\pi/6}^{\pi/3}$

$= \frac{9}{4}\left[\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\cdot\frac{1}{2}\right) - \left(\frac{\pi}{6} - \frac{\sqrt{3}}{2}\cdot\frac{1}{2}\right)\right]$

$= \frac{9}{4}\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{9}{4}\cdot\frac{\pi}{6} = \frac{3\pi}{8}$



~ #22
§ 11.4

3. The **area of the parallelogram** that has the vectors

$$\vec{u} = \langle 1, -2, 3 \rangle,\ \vec{v} = \langle 2, -1, 0 \rangle$$

as adjacent sides is equal to

(a) $3\sqrt{6}$ ______ (correct)

(b) $4\sqrt{6}$

(c) $5\sqrt{6}$

(d) $2\sqrt{6}$

(e) $6\sqrt{6}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ 2 & -1 & 0 \end{vmatrix}$$

$$= \langle 3, 6, 3 \rangle$$

$$\Rightarrow \text{area} = \| \vec{u} \times \vec{v} \| = \sqrt{9+36+9} = \sqrt{9(1+4+1)} = 3\sqrt{6}$$

#48
§ 11.5

4. An **equation for the plane** that passes through the point $(1, 2, 3)$ and is parallel to the yz-plane is

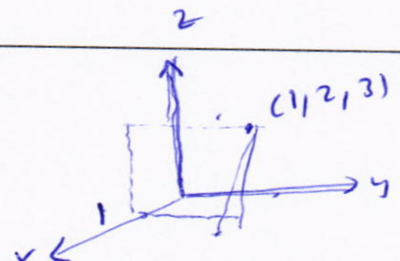
(a) $x = 1$ ______ (correct)

(b) $y = 2$

(c) $z = 3$

(d) $y + z = 5$

(e) $x + y = 3$



$\boxed{x = 1}$

~#59
§13.2

5. $\lim_{(x,y)\to(0,0)} \dfrac{1-\sqrt{1-x^2-y^2}}{x^2+y^2} = \lim_{r\to 0} \dfrac{1-\sqrt{1-r^2}}{r^2}$

$\overset{LR}{=} \lim_{r\to 0} \dfrac{0 - \frac{-2r}{2\sqrt{1-r^2}}}{2r}$

$= \lim_{r\to 0} \dfrac{r}{\sqrt{1-r^2}} \cdot \dfrac{1}{2r}$

$= \lim_{r\to 0} \dfrac{1}{2\sqrt{1-r^2}} = \dfrac{1}{2\sqrt{1-0}} = \dfrac{1}{2}$

(a) $\frac{1}{2}$ ______ (correct)
(b) 0
(c) $-\frac{1}{4}$
(d) -2
(e) does not exist

~#63
§13.2

6. The function $f(x,y,z) = \dfrac{\ln(1-z)}{e^x - e^y}$ is **continuous** on

(a) $\{(x,y,z) : x \neq y,\ z < 1\}$ ______ (correct)
(b) $\{(x,y,z) : x \neq y,\ z \leq 1\}$
(c) $\{(x,y,z) : x < y,\ z < 1\}$
(d) $\{(x,y,z) : x > y,\ z > 1\}$
(e) $\{(x,y,z) : x \neq y,\ z > 1\}$

- $1 - z > 0 \Rightarrow z < 1$
- $e^x - e^y \neq 0 \Rightarrow e^x \neq e^y \Rightarrow x \neq y$

#30
§13.6

7. If $f(x,y) = \dfrac{y+x}{y+1}$, then $\nabla f(0,1) =$

(a) $\frac{1}{2}\vec{i} + \frac{1}{4}\vec{j}$ ______ (correct)

