King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 201
Final Exam
251
16 December 2025
Net Time Allowed: 120 Minutes

MASTER VERSION

1. $\int_0^{\sqrt{\pi/2}} \int_x^{\sqrt{\pi/2}} \int_2^{12} \sin(y^2)\, dz\ dy\ dx =$

   (a) 5 (correct)

   (b) $2\sqrt{\pi} + 12$

   (c) $2\sqrt{\frac{\pi}{2}} + 12$

   (d) 10

   (e) $12 - \sqrt{\frac{\pi}{2}}$

2. The **maximum value** of $f(x, y) = 6xy$, where $x > 0$ and $y > 0$, subject to the constraint $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is equal to

   (a) 18 (correct)

   (b) 36

   (c) 26

   (d) 44

   (e) 47

3. Let (a, b, c) be a point on the line

$$x = 3t + 1, y = t - 1, z = 4t + 2.$$

If the distance from this point to the plane $2x + 2y + z = 2$ is 8, then a possible value of $a^2 + b^2 + c^2 =$

(a) 150 _____ (correct)

(b) 79

(c) 97

(d) 158

(e) 12

4. If D is the **distance** between the point $Q(0, -2, 2)$ and the line given by the set of parametric equations $x = t$, $y = 2t - 3$, $z = 2t$, then $D =$

(a) 1 _____ (correct)

(b) 8

(c) 9

(d) 3

(e) 2

5. The point with rectangular coordinates $(x, y, z) = (\sqrt{3}, 1, 2\sqrt{3})$ is represented in spherical coordinates by $(\rho, \theta, \phi) =$

(a) $\left(4, \frac{\pi}{6}, \frac{\pi}{6}\right)$ _____(correct)

(b) $\left(4, \frac{\pi}{3}, \frac{\pi}{6}\right)$

(c) $\left(4, \frac{\pi}{6}, \frac{\pi}{3}\right)$

(d) $\left(4, \frac{\pi}{6}, \frac{7\pi}{6}\right)$

(e) $\left(4, \frac{7\pi}{6}, \frac{\pi}{6}\right)$

6. The area of the region that lies inside $r = 1 + \cos\theta$ and outside $r = \cos\theta$ is

(a) $\frac{5\pi}{4}$ _____(correct)

(b) $\frac{7\pi}{4}$

(c) $\frac{3\pi}{2}$

(d) $\frac{5\pi}{2}$

(e) $\frac{\pi}{2}$

7. If $w = x^2 + y^2 + z^2$, $x = t \sin s$, $y = t \cos s$, $z = s\ t^2$, then the value of $\dfrac{\partial w}{\partial t}$, when $s = 3$ and $t = 2$, is equal to

   (a) 292 (correct)

   (b) 220

   (c) $288 + \dfrac{\pi^2}{8}$

   (d) $216 + \dfrac{\pi^2}{4}$

   (e) $220 + \dfrac{\pi^2}{9}$

8. The surface of a mountain is modeled by the equation

$$h(x, y) = 5000 - 0.001x^2 - 0.004y^2$$

   A mountain climber is at the point (500, 300, 4390). Which of the following vectors points in the direction of **steepest descent** (descend at the greatest rate) ?

   (a) $5\ \mathbf{i} + 12\ \mathbf{j}$ (correct)

   (b) $5\ \mathbf{i} - 12\ \mathbf{j}$

   (c) $-5\ \mathbf{i} + 3\ \mathbf{j}$

   (d) $-\ \mathbf{i} - 4\ \mathbf{j}$

   (e) $-5\ \mathbf{i} - 3\ \mathbf{j}$

9. If (a, b, c), with $a < 0$, is the point on the surface $4x^2 + 2y^2 - z^2 = 100$ where the **tangent plane** is parallel to the yz-plane , then $a^2 + b^2 + c^2 =$

(a) 25 _____ (correct)

(b) 16

(c) 225

(d) 100

(e) 256

10. The graph of the function

$$f(x, y) = 4x - 4xy + 2y^2 - 4y + x^3 + 10$$

has

(a) one saddle point and one relative minimum _____ (correct)

(b) one saddle point and one relative maximum

(c) one relative minimum and one relative maximum

(d) no saddle point

(e) one saddle point and two relative maxima

11. The **maximum value** of $f(x, y) = 4x^2 + 4y^2 - 8y$ over the region in the xy-plane bounded by the graphs of $y = 2x^2$ and $y = 2$ is equal to

(a) 4 (correct)

(b) 2

(c) -2

(d) 0

(e) 28

12. If (a, b, c) is the point on the paraboloid $z = x^2 + y^2$ that is closest to the point $(3, 3, 1)$, then $a^3 + b^3 + c^3 =$

(a) 10 (correct)

