King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 201

Major Exam II

242

14 April 2025

Net Time Allowed: 120

MASTER VERSION

#82
§11.5

1. Which one of the following statements is **FALSE** about the plane $2x - y + z = 4$?

$\vec{n} = \langle 2, -1, 1 \rangle$

(a) It is parallel to the plane $x - 2y + 2z = 2$ ____________________(correct)

$\vec{m} = \langle 1, -2, 2 \rangle$

$\vec{m} \not\parallel \vec{n}$ as $\vec{m} \times \vec{n} \neq \vec{0}$

(b) It has a normal vector $\vec{n} = < 2, -1, 1 >$

(c) It passes through the point $(2, 1, 1)$

(d) It intersects the x-axis at $(2, 0, 0)$

(e) The point $(0, -4, 0)$ lies in the plane

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§11.5

2. If (a, b, c) is the **point of intersection** between the lines

$$L_1 : x = 1 + 2t,\ y = 1 - t,\ z = 2 - 3t$$

$$L_2 : x = 4 - s,\ y = -1 + 2s,\ z = -4 + 6s$$

then $abc =$

$$\begin{matrix} x = x \\ y = y \\ z = z \end{matrix} \begin{cases} 1 + 2t = 4 - s \\ 1 - t = -1 + 2s \\ 2 - 3t = -4 + 6s \end{cases} \Rightarrow \begin{cases} 2t + s = 3 & \text{—(1)} \\ -t - 2s = -2 & \text{—(2)} \\ -3t - 6s = -6 & \text{—(3)} \end{cases}$$

(a) $\dfrac{22}{9}$ ____________________(correct)

Solve (1) & (2) : $\begin{cases} 2t + s = 3 \\ -t - 2s = -2 \end{cases} \Rightarrow \begin{cases} 4t + 2s = 6 \\ -t - 2s = -2 \end{cases}$

$3t = 4$

$\Rightarrow \boxed{t = \frac{4}{3}}$

(b) $-\dfrac{20}{9}$

(c) $\dfrac{22}{3}$

$\xrightarrow{(1)} \frac{8}{3} + s = 3$

(d) -7

$\Rightarrow \boxed{s = 3 - \frac{8}{3} = \frac{1}{3}}$

(e) $\dfrac{11}{9}$

Sub. in (3) : $-3\left(\frac{4}{3}\right) - 6\left(\frac{1}{3}\right) \stackrel{?}{=} -6$

$-4 - 2 = -6$ ✓

$t = \frac{4}{3} \xrightarrow{L_1} x = 1 + \frac{8}{3} = \frac{11}{3}$; $y = 1 - \frac{4}{3} = -\frac{1}{3}$; $z = 2 - 3 \cdot \frac{4}{3} = -2$

pt of int : $\left(\frac{11}{3}, -\frac{1}{3}, -2\right)$

$abc = \frac{11}{3}\left(-\frac{1}{3}\right)(-2) = \frac{22}{9}$

3. If $ax + by + cz = 5$ is an **equation for the plane** through the points $P(2, 2, 1)$ and $Q(-1, 1, -1)$ and is perpendicular to the plane $2x - 3y + z = 3$, then $a + b + c =$

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§ 11.5

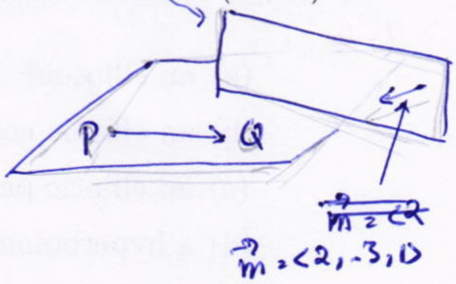
(a) -3 ____(correct)

(b) 4

(c) -2

(d) 1

(e) 0



$\vec{n} = \vec{PQ} \times \vec{m}$

$= \langle -3, -1, -2 \rangle \times \langle 2, -3, 1 \rangle$

$\begin{vmatrix} i & j & k \\ -3 & -1 & -2 \\ 2 & -3 & 1 \end{vmatrix}$

$= -7i - j + 11k$

$P(2,2,1)$

eq: $-7(x-2) - (y-2) + 11(z-1) = 0$

$-7x + 14 - y + 2 + 11z - 11 = 0$

$-7x - y + 11z + 5 = 0$

$\Rightarrow 7x + y - 11z = 5 \quad \Rightarrow a = 7,\ b = 1,\ c = -11$

$\Rightarrow a + b + c = 7 + 1 - 11 = -3$

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§ 11.6

4. The set of all points **equidistant** from the point $(0, 1, 0)$ and the xz-plane is

(x, y, z)

