1. [Q 24. sec. 14.1]
$$\int_1^e \int_0^y \frac{4}{x^2+y^2}\,dx\,dy =$$

(a) π (correct)

(b) $\frac{\pi}{2}$

(c) 2π

(d) 3π

(e) 4π

2. [Q. 63 sec. 14.1]
$$\int_0^1 \int_{2x}^2 4e^{y^2}\,dy\,dx =$$

(a) $e^4 - 1$ (correct)

(b) $e^3 - 1$

(c) $e^2 - 1$

(d) e^4

(e) $e^4 + 1$

3. [Q. 56 sec. 14.2]
The average value of $f(x, y) = \sin(x + y)$ over the rectangular region with vertices $(0, 0)$, $(\pi, 0)$, (π, π) and $(0, \pi)$ is equal to

(a) 0 (correct)
(b) π
(c) 2π
(d) 3π
(e) 1

4. [Q. 25 sec. 14.2]
The volume of the solid bounded by $z = 8(1 - xy)$, $z = 0$, $y = x$, $y = 1$ and $x = 0$ is equal to

(a) 3 (correct)
(b) 1
(c) 5
(d) 2
(e) 7

5. [Q. 35 sec. 14.2]
The volume of the solid region bounded above by the paraboloid $z = 4 - x^2 - y^2$ and the plane $z = 4 - 2x$ is equal to

(a) $\displaystyle\int_0^2 \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} (2x - x^2 - y^2)\, dy\, dx$ (correct)

(b) $\displaystyle\int_{-2}^2 \int_0^{\sqrt{2x-x^2}} (2x - x^2 - y^2)\, dy\, dx$

(c) $\displaystyle\int_0^4 \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} (4 - x^2 - y^2)\, dy\, dx$

(d) $\displaystyle\int_{-4}^4 \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} (2x - x^2 + y^2)\, dy\, dx$

(e) $\displaystyle\int_0^{\sqrt{2}} \int_{-\sqrt{2}}^{\sqrt{2}} (2x - x^2 - y^2)\, dy\, dx$

6. [Q. 27 sec. 14.3]
$$\int_0^2 \int_0^x \sqrt{x^2 + y^2}\, dy\, dx \;+\; \int_2^{2\sqrt{2}} \int_0^{\sqrt{8-x^2}} \sqrt{x^2 + y^2}\, dy\, dx =$$

(a) $\dfrac{4\sqrt{2}\pi}{3}$ (correct)

(b) $\dfrac{8\sqrt{2}\pi}{3}$

(c) $4\sqrt{2}\pi$

(d) $\dfrac{2\sqrt{2}\pi}{3}$

(e) $\dfrac{\sqrt{2}\pi}{3}$

7. [≃ Q. 31 sec. 14.7]
The volume of the solid inside the sphere $x^2 + y^2 + z^2 = 4$, outside the cone $z = \sqrt{x^2 + y^2}$, and above the xy-plane is

(a) $\dfrac{8\sqrt{2}\,\pi}{3}$ (correct)

(b) $\dfrac{16\sqrt{2}\,\pi}{3}$

(c) $\dfrac{8\pi}{3}$

(d) $\dfrac{4\sqrt{2}\,\pi}{3}$

(e) $8\sqrt{2}\,\pi$

8. [≃ Q. 15]
The volume of the solid inside both $x^2 + y^2 + z^2 = 4$ and $(x-1)^2 + y^2 = 1$ is equal to

(a) $\dfrac{16}{9}(3\pi - 4)$ (correct)

(b) $\dfrac{4}{9}(3\pi - 4)$

(c) $\dfrac{16}{3}(\pi - 2)$

(d) $\dfrac{32}{9}(\pi - 1)$

(e) $\dfrac{16}{9}(\pi - 4)$

9. [Q. 7 sec. 14.6]

$$\int_1^4 \int_0^1 \int_0^x 2\, z\, e^{-x^2}\, dy\, dx\, dz =$$

(a) $\frac{15}{2}\,(1 - e^{-1})$ (correct)

(b) $15\,(e^{-1} - 1)$

(c) $\frac{7}{2}\,(1 - e^{-1})$

(d) $\frac{15}{2}\,(1 + e^{-1})$

(e) $8\,(1 - e^{-1})$

10. [$\simeq$ Q. 8 Sec. 13.10]
The minimum value of $f(x, y) = \frac{3}{2}x + y + 1$ subject to the constraint $x^2y = 6$ is equal to