(b) $\frac{1}{2}\vec{i}$

(c) $\vec{i} - \frac{1}{4}\vec{j}$

(d) $2\vec{i} + 4\vec{j}$

(e) $-\frac{1}{2}\vec{j}$

$$\nabla f(x,y) = \langle f_x, f_y \rangle$$
$$= \left\langle \frac{1}{y+1}, \frac{(y+1)-(y+x)}{(y+1)^2} \right\rangle$$
$$= \left\langle \frac{1}{y+1}, \frac{1-x}{(y+1)^2} \right\rangle$$
$$\nabla f(0,1) = \left\langle \frac{1}{2}, \frac{1}{4} \right\rangle$$

~#21
§13.6

8. The **directional derivative** of $f(x,y) = xy$ at $P(2,-1)$ in the direction of the vector $\vec{v} = \langle 1, 1 \rangle$ is equal to

(a) $\frac{\sqrt{2}}{2}$ ______ (correct)

(b) $\sqrt{2}$

(c) $2\sqrt{2}$

(d) $\frac{\sqrt{2}}{3}$

(e) 2

$$\nabla f(x,y) = \langle f_x, f_y \rangle = \langle y, x \rangle$$
$$\nabla f(2,-1) = \langle -1, 2 \rangle$$
$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{2}}\langle 1, 1 \rangle = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$
$$D_{\vec{v}} f(2,-1) = \nabla f(2,-1) \cdot \vec{u}$$
$$= \langle -1, 2 \rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$
$$= -\frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{2}$$

~ #7
§13.7

9. If $ax + by - z = d$ is an equation of the **tangent plane** to the paraboloid $z = x^2 + 2y^2 + 1$ at the point $(2, 1, 7)$, then $a + b + d =$

$F(x,y,z) = x^2+2y^2-z+1$

$\nabla F(x,y,z) = \langle 2x, 4y, -1\rangle$

(a) 13 ______ (correct)

(b) 8

(c) 10

(d) -7

(e) -6

$\nabla F(2,1,7) = \langle 4, 4, -1\rangle$

Eq:

$4(x-2)+4(y-1)-(z-7)=0$

$4x-8+4y-4-z+7=0$

$4x+4y-z=5$

$a=4, \quad b=4, \quad d=5$

$a+b+d=4+4+5=13$

#21
§13.8

10. If (a, b) is the only **critical point** of $f(x, y) = x^2 - xy - y^2 - 3x - y$, then

(a) f has a saddle point at (a, b) ______ (correct)

(b) f has a relative minimum at (a, b)

(c) f has a relative maximum at (a, b)

(d) $a + b = 3$

(e) $ab = -2$

$\begin{cases} f_x = 2x-y-3 = 0 \\ f_y = -x-2y-1 = 0 \end{cases} \Rightarrow \begin{cases} 2x-y-3=0 \\ -2x-4y-2=0 \end{cases}$

Sum: $-5y-5=0$

$\Rightarrow y=-1$

$\Rightarrow 2x+1-3=0 \Rightarrow x=1$

Critical pt is $(1,-1)$

$f_{xx} = 2$

$f_{yy} = -2$

$f_{xy} = -1$

$D(x,y) = 2\cdot(-2)-(-1)^2$

$= -4-1$

$= -5 < 0$ for all (x,y)

$\Rightarrow (-1,1)$ is a Saddle pt

(x, y, z)

~#5
§13.9

11. The **minimum distance** from the point $(-1,-2,0)$ to the surface $z=\sqrt{1-2x-2y}$ is equal to
(Hint: To simplify the computations, minimize the square of the distance)

$d^2 = (x+1)^2+(y+2)^2+(z-0)^2$

$f(x,y) = (x+1)^2+(y+2)^2+1-2x-2y$

(a) $\sqrt{5}$ ——— (correct)

(b) $2\sqrt{5}$

(c) $\frac{\sqrt{5}}{2}$

(d) $\sqrt{7}$

(e) $2\sqrt{7}$

$f_x = 2(x+1)-2 = 0 \Rightarrow x+1=1 \Rightarrow x=0$
$f_y = 2(y+2)-2 = 0 \Rightarrow y+2=1 \Rightarrow y=-1$
$\Rightarrow (x,y)=(0,-1)$