(a) an elliptic paraboloid ____(correct)

(b) an elliptic hyperbola

(c) an ellipsoid

(d) a plane

(e) a hyperboloid of one sheet

dist from (x,y,z) to $(0,1,0)$ = dist from (x,y,z) to the xz-plane

$\sqrt{x^2 + (y-1)^2 + z^2} = |y|$

$\Rightarrow x^2 + y^2 - 2y + 1 + z^2 = y^2$

$\Rightarrow x^2 - 2y + 1 + z^2 = 0$

$\Rightarrow 2y = x^2 + z^2 + 1$

an elliptic paraboloid

5. The graph of the equation $z^2 = 1 - 2x^2 - 3y^2$ is

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§11.6

(a) an ellipsoid ______________________________ (correct)
(b) an elliptic cone
(c) an elliptic paraboloid
(d) a hyperboloid of one sheet
(e) a hyperboloid of two sheets

$2x^2 + 3y^2 + z^2 = 1$
an ellipsoid

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§13.1

6. The **domain** of the function $f(x, y) = \dfrac{y-1}{\sqrt{x+1}}$ is

$x+1>0$, $y \in \mathbb{R}$

(a) $\{(x, y) : x \in (-1, \infty),\ y \in (-\infty, \infty)\}$ ______________________________ (correct)
(b) $\{(x, y) : x \in [-1, \infty),\ y \in (-\infty, \infty)\}$
(c) $\{(x, y) : x \in (-1, \infty),\ y \in [1, \infty)\}$
(d) $\{(x, y) : x \in (-\infty, \infty),\ y \in (-\infty, \infty)\}$
(e) $\{(x, y) : x \in (1, \infty),\ y \in (-1, \infty)\}$

$x > -1$, $y \in (-\infty, \infty)$

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§13.1

7. An equation for the **level curve** of $f(x,y) = \ln(x-y)$ that passes through the point $(e^2, 0)$ is given by

(a) $y = x - e^2$ ______ (correct)

(b) $y = x - 2$

(c) $y = ex - e^3$

(d) $y = x^2$

(e) $y = 2x - 2e^2$

$f(x,y) = c$

$\ln(x-y) = c$

Find c? $x = e^2,\ y = 0$

$\ln(e^2 - 0) = c \implies c = \ln(e^2) = 2.$

So $\ln(x-y) = 2$

$\implies x - y = e^2$

$\implies y = x - e^2$

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§13.2

8. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^3}{x^2+y^2} =$ $\displaystyle\lim_{r\to 0^+} \frac{r^3 \sin^3\theta}{r^2} = \lim_{r\to 0^+} r\sin^3\theta$

$-1 \le \sin^3\theta \le 1$

$-r \le r\sin^3\theta \le r$ $(r>0)$

$\downarrow_0 \qquad \downarrow_0$

(a) 0 ______ (correct)

(b) 1

(c) $\frac{1}{2}$

(d) DNE

(e) 2

$= 0$

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§13.3

9. If $z = \ln\sqrt{x^2+y^2}$, then $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} =$

(a) 1 ______________________________ (correct)

(b) z

(c) 0

(d) $-z$

(e) $\dfrac{z}{2}$

$z = \frac{1}{2}\ln(x^2+y^2)$

$\frac{\partial z}{\partial x} = \frac{1}{2}\cdot\frac{2x}{x^2+y^2} = \frac{x}{x^2+y^2}$

$\frac{\partial z}{\partial y} = \frac{1}{2}\cdot\frac{2y}{x^2+y^2} = \frac{y}{x^2+y^2}$

$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = x\cdot\frac{x}{x^2+y^2} + y\cdot\frac{y}{x^2+y^2}$

$= \frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2}$

$= \frac{x^2+y^2}{x^2+y^2} = 1$

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§13.3

10. If $f(x,y,z) = x^4 - 2xy + 3y^3 + xy^2z^4$, then $f_{xyzz}(x,y,z) =$

(a) $24yz^2$ ______________________________ (correct)

