King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 202
Final Exam
211
22 December, 2021

EXAM COVER

Number of versions: 4
Number of questions: 10
Number of Answers: 5

King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 202

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211

22 December, 2021

Net Time Allowed: 150 Minutes

MASTER VERSION

1. If the differential equation

$$\left(g(x)y^3 - \frac{1}{1+9x^2}\right)\frac{dx}{dy} + x^3y^2 = 0$$

is exact, then $g(1) =$

(a) 1 (correct)

(b) 2

(c) -2

(d) 3

(e) -3

2. The solution of the differential equation

$$\frac{dy}{dx} = (x+y+1)^2$$

is given by

(a) $y = -x - 1 + \tan(x+c)$ (correct)

(b) $y = x - 1 + \tan(x+c)$

(c) $y = 2x - 1 + \tan(x+c)$

(d) $y = -2x + 1 + \tan(x+c)$

(e) $y = x + 1 - \tan(2x+c)$

3. The function $y_1 = x + 1$ is a solution of the differential equation

$$(1 - 2x - x^2)y'' + 2(1 + x)y' - 2y = 0$$

The method of Reduction of order produces the second solution $y_2 =$

(a) $x^2 + x + 2$ (correct)
(b) $2x^2 - x + 1$
(c) $x^2 - x + 3$
(d) $x^2 + x + 3$
(e) $x^2 + 2$

4. A linear differential operator that annihilates the function

$$e^{-x} \sin x - e^{2x} \cos x$$

is give by

(a) $D^4 - 2D^3 - D^2 + 2D + 10$ (correct)
(b) $D^4 + 2D^3 - D^2 + 2D + 10$
(c) $D^4 - 2D^3 + D^2 - 2D + 10$
(d) $D^4 + 2D^3 + D^2 + 2D + 10$
(e) $D^4 - 2D^3 + D^2 + 2D - 10$

5. The solution $y(x)$ of the third order initial value problem

$$y''' + 36y' = 0, \ y(0) = 0, \ y'(0) = -6, \ y''(0) = -36$$

satisfies $y\left(\frac{\pi}{2}\right) =$

(a) -2 (correct)

(b) 2

(c) 3

(d) -3

(e) 0

6. If the particular solution of the differential equation

$$y'' + 3y' + 2y = \frac{1}{1+e^x}$$

has the form $y_p(x) = e^{-x}u_1(x) + e^{-2x}u_2(x)$, then $u_1(0) =$

(a) $\ln 2$ (correct)

(b) $-\ln 2$

(c) $-\ln 3$

(d) $\ln 3$

(e) 0

7. The general solution of the Cauchy-Euler differential equation $x^3y''' - 6y = 0$ is given by

(a) $c_1x^3 + c_2\cos(\sqrt{2}\ln x) + c_3\sin(\sqrt{2}\ln x)$ (correct)

(b) $c_1x^3 + c_2\cos(\ln x) + c_3\sin(\ln x)$

(c) $c_1x^{-3} + c_2\cos(\sqrt{2}\ln x) + c_3\sin(\sqrt{2}\ln x)$

(d) $c_1x^{-3} + c_2\cos(\ln x) + c_3\sin(\ln x)$

(e) $c_1x^3 + c_2\cos(2\ln x) + c_3\sin(2\ln x)$

8. The indicial roots of the singular point $x_0 = 0$ of the differential equation

$$3x^2y'' + 9xy' - (5x + 9)\,y = 0$$

are

(a) $r = 1$ and $r = -3$ (correct)

(b) $r = 1$ and $r = 2$

(c) $r = 1$ and $r = -2$

(d) $r = 2$ and $r = 3$

(e) $r = 2$ and $r = -3$

9. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution about the ordinary point $x_0 = 0$ of the differential equation $y'' - (3+x)y = 0$, then the coefficients c_n satisfy

(a) $c_{n+2} = \dfrac{3c_n + c_{n-1}}{(n+1)(n+2)},\ n \geq 1$ (correct)