$f_{xx}=2$; $f_{xy}=0$; $f_{yy}=2$

$D(x,y) = 4$

$D(0,-1) = 4>0$, $f_{xx}(0,-1)=2>0$ $\Rightarrow$ min at $(0,-1)$
$\Rightarrow z=\sqrt{1-0+2}=\sqrt{3}$
$\Rightarrow (x,y,z)=(0,-1,\sqrt{3})$

$d^2 = (0+1)^2+(-1+2)^2+(\sqrt{3}-0)^2$
$= 1+1+3$
$= 5$
$\Rightarrow d=\sqrt{5}$

~#7
§13.10

12. The **maximum value** of

$$f(x,y) = 2x+2xy+y$$

subject to the constraint $2x+y=20$ is
$g(x,y)$

$f_x = \lambda g_x$, $f_y = \lambda g_y$, $g=20$ $\Rightarrow$
$2+2y = \lambda \cdot 2$ — (1)
$2x+1 = \lambda \cdot 1$ — (2)
$2x+y = 20$ — (3)

(1) & (2) $\Rightarrow 2+2y = 2(2x+1)$
$\Rightarrow y=2x$ — (4)
$\overset{(3)}{\Rightarrow} 2x+2x=20$

(a) 120 ——— (correct)

(b) 100

(c) 80

(d) 140

(e) 60

$\Rightarrow x=5$
$\overset{(4)}{\Rightarrow} y=10$
$\Rightarrow (x,y)=(5,10)$
$\Rightarrow f(5,10) = 10+100+10 = 120$

• $(0,20)$ on the graph of g &

$f(0,20) = 0+0+20$
$= 20 < 120$

So f is max at $(5,10)$.

#17
§14.1

13. $\int_0^{\frac{\pi}{2}} \int_0^{\cos x} (1 + \sin x)\, dy\, dx = \int_0^{\pi/2} (1+\sin x)\cdot y \Big]_{y=0}^{y=\cos x} dx$

$= \int_0^{\pi/2} (1+\sin x)\cdot \cos x \; dx$

$u = 1+\sin x \Rightarrow du = \cos x \; dx$

$x=0 \Rightarrow u=1$

$x=\frac{\pi}{2} \Rightarrow u=2$

$= \int_1^2 u \, du$

$= \frac{1}{2} u^2 \Big]_1^2 = \frac{1}{2}(4-1) = \frac{3}{2}$

(a) $\frac{3}{2}$ (correct)

(b) $\frac{1}{2}$

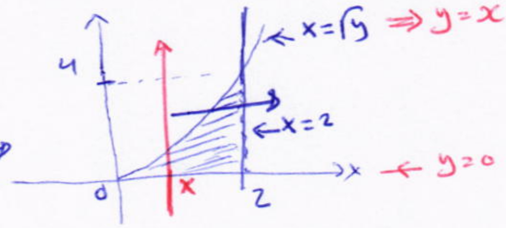
(c) 2

(d) $\frac{5}{2}$

(e) 1

#44
§14.1

14. $\int_0^4 \int_{\sqrt{y}}^2 f(x,y)\, dx\, dy =$ $\longrightarrow$



R: $0 \le x \le 2,\ 0 \le y \le x^2$

(a) $\int_0^2 \int_0^{x^2} f(x,y)\, dy\, dx$ (correct)

(b) $\int_0^2 \int_0^{\sqrt{x}} f(x,y)\, dy\, dx$

(c) $\int_0^4 \int_{\sqrt{x}}^2 f(x,y)\, dy\, dx$

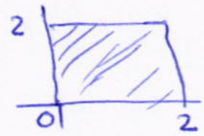
(d) $\int_0^2 \int_{x^2}^0 f(x,y)\, dy\, dx$

