Code-wise Key at the end

1. The parametric curve

~ #43 §10.3

$$x = 2 + t^2,\ y = 1 + t + t^3$$

is

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1+3t^2}{2t} = \frac{1}{2}t^{-1} + \frac{3}{2}t = y'$

$\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} = \frac{-\frac{1}{2}t^{-2} + \frac{3}{2}}{2t} = -\frac{1}{4}t^{-3} + \frac{3}{4}t^{-1}$

$= \frac{t^{-3}}{4}\left[-1 + 3t^2\right]$

$= \frac{3t^2 - 1}{4t^3}$

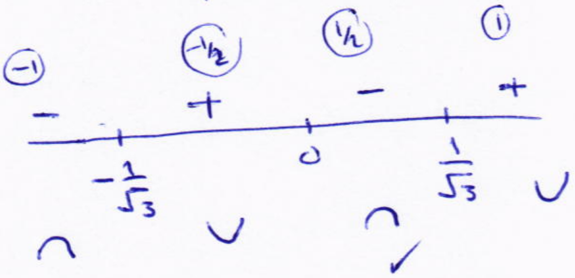
(a) concave downward on $\left(0, \frac{1}{\sqrt{3}}\right)$ ______ (correct)

(b) concave downward on $(0, \infty)$

(c) concave downward on $\left(-\frac{1}{\sqrt{3}}, 0\right)$

(d) concave upward on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(e) concave upward on $(0, \infty)$



2. The graph of the parametric curve

~ #42 §10.2

$$x = \sin t,\ \ y = \cos(2t),\ t \in (-\infty, \infty)$$

is

$\cos(2t) = 1 - 2\sin^2 t$

$\Rightarrow\ y = 1 - 2x^2$

(a) part of a parabola ______ (correct)

(b) a full parabola

(c) an ellipse

(d) part of a line

(e) a hyperbola

For $t \in (-\infty, \infty)$, $-1 \le x \le 1$ & $-1 \le y \le 1$

∴ the graph is part of a parabola.

~#36 §10.3

3. The parametric curve

$$x = t^2 - 2t + 1,\ y = t^3 - 6t^2$$

has a **vertical tangent** line at the point

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

V.T when $\frac{dx}{dt} = 0$ & $\frac{dy}{dt} \neq 0$

- $2t - 2 = 0 \Rightarrow t - 1 = 0 \Rightarrow t = 1$
- $\frac{dy}{dt} = 3t^2 - 12t \Big|_{t=1} \neq 0$ ok

(a) $(x, y) = (0, -5)$ ______ (correct)

$t = 1 \Rightarrow (x, y) = (0, -5)$.

(b) $(x, y) = (4, -7)$

(c) $(x, y) = (1, -16)$

(d) $(x, y) = (9, -32)$

(e) $(x, y) = (1, 0)$

~#50 §10.3

4. The **length** of the parametric curve

$$x = \frac{1}{3}t^3 - t,\ y = t^2,\ 0 \leq t \leq 3$$

is equal to

- $\frac{dx}{dt} = t^2 - 1$; $\frac{dy}{dt} = 2t$
- $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (t^2-1)^2 + (2t)^2 = t^4 - 2t^2 + 1 + 4t^2 = t^4 + 2t^2 + 1 = (t^2+1)^2$

(a) 12 ______ (correct)

$L = \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^3 (t^2+1)\, dt = \frac{t^3}{3} + t \Big]_0^3 = \frac{27}{3} + 3 - 0 = 9 + 3 = 12.$

(b) $\frac{14}{3}$

(c) $\frac{5}{3}$

(d) 9

(e) 15

5. The **slope** of the tangent line to the polar curve

~#64
§ 10.4

$$r = 2 + \cos(3\theta)$$

at the point corresponding to $\theta = \frac{\pi}{2}$ is equal to

(a) $-\frac{3}{2}$ ———— (correct)

(b) -1

(c) -3

(d) 3

(e) $\frac{1}{2}$

$$\frac{dy}{dx} = \frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{-r\sin\theta + \frac{dr}{d\theta}\cos\theta}$$

$$= \frac{[2+\cos(3\theta)]\cos\theta + [-3\sin(3\theta)]\sin\theta}{-[2+\cos(3\theta)]\sin\theta + [-3\sin(3\theta)]\cos\theta}$$

$$\text{Slope} = \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{(2+0)\cdot 0 + (3)\cdot 1}{-(2+0)\cdot 1 + (3)\cdot 0} = \frac{0+3}{-2+0} = -\frac{3}{2}$$