(b) $12yz^2$

(c) $6xyz^2$

(d) $24xyz^2$

(e) $12xz^2$

Switch the order of differentiation

$f_{xyzz}(x,y,z) = f_{zzxy}$

$f_z = 0 + 4xy^2z^3$

$f_{zz} = 12xy^2z^2$

$f_{zzx} = 12y^2z^2$

$f_{zzxy}(x,y,z) = 24yz^2$

11. Let $z = x^2 + y^2$. If (x, y) changes from $(2, 1)$ to $(2.1, 1.05)$, then using **differentials**, the change in z is approximately equal to

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§13.4

$\Delta z \approx dz$

(a) 0.5 ________________________________ (correct)
(b) 0.03
(c) -0.2
(d) 0.1
(e) 0.25

$dz = f_x\,dx + f_y\,dy$

$= (2x)\,dx + (2y)\,dy$; $x = 2,\ y = 1$

$= (2)(2)(0.1) + (2)(1)(0.05)$; $dx = \Delta x = 2.1 - 2 = 0.1$, $dy = \Delta y = 1.05 - 1 = 0.05$

$= 0.4 + 0.1$

$= 0.5$

So $\Delta z \approx 0.5$

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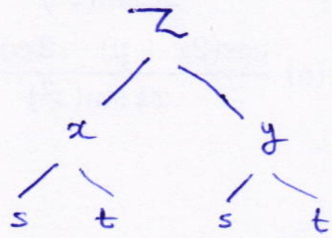
§13.5

12. If $z = \sin(x^2 y)$, $x = s^2 t$, $y = s + \dfrac{1}{t^2}$, then $\dfrac{\partial z}{\partial t} =$



(a) $2s^5 t \cos(x^2 y)$ ________________________________ (correct)
(b) $\left(2s^5 t - \dfrac{s^5}{t}\right) \cos(x^2 y)$
(c) $3s^4 t^2 \cos(x^2 y)$
(d) $\left(s^3 - \frac{s^2}{t}\right) \cos(x^2 y)$
(e) $2st^2 \cos(x^2 y)$

$\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x} \cdot \dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y} \cdot \dfrac{\partial y}{\partial t}$

$= 2xy \cos(x^2 y) \cdot s^2 + x^2 \cos(x^2 y) \cdot (-2t^{-3})$

$= (2xy\, s^2 - 2x^2 t^{-3}) \cos(x^2 y)$

$= \left[2 s^2 t \left(s + \frac{1}{t^2}\right) s^2 - 2 s^4 t^2 \cdot t^{-3}\right] \cos(x^2 y)$

$= \left[2 s^5 t + 2 s^4 \cdot \frac{1}{t} - 2 s^4 \cdot \frac{1}{t}\right] \cos(x^2 y)$

$= 2 s^5 t \cos(x^2 y)$

13. If z is defined **implicitly** as a differentiable function of x and y by the equation

$F = \sin(2x+y) - 2x\cos(z^2) = 1,$

then $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{2\cos(2x+y) - 2\cos(z^2)}{0 - 2x\cdot(-\sin(z^2)2z)}$

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§14.5

(a) $\dfrac{\cos(z^2)-\cos(2x+y)}{2xz\sin(z^2)}$ ______ (correct)

$= -\dfrac{2\cos(2x+y) - 2\cos(z^2)}{4xz\sin(z^2)}$

(b) $\dfrac{\cos(z^2)+\cos(2x+y)}{xz\sin(z^2)}$

$= \dfrac{2[\cos(z^2) - \cos(2x+y)]}{4xz\sin(z^2)}$

(c) $\dfrac{\cos(2x+y)-\cos(z^2)}{2xz\cos(z^2)}$

(d) $\dfrac{\cos(2x+y)-\cos(z^2)}{2z\sin(z^2)}$

$= \dfrac{\cos(z^2) - \cos(2x+y)}{2xz\sin(z^2)}$

(e) $\dfrac{\cos(2x+y)-2\cos(z^2)}{xz\sin(z^2)}$

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§13.6

14. The **maximum value** of the directional derivative of $f(x,y) = xe^{\frac{y}{x}}$ at the point $(1,2)$ is equal to

$\nabla f = \langle f_x, f_y \rangle = \langle x\cdot e^{y/x}\cdot(-\frac{y}{x^2}) + e^{y/x},\ x e^{y/x}\cdot\frac{1}{x} \rangle$

(a) $e^2\sqrt{2}$ ______ (correct)