(e) $\int_{\sqrt{y}}^2 \int_0^4 f(x,y)\, dy\, dx$

\#53
§ 14.2

15. The **average value** of $f(x, y) = x^2 + y^2$ over the square region with vertices $(0, 0)$, $(0, 2)$, $(2, 0)$, $(2, 2)$ is equal to

area $= 2 \cdot 2$
$= 4$



(a) $\frac{8}{3}$ ——————————————————(correct)

(b) $\frac{16}{3}$

(c) 5

(d) $\frac{10}{3}$

(e) $\frac{20}{3}$

$$f_{ave} = \frac{1}{\text{area}(R)} \iint_R f \, dA$$

$$= \frac{1}{4} \int_0^2 \underbrace{\int_0^2 (x^2+y^2)\, dy}_{x^2 y + \frac{1}{3}y^3 \big]_{y=0}^{y=2} = 2x^2 + \frac{8}{3}} dx$$

$$= \frac{1}{4} \int_0^2 \left(2x^2 + \frac{8}{3}\right) dx$$

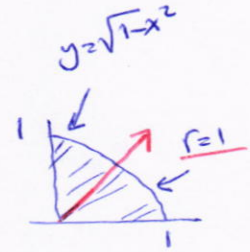
$$= \frac{1}{4} \cdot \left(\frac{2}{3}x^3 + \frac{8}{3}x\right)\Big]_0^2$$

$$= \frac{1}{4} \cdot \left(\frac{16}{3} + \frac{16}{3}\right) = \frac{1}{4} \cdot \frac{32}{3} = \frac{8}{3}$$

\#21
§ 14.3

16. $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2+y^2)^{\frac{3}{2}} \, dy \, dx =$

$R: \; 0 \le r \le 1$
$\quad 0 \le \theta \le \frac{\pi}{2}$



(a) $\frac{\pi}{10}$ ——————————————————(correct)

(b) $\frac{2\pi}{5}$

(c) $\frac{\pi}{5}$

(d) $\frac{3\pi}{10}$

(e) π

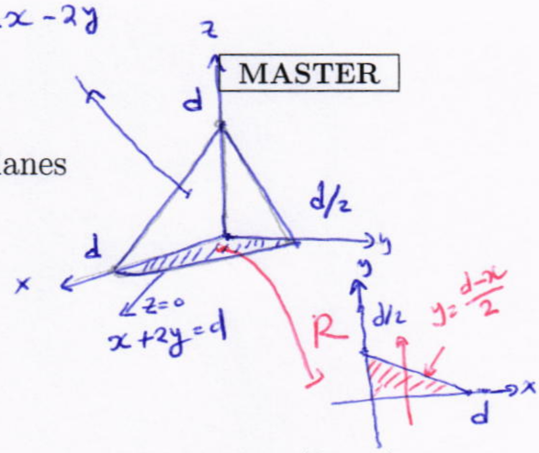
$$= \int_0^{\pi/2} \int_0^1 (r^2)^{3/2} \cdot r \, dr \, d\theta$$

$$= \int_0^{\pi/2} \underbrace{\int_0^1 r^4 \, dr}_{\frac{r^5}{5}\big]_{r=0}^{r=1} = \frac{1}{5}} d\theta$$

$$= \int_0^{\pi/2} \frac{1}{5} \, d\theta = \frac{1}{5}\theta\Big]_0^{\pi/2}$$

$$= \frac{\pi}{10}$$

$z = d - x - 2y$



~#13
§14.6

17. Let $d > 0$. If the **volume** of the solid bounded by the planes

$$x + 2y + z = d,\ x = 0,\ y = 0,\ z = 0$$

is equal to $\dfrac{16}{3}$, then $d =$

$V = \iiint dV$

$= \iint_R \left[\int_0^{d-x-2y} dz \right] dA$

(a) 4 ______________________________ (correct)