#40
§ 10.4

6. The polar equation $\theta = \frac{5\pi}{6}$ can be converted to the rectangular equation

$$\tan\theta = \tan\left(\frac{5\pi}{6}\right)$$

(a) $y = -\frac{\sqrt{3}}{3}x$ ———— (correct)

(b) $y = -\sqrt{3}x$

(c) $y = \sqrt{3}x$

(d) $y = -2\sqrt{3}x$

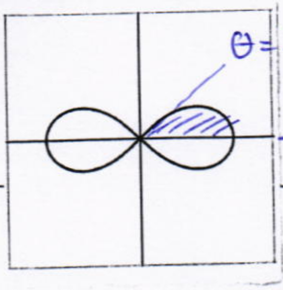
(e) $y = \frac{\sqrt{3}}{2}x$

$$\Rightarrow \frac{y}{x} = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow y = -\frac{1}{\sqrt{3}}x = -\frac{\sqrt{3}}{3}x$$

~#17

§10.5 7. The **area** of the region lying in the first quadrant and inside the polar curve $r^2 = 2\cos(2\theta)$ is equal to



(a) $\frac{1}{2}$ ——— (correct)

(b) 1

(c) $\frac{1}{\sqrt{2}}$

(d) $\sqrt{2}$

(e) $\frac{1}{4}$

$r = 0 \Rightarrow 2\cos(2\theta) = 0$

$\Rightarrow \cos(2\theta) = 0$

$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \dots$

↑ 1st quadrant

$A = \int_0^{\pi/4} \frac{1}{2} r^2 \, d\theta$

$= \frac{1}{2} \int_0^{\pi/4} 2\cos(2\theta) \, d\theta$

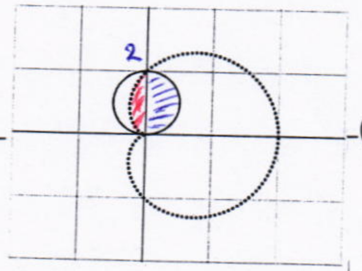
$= \int_0^{\pi/4} \cos(2\theta) \, d\theta = \frac{1}{2} \sin(2\theta) \Big]_0^{\pi/4}$

$= \frac{1}{2}(1 - 0) = \frac{1}{2}$.

#47

§10.5

8. The **area** of the region in the common interior of the polar curves $r = 2(1 + \cos\theta)$ and $r = 2\sin\theta$ is equal to



(a) $2\pi - 4$ ——— (correct)

(b) $2\pi - 2$

(c) $2\pi + 3$

(d) $2\pi + 5$

(e) $\pi + 6$

A_1 = area inside half the circle $r = 2\sin\theta$

$= \frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$

$A_2 = \int_{\pi/2}^{\pi} \frac{1}{2}[2(1+\cos\theta)]^2 \, d\theta = 2\int_{\pi/2}^{\pi} 1 + 2\cos\theta + \cos^2\theta \, d\theta$

$= 2\int_{\pi/2}^{\pi} 1 + 2\cos\theta + \frac{1+\cos(2\theta)}{2} \, d\theta$

$= 2\int_{\pi/2}^{\pi} \frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos(2\theta) \, d\theta$

$= 2\left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin(2\theta)\right]_{\pi/2}^{\pi}$

$= 2\left[\left(\frac{3}{2}\pi\right) - \left(\frac{3\pi}{4} + 2\right)\right] = 2\left(\frac{3\pi}{4} - 2\right) = \frac{3\pi}{2} - 4$

Total area

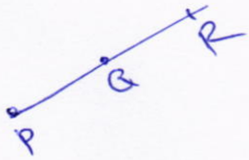
$= A_1 + A_2$

$= \frac{\pi}{2} + \frac{3\pi}{2} - 4$

$= 2\pi - 4$

~ Example 5

§11.2 9. If the points



$$P(1,-2,k),\ Q(k-1,1,0),\ R(4,7,-2k)$$

are **collinear**, then $k^3 - 2k + 10 =$

$\vec{PQ} = \langle k-2, 3, -k \rangle$

$\vec{PR} = \langle 3, 9, -3k \rangle$

(a) 31 ______ the points are collinear $\iff \vec{PQ} \parallel \vec{PR}$ ______ (correct)