(b) $c_{n+1} = \dfrac{3c_n + c_{n-1}}{(n+1)(n+2)},\ n \geq 1$

(c) $c_{n+2} = \dfrac{3c_{n+1} + c_{n-1}}{n(n+2)},\ n \geq 1$

(d) $c_{n+1} = \dfrac{3c_{n+1} + c_{n-1}}{n(n+2)},\ n \geq 1$

(e) $c_{n+1} = \dfrac{3c_n + c_{n-1}}{(n+2)(n+3)},\ n \geq 1$

10. Consider the nonhomogeneous system

$$X' = AX + \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^t,$$

then the particular solution, $X_p(t)$ at $t = 1$ equals:

(a) $\begin{pmatrix} -26 \\ -21 \end{pmatrix}$ (correct)

(b) $\begin{pmatrix} 13 \\ 12 \end{pmatrix}$

(c) $\begin{pmatrix} 11 \\ 13 \end{pmatrix}$

(d) $\begin{pmatrix} -21 \\ -3 \end{pmatrix}$

(e) $\begin{pmatrix} 3 \\ 12 \end{pmatrix}$

11. **(10 points)** Solve the following initial-value problem

$$X' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} X, \ X(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

12. **(10 points)** Use the matrix exponential method to find the general solution of the following system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix} X.$$

13. **(12 points)** Find the general solution of the system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} X.$$

14. **(13 points)** Find the first three nonzero terms of the series solution of the equation $2xy'' - y' + 2y = 0$ which corresponds to the larger indicial root of the differential equation around the regular singular point $x = 0$.

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE01 **CODE01**

Math 202
Final Exam
211
22 December, 2021
Net Time Allowed: 150 Minutes

Name: ______________________________

ID: ______________ Sec: ______________.

Check that this exam has 14 questions.

1-10 questions are MCQ's and 11-14 are written questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.
9. For the written part, show all your work. No points for answers without justification.

1. The indicial roots of the singular point $x_0 = 0$ of the differential equation

$$3x^2y'' + 9xy' - (5x+9)\,y = 0$$

are

(a) $r = 1$ and $r = 2$
(b) $r = 2$ and $r = -3$
(c) $r = 1$ and $r = -3$
(d) $r = 2$ and $r = 3$
(e) $r = 1$ and $r = -2$

2. The solution $y(x)$ of the third order initial value problem

$$y''' + 36y' = 0,\ \ y(0) = 0,\ \ y'(0) = -6,\ \ y''(0) = -36$$

satisfies $y\left(\frac{\pi}{2}\right) =$

(a) 3
(b) -2
(c) 2
(d) -3
(e) 0

3. Consider the nonhomogeneous system

$$X' = AX + \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^t,$$

then the particular solution, $X_p(t)$ at $t = 1$ equals:

(a) $\begin{pmatrix} -21 \\ -3 \end{pmatrix}$

(b) $\begin{pmatrix} 11 \\ 13 \end{pmatrix}$

(c) $\begin{pmatrix} 3 \\ 12 \end{pmatrix}$

(d) $\begin{pmatrix} -26 \\ -21 \end{pmatrix}$

(e) $\begin{pmatrix} 13 \\ 12 \end{pmatrix}$

4. If the particular solution of the differential equation

$$y'' + 3y' + 2y = \frac{1}{1 + e^x}$$

has the form $y_p(x) = e^{-x}u_1(x) + e^{-2x}u_2(x)$, then $u_1(0) =$

(a) $-\ln 2$

(b) $-\ln 3$

(c) 0

(d) $\ln 3$

(e) $\ln 2$

5. The general solution of the Cauchy-Euler differential equation $x^3y''' - 6y = 0$ is given by

(a) $c_1x^{-3} + c_2\cos(\sqrt{2}\ln x) + c_3\sin(\sqrt{2}\ln x)$
(b) $c_1x^3 + c_2\cos(\sqrt{2}\ln x) + c_3\sin(\sqrt{2}\ln x)$
(c) $c_1x^{-3} + c_2\cos(\ln x) + c_3\sin(\ln x)$
(d) $c_1x^3 + c_2\cos(2\ln x) + c_3\sin(2\ln x)$
(e) $c_1x^3 + c_2\cos(\ln x) + c_3\sin(\ln x)$