(b) 6

(c) $64 + 4\sqrt{2}$

(d) 0

(e) $64 - 4\sqrt{2}$

13. The **highest** point on the curve of intersection of the cone $x^2 + y^2 - z^2 = 0$ and the plane $x + 4z = 9$ is

(a) $(-3, 0, 3)$ (correct)
(b) $(-4, 0, 4)$
(c) $(1, \sqrt{3}, 2)$
(d) $(5, 2, 1)$
(e) $(3, -3, 3)$

14. If $R = \{(x, y) : 0 \leq x \leq \frac{\pi}{4},\ 0 \leq y \leq 1\}$, then $\iint_R 4\sqrt{2}\ y\ \cos x\, dA =$

(a) 2 (correct)
(b) $\dfrac{\sqrt{2}}{4}$
(c) $\dfrac{\sqrt{2}}{2}$
(d) $4\sqrt{2}$
(e) 4

15. $\int_0^4 \int_0^y f(x,y)\,dx\,dy =$

(a) $\int_0^4 \int_x^4 f(x,y)\,dy\,dx$ _____(correct)

(b) $\int_0^4 \int_4^x f(x,y)\,dy\,dx$

(c) $\int_{-4}^0 \int_x^4 f(x,y)\,dy\,dx$

(d) $\int_{-4}^0 \int_0^x f(x,y)\,dy\,dx$

(e) $\int_0^4 \int_{4x}^0 f(x,y)\,dy\,dx$

16. The **volume** of the solid region bounded by the graphs of the equations

$$z = 4 - x^2 - y^2 \qquad \text{and} \qquad z = 4 - 2x$$

is equal to

(a) $\int_0^2 \int_{-\sqrt{1-(x-1)^2}}^{\sqrt{1-(x-1)^2}} (2x - x^2 - y^2)\,dy\,dx$ _____(correct)

(b) $\int_0^2 \int_{-\sqrt{1-(x+1)^2}}^{\sqrt{1-(x+1)^2}} (x^2 + y^2 + 2x)\,dy\,dx$

(c) $-\int_{-2}^0 \int_{-\sqrt{1-(x-1)^2}}^{\sqrt{1-(x-1)^2}} (x^2 + y^2 + 2x)\,dy\,dx$

(d) $\int_{-2}^2 \int_0^{\sqrt{1-(x+1)^2}} (2x - x^2 - y^2)\,dy\,dx$

(e) $\int_0^2 \int_{-\sqrt{1+(x-1)^2}}^{\sqrt{1+(x-1)^2}} (2x - x^2 - y^2)\,dy\,dx$

17. Let $f(x,y) = 2xy$ and R be the triangular region with vertices $(0,0), (-3,-1)$, and $(0,-1)$. Then the **average value** of f over R is equal to

(a) $\frac{3}{2}$ (correct)

(b) $\frac{8}{3}$

(c) $\frac{2}{3}$

(d) $\frac{9}{4}$

(e) $\frac{27}{8}$

18. $\int_0^{3/\sqrt{2}} \int_y^{\sqrt{9-y^2}} \sqrt{x^2+y^2}\, dx\ dy =$

(a) $\frac{9\pi}{4}$ (correct)

(b) $\frac{13\pi}{9}$

(c) $\frac{15\pi}{4}$

(d) $\frac{7\pi}{2}$

(e) $\frac{8\pi}{5}$

19. The **volume** of the solid region bounded below by the upper nappe of the cone $z^2 = x^2 + y^2$ and above by the sphere $x^2 + y^2 + z^2 = 4$ is equal to

(a) $\frac{8\pi}{3}(2 - \sqrt{2})$ _____(correct)

(b) $\frac{26\pi}{3}(2 - \sqrt{2})$

(c) $9\pi(2 - \sqrt{2})$

(d) $\frac{8\pi}{3}(2 + \sqrt{2})$

(e) $\frac{8\pi}{3}(5 + \sqrt{2})$

20.

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx =$$

(a) $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^3 \rho^3 \sin\phi \; d\rho\, d\phi\, d\theta$ _____(correct)

(b) $\int_0^{\pi} \int_0^{\frac{\pi}{2}} \int_0^3 \rho^3 \sin\phi \; d\rho\, d\phi\, d\theta$

(c) $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^3 \rho^{5/2} \sin\phi \; d\rho\, d\phi\, d\theta$

(d) $\int_0^{\pi} \int_0^{\pi} \int_0^3 \rho^3 \sin\phi \; d\rho\, d\phi\, d\theta$

(e) $\int_0^{\frac{\pi}{2}} \int_0^{\pi} \int_0^3 \rho^{5/2} \sin\phi \; d\rho\, d\phi\, d\theta$

21. Let L be the arc length of the curve represented by the parametric equations

$$x = ae^{-t}\cos t, \qquad y = ae^{-t}\sin t, \qquad 0 \le t \le \frac{\pi}{2}.$$

If $L = 4(1 - e^{-\frac{\pi}{2}})$, then $a =$

(a) $\sqrt{8}$ (correct)
(b) $\sqrt{2}$
(c) $\sqrt{18}$
(d) 4
(e) 2