(b) 29 $\iff$ one vector is a multiple of the other

(c) −21 $\Rightarrow \vec{PR} = 3\vec{PQ}$ " Compare the middle component

(d) −15

(e) 0

$\Rightarrow 3 = 3(k-2)$

$\Rightarrow k-2 = 1$

$\Rightarrow k = 3$

So $k^3 - 2k + 10 = 27 - 6 + 10 = 27 + 4 = 31.$

~ #67

§11.1

10. If $\vec{\mathbf{u}} = \langle a, b \rangle$, with $a < 0$, is a unit vector that is parallel to the tangent line to the curve $y = -x^3 + x^2 - 1$ at the point $(2, -5)$, then $ab =$

$y' = -3x^2 + 2x$

Slope $= -3x^2 + 2x \,\Big|_{x=2} = -12 + 4 = -8$

(a) $-\dfrac{8}{65}$ ______ (correct)

(b) $\dfrac{8}{65}$

(c) $-\dfrac{4}{63}$

(d) $\dfrac{8}{63}$

(e) $\dfrac{2}{65}$

- a vector parallel to the tangent line is $\langle 1, -8 \rangle$
- Unit vectors parallel to the tangent line are

$$\frac{\langle 1,-8\rangle}{\sqrt{1+64}} \text{ or } -\frac{\langle 1,-8\rangle}{\sqrt{1+64}}$$

$$= \left\langle \frac{1}{\sqrt{65}}, \frac{-8}{\sqrt{65}} \right\rangle \text{ or } \left\langle -\frac{1}{\sqrt{65}}, \frac{8}{\sqrt{65}} \right\rangle$$

$\big\} \; a < 0$

$$ab = \left(-\frac{1}{\sqrt{65}}\right)\left(\frac{8}{\sqrt{65}}\right) = -\frac{8}{\sqrt{65}}$$

~#73
§11.1

11. In the xy-plane, if the vector $\vec{\mathbf{u}}$ is of magnitude 2 and makes an angle of $\frac{\pi}{3}$ with the positive x-axis, and the vector $\vec{\mathbf{u}}+\vec{\mathbf{v}}$ is of magnitude $2\sqrt{3}$ and makes an angle of $\frac{\pi}{2}$ with the positive x axis, then

(a) $\vec{\mathbf{v}} = < -1, \sqrt{3} >$ ____________ (correct)

(b) $\vec{\mathbf{v}} = < 1, 2\sqrt{3} >$

(c) $\vec{\mathbf{v}} = < -1, 0 >$

(d) $\vec{\mathbf{v}} = < 0, -\sqrt{3} >$

(e) $\vec{\mathbf{v}} = < 1, \sqrt{3} >$

- $\vec{u} = 2 \langle \cos\frac{\pi}{3}, \sin\frac{\pi}{3} \rangle = 2\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle = \langle 1, \sqrt{3} \rangle$
- $\vec{u}+\vec{v} = 2\sqrt{3} \langle \cos\frac{\pi}{2}, \sin\frac{\pi}{2} \rangle = 2\sqrt{3}\langle 0, 1 \rangle = \langle 0, 2\sqrt{3} \rangle$

$\vec{v} = \vec{u}+\vec{v} - \vec{u} = \langle 0, 2\sqrt{3} \rangle - \langle 1, \sqrt{3} \rangle = \langle -1, \sqrt{3} \rangle$

~#41
§11.3

12. The **projection** of $\vec{\mathbf{u}} = < 0, 2, -3 >$ onto $\vec{\mathbf{v}} = < 1, -1, 2 >$ is

(a) $\left\langle -\frac{4}{3}, \frac{4}{3}, -\frac{8}{3} \right\rangle$ ____________ (correct)

(b) $\left\langle -1, 1, -\frac{2}{3} \right\rangle$

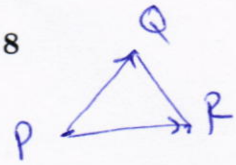
(c) $\left\langle \frac{2}{3}, -\frac{2}{3}, \frac{4}{3} \right\rangle$

