#29
§10.2

1. The parametric curve $x = 4\sec t$, $y = 3\tan t$ is represented in rectangular form by the equation

(a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$ ____________________(correct)

(b) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(c) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(d) $\frac{x}{4} + \frac{y}{3} = 1$

(e) $y = x^2 - 144$

$$1 + \tan^2 t = \sec^2 t$$
$$\Longrightarrow 1 + \left(\frac{y}{3}\right)^2 = \left(\frac{x}{4}\right)^2$$
$$\Longrightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

~#67
§10.5

2. The **length** of the polar curve

$$r = e^{3\theta},\ 0 \le \theta \le \frac{\pi}{3}$$

is equal to

$$L = \int_0^{\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \quad ; \quad \frac{dr}{d\theta} = 3e^{3\theta}$$
$$= \int_0^{\pi/3} \sqrt{e^{6\theta} + 9e^{6\theta}}\, d\theta$$
$$= \int_0^{\pi/3} \sqrt{10 e^{6\theta}}\, d\theta$$
$$= \int_0^{\pi/3} \sqrt{10}\, e^{3\theta}\, d\theta$$
$$= \frac{\sqrt{10}}{3} e^{3\theta}\Big]_0^{\pi/3}$$
$$= \frac{\sqrt{10}}{3}\left(e^{\pi} - 1\right)$$

(a) $\frac{\sqrt{10}}{3}(e^\pi - 1)$ ____________________(correct)

(b) $\frac{\sqrt{10}e^\pi}{3}$

(c) $e^\pi - 1$

(d) $3(e^\pi - 1)$

(e) $\sqrt{10}\,(e^\pi - 1)$

#19
§11.5

3. Which one of the following points lies on the line that passes through the point $(2, 3, 4)$ and is perpendicular to the plane given by $3x + 2y - z = 6$?

(a) $(-1, 1, 5)$ ______ (correct)
(b) $(0, 0, 0)$
(c) $(8, 7, 1)$
(d) $(4, 5, 3)$
(e) $(16, 8, 0)$

~ #91
§11.5

4. The **distance** between the planes

$$\mathcal{P}_1 : 2x - y + 3z = 1 \quad \ni (0,-1,0)$$
$$\mathcal{P}_2 : -4x + 2y - 6z = 3$$

is equal to

distance between $\mathcal{P}_1$ & $\mathcal{P}_2$
= distance from $(0,-1,0)$ to $\mathcal{P}_2$

$$= \frac{|-4(0)+2(-1)-6(0)-3|}{\sqrt{(-4)^2+(2)^2+(-6)^2}} = \frac{5}{\sqrt{16+4+36}}$$

$$= \frac{5}{\sqrt{56}} = \frac{5}{\sqrt{8\cdot 7}} = \frac{5}{2\sqrt{14}}$$

(a) $\dfrac{5}{2\sqrt{14}}$ ______ (correct)

(b) $\dfrac{3}{\sqrt{14}}$

(c) $\dfrac{1}{2\sqrt{14}}$

(d) $\dfrac{1}{\sqrt{14}}$

(e) $\dfrac{2}{\sqrt{14}}$

#45
§13.2

5. Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y}{x^2+y^2}$, if it exists.

(a) DNE ____ (correct)

(b) 0

(c) $\frac{1}{2}$

(d) -1

(e) 2

#86
§13.3

6. If $z = \arctan\left(\frac{y}{x}\right)$, then $z_{xx} - z_x z_y + z_{yy} =$

(a) $\frac{xy}{(x^2+y^2)^2}$ ____ (correct)

(b) $\frac{5xy}{(x^2+y^2)^2}$

(c) $\frac{3xy}{(x^2+y^2)^2}$

(d) $\frac{x-y}{(x^2+y^2)^2}$

(e) 0

~#20
§13.5

7. If $w = x^2 + y^2 + z^2$, $x = t\sin(2r)$, $y = t\cos(2r)$, $z = rt^2$, then the value of $\dfrac{\partial w}{\partial r}$, when $r = \dfrac{\pi}{8}$ and $t = 1$, is equal to

(a) $\dfrac{\pi}{4}$ ____ (correct)