6. If the differential equation

$$\left(g(x)y^3 - \frac{1}{1+9x^2}\right)\frac{dx}{dy} + x^3y^2 = 0$$

is exact, then $g(1) =$

(a) 3
(b) -3
(c) -2
(d) 2
(e) 1

7. The solution of the differential equation

$$\frac{dy}{dx} = (x + y + 1)^2$$

is given by

(a) $y = -2x + 1 + \tan(x + c)$

(b) $y = 2x - 1 + \tan(x + c)$

(c) $y = -x - 1 + \tan(x + c)$

(d) $y = x + 1 - \tan(2x + c)$

(e) $y = x - 1 + \tan(x + c)$

8. The function $y_1 = x + 1$ is a solution of the differential equation

$$(1 - 2x - x^2)y'' + 2(1 + x)y' - 2y = 0$$

The method of Reduction of order produces the second solution $y_2 =$

(a) $2x^2 - x + 1$

(b) $x^2 + x + 3$

(c) $x^2 - x + 3$

(d) $x^2 + x + 2$

(e) $x^2 + 2$

9. A linear differential operator that annihilates the function

$$e^{-x}\sin x - e^{2x}\cos x$$

is give by

(a) $D^4 + 2D^3 + D^2 + 2D + 10$
(b) $D^4 - 2D^3 + D^2 - 2D + 10$
(c) $D^4 + 2D^3 - D^2 + 2D + 10$
(d) $D^4 - 2D^3 - D^2 + 2D + 10$
(e) $D^4 - 2D^3 + D^2 + 2D - 10$

10. If $y = \sum\limits_{n=0}^{\infty} c_n x^n$ is a power series solution about the ordinary point $x_0 = 0$ of the differential equation $y'' - (3+x)y = 0$, then the coefficients c_n satisfy

(a) $c_{n+1} = \dfrac{3c_n + c_{n-1}}{(n+1)(n+2)},\ n \geq 1$

(b) $c_{n+2} = \dfrac{3c_n + c_{n-1}}{(n+1)(n+2)},\ n \geq 1$

(c) $c_{n+1} = \dfrac{3c_{n+1} + c_{n-1}}{n(n+2)},\ n \geq 1$

(d) $c_{n+1} = \dfrac{3c_n + c_{n-1}}{(n+2)(n+3)},\ n \geq 1$

(e) $c_{n+2} = \dfrac{3c_{n+1} + c_{n-1}}{n(n+2)},\ n \geq 1$

11. **(10 points)** Solve the following initial-value problem

$$X' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} X, \ X(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

12. **(10 points)** Use the matrix exponential method to find the general solution of the following system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix} X.$$

13. **(12 points)** Find the general solution of the system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} X.$$

14. **(13 points)** Find the first three nonzero terms of the series solution of the equation $2xy'' - y' + 2y = 0$ which corresponds to the larger indicial root of the differential equation around the regular singular point $x = 0$.

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CODE02 **CODE02**

Math 202
Final Exam
211
22 December, 2021
Net Time Allowed: 150 Minutes

Name: ______________________________

ID: ____________________ Sec: ____________________.

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8. When erasing a bubble, make sure that you do not leave any trace of penciling.
9. For the written part, show all your work. No points for answers without justification.

1. If the particular solution of the differential equation

$$y'' + 3y' + 2y = \frac{1}{1+e^x}$$

has the form $y_p(x) = e^{-x}u_1(x) + e^{-2x}u_2(x)$, then $u_1(0) =$

(a) 0
(b) $-\ln 3$
(c) $\ln 2$
(d) $-\ln 2$
(e) $\ln 3$

2. The general solution of the Cauchy-Euler differential equation $x^3y''' - 6y = 0$ is given by

(a) $c_1x^3 + c_2\cos(2\ln x) + c_3\sin(2\ln x)$
(b) $c_1x^{-3} + c_2\cos(\sqrt{2}\ln x) + c_3\sin(\sqrt{2}\ln x)$
(c) $c_1x^3 + c_2\cos(\ln x) + c_3\sin(\ln x)$
(d) $c_1x^{-3} + c_2\cos(\ln x) + c_3\sin(\ln x)$
(e) $c_1x^3 + c_2\cos(\sqrt{2}\ln x) + c_3\sin(\sqrt{2}\ln x)$

3. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution about the ordinary point $x_0 = 0$ of the differential equation $y'' - (3+x)y = 0$, then the coefficients c_n satisfy

(a) $c_{n+1} = \dfrac{3c_n + c_{n-1}}{(n+1)(n+2)}, \ n \geq 1$

(b) $c_{n+2} = \dfrac{3c_n + c_{n-1}}{(n+1)(n+2)}, \ n \geq 1$

(c) $c_{n+1} = \dfrac{3c_n + c_{n-1}}{(n+2)(n+3)}, \ n \geq 1$

(d) $c_{n+2} = \dfrac{3c_{n+1} + c_{n-1}}{n(n+2)}, \ n \geq 1$

(e) $c_{n+1} = \dfrac{3c_{n+1} + c_{n-1}}{n(n+2)}, \ n \geq 1$

4. A linear differential operator that annihilates the function

$$e^{-x} \sin x - e^{2x} \cos x$$

is give by

(a) $D^4 + 2D^3 + D^2 + 2D + 10$

(b) $D^4 - 2D^3 - D^2 + 2D + 10$

(c) $D^4 + 2D^3 - D^2 + 2D + 10$

(d) $D^4 - 2D^3 + D^2 + 2D - 10$

(e) $D^4 - 2D^3 + D^2 - 2D + 10$

5. The indicial roots of the singular point $x_0 = 0$ of the differential equation

$$3x^2y'' + 9xy' - (5x + 9)\, y = 0$$

are

(a) $r = 1$ and $r = -3$
(b) $r = 2$ and $r = 3$
(c) $r = 1$ and $r = -2$
(d) $r = 1$ and $r = 2$
(e) $r = 2$ and $r = -3$

6. If the differential equation

$$\left(g(x)y^3 - \frac{1}{1 + 9x^2}\right) \frac{dx}{dy} + x^3y^2 = 0$$

is exact, then $g(1) =$

(a) -2
(b) 3
(c) 2
(d) -3
(e) 1

7. Consider the nonhomogeneous system

$$X' = AX + \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^t,$$

then the particular solution, $X_p(t)$ at $t = 1$ equals:

(a) $\begin{pmatrix} 3 \\ 12 \end{pmatrix}$

(b) $\begin{pmatrix} -26 \\ -21 \end{pmatrix}$

(c) $\begin{pmatrix} -21 \\ -3 \end{pmatrix}$

(d) $\begin{pmatrix} 11 \\ 13 \end{pmatrix}$

(e) $\begin{pmatrix} 13 \\ 12 \end{pmatrix}$

8. The solution $y(x)$ of the third order initial value problem

$$y''' + 36y' = 0, \ y(0) = 0, \ y'(0) = -6, \ y''(0) = -36$$

satisfies $y\left(\frac{\pi}{2}\right) =$

(a) 2

(b) 3

(c) 0

(d) -2

(e) -3

9. The function $y_1 = x + 1$ is a solution of the differential equation

$$(1 - 2x - x^2)y'' + 2(1 + x)y' - 2y = 0$$

The method of Reduction of order produces the second solution $y_2 =$

(a) $x^2 - x + 3$
(b) $x^2 + 2$
(c) $x^2 + x + 3$
(d) $2x^2 - x + 1$
(e) $x^2 + x + 2$

10. The solution of the differential equation

$$\frac{dy}{dx} = (x + y + 1)^2$$

is given by

(a) $y = x - 1 + \tan(x + c)$
(b) $y = 2x - 1 + \tan(x + c)$
(c) $y = x + 1 - \tan(2x + c)$
(d) $y = -x - 1 + \tan(x + c)$
(e) $y = -2x + 1 + \tan(x + c)$

11. **(10 points)** Solve the following initial-value problem

$$X' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} X, \; X(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

12. **(10 points)** Use the matrix exponential method to find the general solution of the following system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix} X.$$

13. **(12 points)** Find the general solution of the system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} X.$$

14. **(13 points)** Find the first three nonzero terms of the series solution of the equation $2xy'' - y' + 2y = 0$ which corresponds to the larger indicial root of the differential equation around the regular singular point $x = 0$.