(d) $\left\langle \frac{1}{3}, -\frac{1}{3}, \frac{1}{3} \right\rangle$

(e) $\left\langle 0, -\frac{8}{3}, 4 \right\rangle$

$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2} \vec{v}$: $\vec{u}\cdot\vec{v} = 0 - 2 - 6 = -8$, $\|\vec{v}\|^2 = 1 + 1 + 4 = 6$

$= \frac{-8}{6} \langle 1, -1, 2 \rangle = -\frac{4}{3} \langle 1, -1, 2 \rangle = \langle -\frac{4}{3}, \frac{4}{3}, \frac{-8}{3} \rangle$



~#26
§11.4

13. The **area** of the triangle with vertices

$$P(2,2,2),\ Q(5,4,0),\ R(-1,1,1)$$

is equal to

$A = \frac{1}{2}\|\vec{PQ}\times\vec{PR}\|$

• $\vec{PQ} = \langle 3, 2, -2\rangle$; $\vec{PR} = \langle -3,-1,-1\rangle$

(a) $\frac{\sqrt{106}}{2}$ ______ (correct)

(b) $\frac{3\sqrt{10}}{2}$

(c) 5

(d) $\frac{\sqrt{97}}{2}$

(e) $\frac{\sqrt{101}}{2}$

• $\vec{PQ}\times\vec{PR} = \begin{vmatrix} i & j & k \\ 3 & 2 & -2 \\ -3 & -1 & -1 \end{vmatrix} = -4i + 9j + 3k$

$A = \frac{1}{2}\sqrt{16+81+9} = \frac{1}{2}\sqrt{106}$

~#36
§11.4

14. Let $x > 0$. If the **volume** of the parallelepiped with adjacent edges

$$\vec{\mathbf{u}} = <2, x, -x>,\ \vec{\mathbf{v}} = <3, x, 4>,\ \vec{\mathbf{w}} = <5, -3, 3>$$

is equal to 55, then $x^3 + x^2 - 4 =$

$u\cdot(v\times w) = \begin{vmatrix} 2 & x & -x \\ 3 & x & 4 \\ 5 & -3 & 3 \end{vmatrix}$

(a) -2 ______ (correct)

(b) 2

(c) 0

(d) 3

(e) -5

$= 2(3x+12) - x(9-20) - x(-9-5x)$

$= 6x + 24 + 11x + 9x + 5x^2$

$= 5x^2 + 26x + 24 > 0$ as $x > 0$

Vol. $= 55 \Rightarrow |u\cdot(v\times w)| = 55$

$\Rightarrow 5x^2 + 26x + 24 = 55$

$\Rightarrow 5x^2 + 26x - 31 = 0$

$\Rightarrow (5x+31)(x-1) = 0$

$\Rightarrow x = -\frac{31}{5},\ x = 1$

$\Rightarrow x = 1$ as $x > 0$

so $x^3 + x^2 - 4 = 1 + 1 - 4 = -2$

15. Which one of the following statements is TRUE:

($\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{w}}$ are vectors in 3D-space, $\vec{\mathbf{o}}$ is the zero vector)

(a) $(\vec{\mathbf{u}}+\vec{\mathbf{v}})\times(\vec{\mathbf{u}}-\vec{\mathbf{v}}) = 2(\vec{\mathbf{v}}\times\vec{\mathbf{u}})$ ________________ (correct)

(b) If $\vec{\mathbf{u}}\cdot\vec{\mathbf{v}} = \vec{\mathbf{u}}\cdot\vec{\mathbf{w}}$ and $\vec{\mathbf{u}} \neq \vec{\mathbf{o}}$, then $\vec{\mathbf{v}} = \vec{\mathbf{w}}$

(c) $\| \vec{\mathbf{u}}+\vec{\mathbf{v}} \|^2 = \| \vec{\mathbf{u}} \|^2 + \| \vec{\mathbf{v}} \|^2$

(d) If $\vec{\mathbf{u}} \neq \vec{\mathbf{o}}$ and $\vec{\mathbf{v}} \neq \vec{\mathbf{o}}$, then $\vec{\mathbf{u}}\times\vec{\mathbf{v}} \neq \vec{\mathbf{o}}$