(b) $2 + \dfrac{\pi}{4}$

(c) $2 + \dfrac{\pi}{8}$

(d) $\dfrac{\pi}{8}$

(e) 0

~#25
§13.6

8. Let $f(x, y) = 3x^2 - xy^2 + y^3$, $P(-1, 1)$ and $Q(2, -2)$. The **directional derivative** of f at P in the direction of $\overrightarrow{PQ}$ is equal to

(a) $-6\sqrt{2}$ ____ (correct)

(b) $-3\sqrt{2}$

(c) $4\sqrt{2}$

(d) $2\sqrt{2}$

(e) $-\dfrac{\sqrt{2}}{3}$

~#50

§ 13.7

9. If (a, b, c), with $a < 0$, is the point on the surface $x^2 + 4y^2 - z^2 = -4$ where the **tangent plane** is parallel to the plane $2x - 4y + 3z = 1$, then $a + b + c =$

(a) 4 ____________________ (correct)

(b) -3

(c) 6

(d) -1

(e) 0

~Example 3

§ 13.8

10. The graph of the function

$$f(x, y) = -2x^3 + 8xy - 4y^2 + 5$$

has

(a) one saddle point and one relative maximum ____________________ (correct)

(b) one saddle point and one relative minimum

(c) one relative minimum and one relative maximum

(d) no saddle point

(e) one saddle point and two relative maxima

$f_x = -6x^2 + 8y = 0$ — (1)

$f_y = 8x - 8y = 0$ — (2)

$(2) \Rightarrow y = x \overset{(1)}{\Rightarrow} -6x^2 + 8x = 0$

$\Rightarrow -2x(3x - 4) = 0$

$\Rightarrow x = 0, \; x = \frac{4}{3}$

$\Rightarrow (x, y) = (0, 0), \; (\frac{4}{3}, \frac{4}{3})$

• $f_{xx} = -12x$

$f_{yy} = -8$

$f_{xy} = 8$

• $D(x, y) = (-12x)(-8) - 8^2$

$= 8(12x - 8)$

$D(0, 0) = -64 < 0 \Rightarrow$ Saddle pt

$D(\frac{4}{3}, \frac{4}{3}) = 8(12 \cdot \frac{4}{3} - 8) = 8(16 - 8) = 8 \cdot 8 > 0$

$f_{xx}(\frac{4}{3}, \frac{4}{3}) = -12 \cdot \frac{4}{3} = -16 < 0$

$\Rightarrow$ l. max at $(\frac{4}{3}, \frac{4}{3})$

#43
§13.8

11. The **maximum value** of $f(x,y) = 3x^2 + 2y^2 - 4y$ over the region in the xy-plane bounded by the graphs of $y = x^2$ and $y = 4$ is equal to

(a) 28 ____ (correct)

(b) 16

(c) -2

(d) 0

(e) 20

~ #5
§ 13.9

12. If (a, b, c), with $c > 0$, is the point on the cone $z^2 = x^2 + y^2$ that is closest to the point $(1, 2, 0)$, then $8abc =$

(a) $2\sqrt{5}$ ____ (correct)

(b) $\sqrt{5}$

(c) $-3\sqrt{5}$

(d) $4\sqrt{5}$

(e) $5\sqrt{5}$

~ Example 1
§ 13.10

13. The **maximum value** of $f(x,y,z) = xyz$ on the sphere $x^2+y^2+z^2=12$ is equal to

$\Rightarrow x \neq 0, y \neq 0, z \neq 0$

(a) 8 ______ By Lagrange's Mult. ______(correct)

(b) 10

(c) 27

(d) 16

(e) 12

$$\begin{cases} yz = \lambda(2x) & -(1) \\ xz = \lambda(2y) & -(2) \\ xy = \lambda(2z) & -(3) \\ x^2+y^2+z^2 = 12 & -(4) \end{cases}$$

$\frac{(1)}{(2)}: \frac{y}{x} = \frac{x}{y} \Rightarrow y^2 = x^2$

$\frac{(2)}{(3)}: \frac{z}{y} = \frac{y}{z} \Rightarrow z^2 = y^2$

$\xrightarrow{(4)} x^2 + x^2 + x^2 = 12 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