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Department of Mathematics

CODE03 **CODE03**

Math 202
Final Exam
211
22 December, 2021
Net Time Allowed: 150 Minutes

Name: ____

ID: ______________________ Sec: ______________________.

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9. For the written part, show all your work. No points for answers without justification.

1. Consider the nonhomogeneous system

$$X' = AX + \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^t,$$

then the particular solution, $X_p(t)$ at $t = 1$ equals:

(a) $\begin{pmatrix} 13 \\ 12 \end{pmatrix}$

(b) $\begin{pmatrix} 11 \\ 13 \end{pmatrix}$

(c) $\begin{pmatrix} -21 \\ -3 \end{pmatrix}$

(d) $\begin{pmatrix} -26 \\ -21 \end{pmatrix}$

(e) $\begin{pmatrix} 3 \\ 12 \end{pmatrix}$

2. If $y = \sum\limits_{n=0}^{\infty} c_n x^n$ is a power series solution about the ordinary point $x_0 = 0$ of the differential equation $y'' - (3+x)y = 0$, then the coefficients c_n satisfy

(a) $c_{n+2} = \dfrac{3c_n + c_{n-1}}{(n+1)(n+2)}, \ n \geq 1$

(b) $c_{n+1} = \dfrac{3c_{n+1} + c_{n-1}}{n(n+2)}, \ n \geq 1$

(c) $c_{n+1} = \dfrac{3c_n + c_{n-1}}{(n+2)(n+3)}, \ n \geq 1$

(d) $c_{n+2} = \dfrac{3c_{n+1} + c_{n-1}}{n(n+2)}, \ n \geq 1$

(e) $c_{n+1} = \dfrac{3c_n + c_{n-1}}{(n+1)(n+2)}, \ n \geq 1$

3. If the differential equation

$$\left(g(x)y^3 - \frac{1}{1+9x^2}\right)\frac{dx}{dy} + x^3y^2 = 0$$

is exact, then $g(1) =$

(a) -3
(b) 2
(c) -2
(d) 3
(e) 1

4. If the particular solution of the differential equation

$$y'' + 3y' + 2y = \frac{1}{1+e^x}$$

has the form $y_p(x) = e^{-x}u_1(x) + e^{-2x}u_2(x)$, then $u_1(0) =$

(a) $-\ln 3$
(b) 0
(c) $\ln 3$
(d) $\ln 2$
(e) $-\ln 2$

5. The function $y_1 = x + 1$ is a solution of the differential equation

$$(1 - 2x - x^2)y'' + 2(1 + x)y' - 2y = 0$$

The method of Reduction of order produces the second solution $y_2 =$

(a) $x^2 + 2$
(b) $x^2 + x + 3$
(c) $2x^2 - x + 1$
(d) $x^2 + x + 2$
(e) $x^2 - x + 3$

6. The solution $y(x)$ of the third order initial value problem

$$y''' + 36y' = 0,\ y(0) = 0,\ y'(0) = -6,\ y''(0) = -36$$

satisfies $y\left(\frac{\pi}{2}\right) =$

(a) 2
(b) -3
(c) 3
(d) 0
(e) -2

7. The indicial roots of the singular point $x_0 = 0$ of the differential equation

$$3x^2y'' + 9xy' - (5x + 9)\, y = 0$$

are

(a) $r = 1$ and $r = 2$
(b) $r = 2$ and $r = -3$
(c) $r = 1$ and $r = -3$
(d) $r = 2$ and $r = 3$
(e) $r = 1$ and $r = -2$