(b) 3

(c) 5

(d) 2

(e) 6

$R: 0 \le x \le d$
$0 \le y \le \frac{d-x}{2}$

$= \iint (d-x-2y)\, dA$

$= \int_0^d \underbrace{\int_0^{\frac{d-x}{2}} (d-x-2y)\, dy}\, dx$

$dy - xy - y^2 \Big]_{y=0}^{y=\frac{d-x}{2}}$

$d \cdot \frac{d-x}{2} - x \cdot \frac{d-x}{2} - \left(\frac{d-x}{2}\right)^2 = \frac{d-x}{2}(d-x) - \left(\frac{d-x}{2}\right)^2 = \frac{(d-x)^2}{2} - \frac{(d-x)^2}{4}$

$= \frac{(d-x)^2}{4}$

$= \frac{1}{4} \int_0^d (d-x)^2\, dx = -\frac{1}{4} \frac{(d-x)^3}{3} \Big]_{x=0}^{x=d}$

$= -\frac{1}{12}(0 - d^3) = \frac{d^3}{12} = \frac{16}{3} \Rightarrow d^3 = \frac{16 \cdot 12}{3} = 4 \cdot 4 \cdot 4 \Rightarrow d = 4$

#37
§ 11.7

18. The point with spherical coordinates $(\rho, \theta, \phi) = \left(4, \frac{\pi}{6}, \frac{\pi}{4}\right)$ is represented in rectangular coordinates by

(a) $(x, y, z) = (\sqrt{6}, \sqrt{2}, 2\sqrt{2})$ ______________________________ (correct)

(b) $(x, y, z) = (2\sqrt{6}, \sqrt{2}, \sqrt{2})$

(c) $(x, y, z) = (\sqrt{6}, 2\sqrt{2}, 1)$

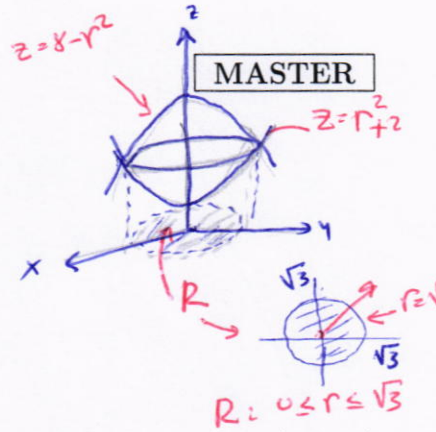
(d) $(x, y, z) = (\sqrt{6}, \sqrt{2}, \sqrt{2})$

(e) $(x, y, z) = (2\sqrt{6}, 2\sqrt{2}, 2\sqrt{2})$

$x = \rho \sin\phi \cos\theta = 4 \cdot \sin\frac{\pi}{4} \cos\frac{\pi}{6} = 4 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \sqrt{6}$

$y = \rho \sin\phi \sin\theta = 4 \cdot \sin\frac{\pi}{4} \sin\frac{\pi}{6} = 4 \cdot \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \sqrt{2}$

$z = \rho \cos\phi = 4 \cdot \cos\frac{\pi}{4} = 4 \cdot \frac{\sqrt{2}}{2} = 2\sqrt{2}$



~#18
§14.7

19. The **volume** of the solid bounded by the paraboloids

$$z = x^2 + y^2 + 2 \text{ and } z = 8 - x^2 - y^2$$

is equal to $x^2+y^2+2 = 8-x^2-y^2$
$\Rightarrow 2x^2+2y^2=6 \Rightarrow x^2+y^2=3$

(a) 9π ______ (correct)

(b) $4\pi\sqrt{3}$

(c) $2\pi\sqrt{3}$

(d) 6π

(e) $\frac{5\pi}{2}$

$$V = \iiint_D 1\, dV$$

$$= \int_0^{2\pi}\int_0^{\sqrt{3}} \Big[\underbrace{\int_{r^2+2}^{8-r^2} dz}_{z\big]_{r^2+2}^{8-r^2} = (8-r^2)-(r^2+2) = 6-2r^2} \Big] r\,dr\,d\theta$$

$$= \int_0^{2\pi} \underbrace{\int_0^{\sqrt{3}} (6r-2r^3)\,dr}_{3r^2-\frac{1}{2}r^4\big]_{r=0}^{r=\sqrt{3}} = 3\cdot 3-\frac{9}{2} = \frac{9}{2}}\, d\theta$$

$$= \int_0^{2\pi} \frac{9}{2}\,d\theta = \frac{9}{2}\theta\Big]_0^{2\pi} = \frac{9}{2}\cdot 2\pi = 9\pi$$