$\Rightarrow (x,y,z) = (\pm 2, \pm 2, \pm 2)$ ← all possible combinations

$f(2,2,2) = 8$

$f(2,-2,2) = -8$. Max values is $\underline{\underline{8}}$

$\vdots$

$f \longrightarrow 8$ or -8

~ #14
§ 14.1

14. If $R = \{(x,y) : 0 \le x \le 1,\ 0 \le y \le \ln 3\}$, then $\iint_R ye^{xy}\, dA =$

(a) $2 - \ln 3$ ______(correct)

(b) $3 - \ln 3$

(c) $4 - \ln 3$

(d) $1 + \ln 3$

(e) $e - \ln 3$

$$\int_0^{\ln 3} \underbrace{\int_0^1 y e^{xy}\, dx}_{e^{xy}\big]_{x=0}^{x=1}}\, dy$$

$$\int_0^{\ln 3} (e^y - 1)\, dy$$

$$= e^y - y \Big]_{y=0}^{y=\ln 3}$$

$$= (3 - \ln 3) - (1 - 0)$$

$$= 3 - \ln 3 - 1$$

$$= 2 - \ln 3$$

~#49
§14.1

15. $\displaystyle\int_0^2 \int_{x^2}^{2x} f(x,y)\,dy\,dx =$

(a) $\displaystyle\int_0^4 \int_{y/2}^{\sqrt{y}} f(x,y)\,dx\,dy$ ——— (correct)

(b) $\displaystyle\int_0^2 \int_{y/2}^{\sqrt{y}} f(x,y)\,dx\,dy$

(c) $\displaystyle\int_0^4 \int_{y}^{2y^2} f(x,y)\,dx\,dy$

(d) $\displaystyle\int_0^2 \int_{y}^{2\sqrt{y}} f(x,y)\,dx\,dy$

(e) $\displaystyle\int_{x^2}^{2x} \int_0^2 f(x,y)\,dx\,dy$

~#26
§14.2

16. The **volume** of the solid lying in the **first octant** and bounded by the graphs of $z = 4 - y^2$, $y = x$, $y = 1$ is equal to

(a) $\dfrac{7}{4}$ ——— (correct)

(b) $\dfrac{5}{12}$

(c) $\dfrac{9}{14}$

(d) $\dfrac{11}{12}$

(e) 2

~#51

§14.2 17. Let $f(x,y) = 2x + y$ and $R = \{(x,y) : 0 \le x \le a,\ 0 \le y \le a\}$. If the **average value** of f over R is 6, then $a =$

$$f_{ave} = \frac{1}{\text{area}(R)} \iint_R f\, dA$$

(a) 4 ____ (correct)

(b) 6

(c) $\dfrac{3}{2}$

(d) $\dfrac{12}{5}$

(e) 2

$$6 = \frac{1}{a^2}\int_0^a \underbrace{\int_0^a (2x+y)\,dx}_{x^2+yx\big]_{x=0}^{x=a} = a^2+ay}\,dy$$

$$= \frac{1}{a^2}\int_0^a (a^2+ay)\,dy$$

$$= \frac{1}{a^2}\cdot a^2y + a\frac{y^2}{2}\Big]_{y=0}^{y=a} = \frac{1}{a^2}\left(a^3 + \frac{a^3}{2}\right)$$

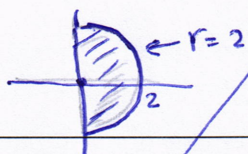
$$= \frac{a^3}{a^2}\cdot\frac{3}{2}$$

$$\Rightarrow 12 = 3a \Rightarrow a = 4$$

~#30

§14.3

18. If $R = \{(x,y) : x^2 + y^2 \le 4,\ x \ge 0\}$, then $\displaystyle\iint_R e^{-x^2-y^2}\, dA =$



$R: \ 0 \le r \le 2,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(a) $\dfrac{\pi}{2}(1 - e^{-4})$ ____ (correct)