8. A linear differential operator that annihilates the function

$$e^{-x} \sin x - e^{2x} \cos x$$

is give by

(a) $D^4 - 2D^3 + D^2 + 2D - 10$
(b) $D^4 + 2D^3 - D^2 + 2D + 10$
(c) $D^4 - 2D^3 - D^2 + 2D + 10$
(d) $D^4 - 2D^3 + D^2 - 2D + 10$
(e) $D^4 + 2D^3 + D^2 + 2D + 10$

9. The general solution of the Cauchy-Euler differential equation $x^3y''' - 6y = 0$ is given by

(a) $c_1x^{-3} + c_2 \cos(\ln x) + c_3 \sin(\ln x)$
(b) $c_1x^3 + c_2 \cos(2 \ln x) + c_3 \sin(2 \ln x)$
(c) $c_1x^{-3} + c_2 \cos(\sqrt{2} \ln x) + c_3 \sin(\sqrt{2} \ln x)$
(d) $c_1x^3 + c_2 \cos(\sqrt{2} \ln x) + c_3 \sin(\sqrt{2} \ln x)$
(e) $c_1x^3 + c_2 \cos(\ln x) + c_3 \sin(\ln x)$

10. The solution of the differential equation

$$\frac{dy}{dx} = (x + y + 1)^2$$

is given by

(a) $y = -2x + 1 + \tan(x + c)$
(b) $y = x - 1 + \tan(x + c)$
(c) $y = -x - 1 + \tan(x + c)$
(d) $y = x + 1 - \tan(2x + c)$
(e) $y = 2x - 1 + \tan(x + c)$

11. **(10 points)** Solve the following initial-value problem

$$X' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} X, \; X(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

12. **(10 points)** Use the matrix exponential method to find the general solution of the following system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix} X.$$

13. **(12 points)** Find the general solution of the system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} X.$$

14. **(13 points)** Find the first three nonzero terms of the series solution of the equation $2xy'' - y' + 2y = 0$ which corresponds to the larger indicial root of the differential equation around the regular singular point $x = 0$.

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE04 CODE04

Math 202
Final Exam
211
22 December, 2021
Net Time Allowed: 150 Minutes

Name: ____

ID: ______________________ Sec: ______________________.

Check that this exam has 14 questions.

1-10 questions are MCQ's and 11-14 are written questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.
9. For the written part, show all your work. No points for answers without justification.

1. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution about the ordinary point $x_0 = 0$ of the differential equation $y'' - (3+x)y = 0$, then the coefficients c_n satisfy

(a) $c_{n+1} = \dfrac{3c_{n+1} + c_{n-1}}{n(n+2)}, \; n \geq 1$

(b) $c_{n+1} = \dfrac{3c_n + c_{n-1}}{(n+2)(n+3)}, \; n \geq 1$

(c) $c_{n+1} = \dfrac{3c_n + c_{n-1}}{(n+1)(n+2)}, \; n \geq 1$

(d) $c_{n+2} = \dfrac{3c_{n+1} + c_{n-1}}{n(n+2)}, \; n \geq 1$

(e) $c_{n+2} = \dfrac{3c_n + c_{n-1}}{(n+1)(n+2)}, \; n \geq 1$

2. The solution $y(x)$ of the third order initial value problem

$$y''' + 36y' = 0, \; y(0) = 0, \; y'(0) = -6, \; y''(0) = -36$$

satisfies $y\left(\frac{\pi}{2}\right) =$

(a) 0
(b) 3
(c) 2
(d) -2
(e) -3

3. The indicial roots of the singular point $x_0 = 0$ of the differential equation

$$3x^2y'' + 9xy' - (5x + 9)\,y = 0$$

are

(a) $r = 1$ and $r = 2$
(b) $r = 1$ and $r = -2$
(c) $r = 1$ and $r = -3$
(d) $r = 2$ and $r = -3$
(e) $r = 2$ and $r = 3$