#69
Review
Ch 14

20. The **volume** of the solid bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the cone $z^2 = 3x^2 + 3y^2$ is equal to

$\hookrightarrow \rho^2 = 4 \Rightarrow \rho = 2$ (as $\rho \ge 0$)

$\hookrightarrow \tan^2\phi = \frac{1}{3}$
$\Rightarrow \tan\phi = \pm\frac{1}{\sqrt{3}} \Rightarrow \phi = \frac{\pi}{6}$

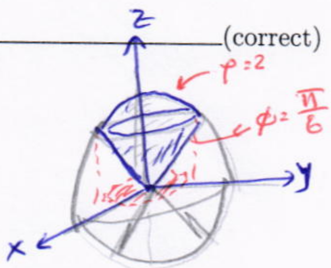
(a) $\frac{8\pi}{3}(2-\sqrt{3})$ ______ (correct)

(b) $\frac{8\pi}{3}\left(1-\frac{\sqrt{3}}{2}\right)$

(c) $\frac{4\pi}{3}(3-\sqrt{3})$

(d) $\frac{16\pi}{3}(\sqrt{3}-1)$

(e) $\frac{8\pi}{3}$



$D: 0\le\rho\le 2$
$0\le\phi\le\frac{\pi}{6}$
$0\le\theta\le 2\pi$

$$V = \iiint_D dV$$

$$= \int_0^{2\pi}\int_0^{\pi/6} \underbrace{\int_0^2 \rho^2 \sin\phi\, d\rho}_{\sin\phi\cdot\frac{1}{3}\rho^3\big]_{\rho=0}^{\rho=2}}\, d\phi\, d\theta$$

$$= \int_0^{2\pi} \underbrace{\int_0^{\pi/6} \frac{8}{3}\sin\phi\, d\phi}_{-\frac{8}{3}\cos\phi\big]_{\phi=0}^{\phi=\frac{\pi}{6}} = -\frac{8}{3}\left(\cos\frac{\pi}{6}-1\right)}\, d\theta$$

$$= \int_0^{2\pi} -\frac{8}{3}\left(\frac{\sqrt{3}}{2}-1\right) d\theta$$

$$= \frac{8}{3}\left(1-\frac{\sqrt{3}}{2}\right)\cdot\theta\Big]_0^{2\pi} = \frac{8}{3}\left(1-\frac{\sqrt{3}}{2}\right)\cdot 2\pi$$
$$= \frac{8\pi}{3}(2-\sqrt{3})$$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	D 19	B 12	D 1	E 4
2	A	A 2	A 5	A 9	B 16
3	A	A 16	A 9	C 5	C 2
4	A	D 13	C 4	A 16	D 15
5	A	B 3	A 8	C 3	D 18
6	A	B 18	A 14	E 18	C 1
7	A	A 10	A 13	E 7	C 8
8	A	C 5	C 11	B 8	E 3
9	A	E 17	B 19	D 20	D 14
10	A	B 1	D 2	A 6	C 6
11	A	D 20	C 10	B 14	B 5
12	A	C 14	C 16	E 11	C 17
13	A	E 15	A 18	E 13	D 9
14	A	B 7	A 15	B 17	B 7
15	A	D 11	D 17	B 19	A 12
16	A	E 12	E 7	D 4	E 13
17	A	C 9	B 3	C 15	E 20
18	A	D 8	D 1	C 2	B 10
19	A	B 4	B 6	C 12	B 19
20	A	E 6	A 20	D 10	B 11