(b) $\dfrac{\pi}{4}(1 - e^{-4})$

(c) $\pi(1 - e^{-4})$

(d) $\dfrac{\pi}{2}(2 + e^{-4})$

(e) $\dfrac{\pi}{4}(1 + e^{-4})$

$$= \int_{-\pi/2}^{\pi/2} \underbrace{\int_0^2 e^{-r^2}\, r\,dr}_{-\frac{1}{2}e^{-r^2}\big]_{r=0}^{r=2}}\, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} -\frac{1}{2}(e^{-4}-1)\,d\theta$$

$$= \frac{(1-e^{-4})}{2}\cdot \theta\Big]_{-\pi/2}^{\pi/2}$$

$$= \frac{1-e^{-4}}{2}\cdot\pi = \frac{\pi}{2}(1-e^{-4})$$

~#18
§14.7 19. The **volume** of the solid bounded above by $z = 3 - x^2 - y^2$ and below by $z = 2x^2 + 2y^2$ is equal to

(a) $\dfrac{3\pi}{2}$ ________________ (correct)

(b) 2π

(c) $\dfrac{5\pi}{2}$

(d) 3π

(e) $\dfrac{7\pi}{2}$

~ # 44
§ 14.7

20. If $D = \{(x, y, z) : 0 \leq z \leq \sqrt{4 - x^2 - y^2},\ 0 \leq y \leq \sqrt{4 - x^2},\ -2 \leq x \leq 2\}$, then $\displaystyle\iiint_D \sin(x^2 + y^2 + z^2)\, dV =$

(a) $\displaystyle\int_0^{\pi}\int_0^{\frac{\pi}{2}}\int_0^{2} \rho^2 \sin(\rho^2) \cdot \sin\phi \; d\rho\, d\phi\, d\theta$ ________________ (correct)

(b) $\displaystyle\int_0^{\pi}\int_0^{\pi}\int_0^{2} \rho^2 \sin\rho \cdot \sin\phi \; d\rho\, d\phi\, d\theta$

(c) $\displaystyle\int_0^{2\pi}\int_0^{\pi}\int_0^{4} \rho \sin(\rho^2) \cdot \sin\phi \; d\rho\, d\phi\, d\theta$

(d) $\displaystyle\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{2} \rho^2 \sin(\rho^2) \cdot \sin\phi \; d\rho\, d\phi\, d\theta$

(e) $\displaystyle\int_0^{\pi}\int_0^{\frac{\pi}{2}}\int_0^{4} \rho \sin(\rho^2) \cdot \sin\phi \; d\rho\, d\phi\, d\theta$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	D $_{2}$	C $_{1}$	D $_{3}$	D $_{4}$
2	A	C $_{1}$	C $_{2}$	B $_{1}$	B $_{3}$
3	A	C $_{4}$	B $_{3}$	C $_{2}$	C $_{1}$
4	A	C $_{3}$	C $_{4}$	B $_{4}$	D $_{2}$
5	A	B $_{5}$	C $_{5}$	E $_{7}$	B $_{6}$
6	A	E $_{7}$	E $_{8}$	A $_{5}$	B $_{5}$
7	A	D $_{6}$	B $_{7}$	B $_{6}$	A $_{8}$
8	A	A $_{8}$	C $_{6}$	B $_{8}$	B $_{7}$
9	A	E $_{12}$	C $_{11}$	C $_{12}$	A $_{10}$
10	A	D $_{13}$	C $_{9}$	D $_{10}$	E $_{13}$
11	A	B $_{11}$	B $_{12}$	E $_{9}$	E $_{12}$
12	A	C $_{10}$	E $_{10}$	D $_{13}$	D $_{9}$
13	A	E $_{9}$	E $_{13}$	B $_{11}$	E $_{11}$
14	A	E $_{17}$	C $_{16}$	A $_{20}$	B $_{17}$
15	A	E $_{16}$	A $_{17}$	E $_{18}$	D $_{14}$
16	A	D $_{19}$	E $_{18}$	B $_{17}$	D $_{18}$
17	A	B $_{18}$	A $_{15}$	B $_{19}$	E $_{19}$
18	A	A $_{15}$	C $_{19}$	B $_{14}$	A $_{15}$
19	A	A $_{20}$	B $_{20}$	D $_{15}$	C $_{20}$
20	A	C $_{14}$	E $_{14}$	E $_{16}$	E $_{16}$