4. If the differential equation

$$\left(g(x)y^3 - \frac{1}{1 + 9x^2}\right)\frac{dx}{dy} + x^3y^2 = 0$$

is exact, then $g(1) =$

(a) 1
(b) -2
(c) 2
(d) 3
(e) -3

5. The function $y_1 = x + 1$ is a solution of the differential equation

$$(1 - 2x - x^2)y'' + 2(1 + x)y' - 2y = 0$$

The method of Reduction of order produces the second solution $y_2 =$

(a) $2x^2 - x + 1$
(b) $x^2 + x + 3$
(c) $x^2 + 2$
(d) $x^2 - x + 3$
(e) $x^2 + x + 2$

6. The solution of the differential equation

$$\frac{dy}{dx} = (x + y + 1)^2$$

is given by

(a) $y = -2x + 1 + \tan(x + c)$
(b) $y = 2x - 1 + \tan(x + c)$
(c) $y = x + 1 - \tan(2x + c)$
(d) $y = x - 1 + \tan(x + c)$
(e) $y = -x - 1 + \tan(x + c)$

7. Consider the nonhomogeneous system

$$X' = AX + \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^t,$$

then the particular solution, $X_p(t)$ at $t = 1$ equals:

(a) $\begin{pmatrix} 13 \\ 12 \end{pmatrix}$

(b) $\begin{pmatrix} -26 \\ -21 \end{pmatrix}$

(c) $\begin{pmatrix} 11 \\ 13 \end{pmatrix}$

(d) $\begin{pmatrix} -21 \\ -3 \end{pmatrix}$

(e) $\begin{pmatrix} 3 \\ 12 \end{pmatrix}$

8. The general solution of the Cauchy-Euler differential equation $x^3y''' - 6y = 0$ is given by

(a) $c_1x^{-3} + c_2\cos(\ln x) + c_3 \sin(\ln x)$

(b) $c_1x^{-3} + c_2 \cos(\sqrt{2} \ln x) + c_3 \sin(\sqrt{2} \ln x)$

(c) $c_1x^3 + c_2\cos(\ln x) + c_3 \sin(\ln x)$

(d) $c_1x^3 + c_2 \cos(\sqrt{2} \ln x) + c_3 \sin(\sqrt{2} \ln x)$

(e) $c_1x^3 + c_2\cos(2\ln x) + c_3 \sin(2\ln x)$

9. If the particular solution of the differential equation

$$y'' + 3y' + 2y = \frac{1}{1+e^x}$$

has the form $y_p(x) = e^{-x}u_1(x) + e^{-2x}u_2(x)$, then $u_1(0) =$

(a) $-\ln 3$
(b) 0
(c) $\ln 2$
(d) $-\ln 2$
(e) $\ln 3$

10. A linear differential operator that annihilates the function

$$e^{-x}\sin x - e^{2x}\cos x$$

is give by

(a) $D^4 + 2D^3 + D^2 + 2D + 10$
(b) $D^4 + 2D^3 - D^2 + 2D + 10$
(c) $D^4 - 2D^3 + D^2 - 2D + 10$
(d) $D^4 - 2D^3 + D^2 + 2D - 10$
(e) $D^4 - 2D^3 - D^2 + 2D + 10$

11. **(10 points)** Solve the following initial-value problem

$$X' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} X, \; X(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

12. **(10 points)** Use the matrix exponential method to find the general solution of the following system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix} X.$$

13. **(12 points)** Find the general solution of the system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} X.$$

14. **(13 points)** Find the first three nonzero terms of the series solution of the equation $2xy'' - y' + 2y = 0$ which corresponds to the larger indicial root of the differential equation around the regular singular point $x = 0$.

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	C	C	D	E
2	A	B	E	A	D
3	A	D	B	E	C
4	A	E	B	D	A
5	A	B	A	D	E
6	A	E	E	E	E
7	A	C	B	C	B
8	A	D	D	C	D
9	A	D	E	D	C
10	A	B	D	C	E

Answer Counts

V	A	B	C	D	E
1	0	3	2	3	2
2	1	3	1	2	3
3	1	0	3	4	2
4	1	1	2	2	4