(b) e^2

$= \langle -\frac{y}{x}e^{y/x} + e^{y/x},\ e^{y/x} \rangle$

(c) $2e^2$

(d) e^4

$\nabla f(1,2) = \langle -2e^2 + e^2,\ e^2 \rangle$

(e) e^2+1

$= \langle -e^2, e^2 \rangle$

max. value of the directional deriv. of f at (1,2) is

$\|\nabla f(1,2)\| = \sqrt{(-e^2)^2+(e^2)^2} = \sqrt{e^4+e^4} = \sqrt{2e^4} = e^2\sqrt{2}$

15. If $ax + by + cz = 10$ is an equation for the **tangent plane** to the surface

$F =$ $$x^2 + y^2 + z^2 - 8x - 12y + 4z + 42 = 0$$

at the point $(2, 3, -3)$, then $a + b + c =$

(a) 6 ____________________ (correct)

(b) 5

(c) 4

(d) 3

(e) 2

$\nabla F = \langle F_x, F_y, F_z \rangle$

$= \langle 2x - 8, 2y - 12, 2z + 4 \rangle$

$\nabla F(2,3,-3) = \langle 4-8, 6-12, -6+4 \rangle$

$= \langle -4, -6, -2 \rangle$

part of #44 § 13.7

$\vec{n} = \nabla F(2,3,-3) = \langle -4, -6, -2 \rangle$

Eq: $-4(x-2) - 6(y-3) - 2(z+3) = 0$

$(\div -2)$: $2(x-2) + 3(y-3) + (z+3) = 0$

$2x - 4 + 3y - 9 + z + 3 = 0$

$2x + 3y + z - 10 = 0$

$2x + 3y + z = 10$

$\Rightarrow a = 2, \; b = 3, \; c = 1$

$\Rightarrow a + b + c = 2 + 3 + 1 = 6$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04	CODE05	CODE06	CODE07
1	A	C $_{3}$	C $_{3}$	A $_{4}$	C $_{1}$	B $_{4}$	E $_{4}$	A $_{1}$
2	A	E $_{5}$	C $_{4}$	C $_{2}$	C $_{3}$	E $_{3}$	D $_{3}$	A $_{2}$
3	A	D $_{2}$	E $_{5}$	D $_{5}$	E $_{4}$	A $_{1}$	E $_{2}$	D $_{3}$
4	A	D $_{4}$	A $_{1}$	B $_{1}$	E $_{5}$	E $_{5}$	D $_{5}$	B $_{5}$
5	A	D $_{1}$	B $_{2}$	E $_{3}$	D $_{2}$	E $_{2}$	D $_{1}$	D $_{4}$
6	A	C $_{8}$	B $_{8}$	C $_{9}$	A $_{9}$	A $_{10}$	E $_{10}$	A $_{10}$
7	A	E $_{9}$	A $_{9}$	E $_{10}$	E $_{7}$	A $_{9}$	C $_{8}$	E $_{7}$
8	A	B $_{10}$	B $_{10}$	B $_{6}$	C $_{8}$	E $_{7}$	D $_{7}$	E $_{9}$
9	A	E $_{6}$	C $_{7}$	D $_{8}$	C $_{10}$	C $_{8}$	E $_{6}$	C $_{8}$
10	A	A $_{7}$	B $_{6}$	A $_{7}$	E $_{6}$	A $_{6}$	B $_{9}$	B $_{6}$
11	A	C $_{12}$	A $_{14}$	A $_{13}$	A $_{13}$	C $_{11}$	B $_{15}$	D $_{11}$
12	A	B $_{13}$	B $_{11}$	B $_{11}$	E $_{12}$	D $_{13}$	A $_{13}$	B $_{15}$
13	A	B $_{15}$	C $_{15}$	A $_{12}$	C $_{14}$	B $_{15}$	B $_{12}$	D $_{14}$
14	A	E $_{14}$	E $_{13}$	C $_{14}$	C $_{15}$	B $_{14}$	E $_{14}$	D $_{12}$
15	A	D $_{11}$	C $_{12}$	A $_{15}$	A $_{11}$	D $_{12}$	E $_{11}$	E $_{13}$

Answer KEY

Q	MASTER	CODE08
1	A	B 5
2	A	E 3
3	A	E 1
4	A	C 4
5	A	E 2
6	A	A 9
7	A	C 6
8	A	D 8
9	A	C 7
10	A	D 10
11	A	C 12
12	A	D 13
13	A	A 15
14	A	A 11
15	A	B 14