(a) $\frac{11}{2}$ (correct)

(b) 9

(c) $\frac{17}{2}$

(d) 14

(e) $\frac{15}{2}$

11. [$\simeq$ Example 1, sec. 13.9]
A rectangular box is resting on the xy-plane with one vertex at the origin and its opposite vertex lying in the plane $5x+3y+2z=30$. The maximum possible volume of the box is equal to

(a) $\dfrac{100}{3}$ (correct)

(b) 50

(c) 90

(d) $\dfrac{200}{3}$

(e) $\dfrac{125}{3}$

12. [Q. 40 sec. 13.8]
Let $f(x,y) = x^2 + xy$, and $R = \{(x,y) : |x| \leq 2,\ |y| \leq 1\}$. If M and m represent respectively the absolute maximum and absolute minimum of f over R, then $m + M =$

(a) $\dfrac{23}{4}$ (correct)

(b) $\dfrac{21}{4}$

(c) $\dfrac{11}{2}$

(d) $\dfrac{25}{4}$

(e) 6

13. [Q. 83 review]
The graph of the function

$$f(x, y) = xy + \frac{1}{x} + \frac{1}{y}$$

has

(a) a relative minimum at $(1, 1)$ (correct)
(b) a relative maximum at $(1, 1)$
(c) a saddle point at $(1, 1)$
(d) no relative extreme values
(e) a relative maximum at $(1, 1)$ and a relative minimum at $(-1, -1)$

14. [Q. 51 sec. 13.7]
Let $P(a, b, c)$, with $a > 0$ be the point on the ellipsoid $x^2 + 4y^2 + z^2 = 9$ where the tangent plane is perpendicular to the line

$$L: \ x = 2 - 4t, \qquad y = 1 + 8t, \qquad z = 3 - 2t.$$

Then $a + b + c =$

(a) 2 (correct)
(b) 0
(c) 1
(d) 5
(e) 6

15. [Q. 35 Sec. 13.6]
The maximum value of the directional derivative of $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point $P(1, 4, 2)$ is

(a) 1 (correct)

(b) 0

(c) $\dfrac{1}{\sqrt{21}}$

(d) $\sqrt{21}$

(e) 2

16. [Q. 58 sec. 13.2]

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2} =$$

(a) 1 (correct)

(b) 0

(c) -1

(d) π

(e) $-\pi$

17. [$\simeq$ Example 1 sec. 13.1]

The domain of the function $f(x, y) = \dfrac{\sqrt{x^2 + y^2 - 9}}{e^{x+y}}$ is

(a) $\{(x, y) : x^2 + y^2 \geq 9\}$ (correct)

(b) $\{(x, y) : x^2 + y^2 \geq 9,\ x + y \neq 0\}$

(c) $\{(x, y) : x^2 + y^2 \geq 9,\ x = 0\}$

(d) $\{(x, y) : x^2 + y^2 > 9,\ y = 0\}$

(e) $\{(x, y) : x^2 + y^2 \leq 9\}$

18. [Q. 42 sec. 11.3]

The vector component of $\vec{u} = \langle 8,\ 2,\ 0 \rangle$ **orthogonal** to $\vec{v} = \langle 2,\ 1,\ -1 \rangle$ is

(a) $\langle 2,\ -1,\ 3 \rangle$ (correct)

(b) $\langle 6,\ 3,\ -3 \rangle$

(c) $\langle 8,\ 2,\ 0 \rangle$

(d) $\langle -2,\ 1,\ -3 \rangle$

(e) $\langle 4,\ 1,\ 1 \rangle$

19. [$\simeq$ Q. 34 Review]
The area of the parallelogram that has the vectors $\vec{u} = \langle 1,\, 2,\, 2\rangle$ and $\vec{v} = \langle 2,\, 1,\, -2\rangle$ as adjacent sides is equal to

(a) 9 (correct)
(b) 4
(c) 6
(d) 8
(e) 10

20. [Q. 91. sec. 10.3]
The slope of the parametric curve $x = 2\big(\theta - \sin\theta\big), \quad y = 2\big(1 - \cos\theta\big)$ at the point corresponding to $\theta = \dfrac{\pi}{6}$ is equal to

(a) $2 + \sqrt{3}$ (correct)
(b) $1 + \sqrt{3}$
(c) $\sqrt{3}$
(d) $2 - \sqrt{3}$
(e) 10