(e) $\vec{\mathbf{u}}\cdot\vec{\mathbf{v}} + \vec{\mathbf{u}}\times\vec{\mathbf{v}} = \vec{\mathbf{v}}\cdot\vec{\mathbf{u}} - (-\vec{\mathbf{v}})\times\vec{\mathbf{u}}$

a) $(\vec{u}+\vec{v})\times(\vec{u}-\vec{v}) = \vec{u}\times\vec{u} + (\vec{u}\times(-\vec{v})) + \vec{v}\times\vec{u} + \vec{v}\times(-v)$

$= \vec{u}\times\vec{u} - (\vec{u}\times\vec{v}) + \vec{v}\times\vec{u} - \vec{v}\times\vec{v}$

$= \vec{0} + \vec{v}\times\vec{u} + \vec{v}\times\vec{u} - \vec{0}$

$= 2(\vec{v}\times\vec{u})$

b) Take $\vec{u}=\vec{i}$, $\vec{v}=\vec{j}$, $\vec{w}=\vec{k}$

c) Take $\vec{u}=\langle 1,0,0\rangle$, $\vec{v}=\langle 1,1,0\rangle$

d) Take $\vec{u}=\vec{i}$, $\vec{v}=2\vec{i}$

e) Take $\vec{u}=\vec{i}$, $\vec{v}=\vec{j}$

or $\vec{u}\cdot\vec{v} + \vec{u}\times\vec{v} = \vec{v}\cdot\vec{u} - (-\vec{v})\times\vec{u}$

$\Rightarrow \vec{u}\times\vec{v} = \vec{v}\times\vec{u}$ False

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04	CODE05	CODE06	CODE07
1	A	E $_{1}$	B $_{1}$	D $_{1}$	A $_{1}$	B $_{3}$	C $_{1}$	E $_{2}$
2	A	E $_{3}$	B $_{3}$	C $_{3}$	D $_{2}$	C $_{4}$	A $_{4}$	E $_{3}$
3	A	B $_{4}$	D $_{4}$	D $_{4}$	B $_{3}$	D $_{1}$	B $_{2}$	D $_{1}$
4	A	B $_{2}$	E $_{2}$	D $_{2}$	C $_{4}$	B $_{2}$	B $_{3}$	D $_{4}$
5	A	E $_{8}$	A $_{7}$	B $_{8}$	D $_{7}$	C $_{7}$	A $_{8}$	B $_{6}$
6	A	E $_{7}$	B $_{6}$	C $_{6}$	A $_{5}$	D $_{6}$	C $_{5}$	C $_{8}$
7	A	E $_{6}$	D $_{5}$	A $_{7}$	A $_{6}$	C $_{5}$	B $_{7}$	B $_{5}$
8	A	D $_{5}$	A $_{8}$	D $_{5}$	B $_{8}$	B $_{8}$	A $_{6}$	C $_{7}$
9	A	D $_{13}$	D $_{15}$	C $_{10}$	E $_{15}$	E $_{15}$	E $_{11}$	B $_{10}$
10	A	A $_{12}$	B $_{9}$	A $_{9}$	A $_{11}$	A $_{12}$	A $_{15}$	A $_{12}$
11	A	D $_{10}$	A $_{13}$	B $_{12}$	D $_{12}$	C $_{14}$	D $_{12}$	B $_{14}$
12	A	B $_{15}$	E $_{10}$	A $_{11}$	B $_{14}$	A $_{10}$	D $_{10}$	A $_{9}$
13	A	B $_{14}$	D $_{14}$	C $_{14}$	E $_{13}$	B $_{11}$	C $_{14}$	B $_{15}$
14	A	E $_{11}$	B $_{12}$	E $_{13}$	C $_{10}$	D $_{9}$	C $_{9}$	E $_{11}$
15	A	C $_{9}$	B $_{11}$	D $_{15}$	D $_{9}$	B $_{13}$	D $_{13}$	E $_{13}$

Answer KEY

Q	MASTER	CODE08
1	A	E 3
2	A	C 2
3	A	B 4
4	A	B 1
5	A	A 8
6	A	C 7
7	A	B 6
8	A	D 5
9	A	E 12
10	A	E 14
11	A	B 9
12	A	A 13
13	A	C 11
14	A	D 10
15	A	C 15