

1. Which one of the following equations is an exact differential equation?

(a)  $2xy dx + (2 + x^2) dy = 0$

(correct)

(b)  $(x^2 + 1) dx - xy dy = 0$

(c)  $x dy + (3x - 2y) dx = 0$

(d)  $x^2 y dy - y dx = 0$

(e)  $2xy dx - (1 + x^2) dy = 0$

2. If we solve the differential equation

$(\cos x \cos y - \cot x) dx - \sin x \sin y dy = 0$ , then which one of the following is a solution (here  $c$  is a constant)

(a)  $\sin x \cos y = \ln(c \sin x)$

(correct)

(b)  $\sin x \cos y = \ln(c \cos x)$

(c)  $\sin x \cos y = -\ln(c \cos x)$

(d)  $\sin x \cos y = -\ln(c \sin y)$

(e)  $\sin x \cos y = \ln(c \tan x)$

3. If we solve the Homogeneous differential equation  $(y - \sqrt{x^2 + y^2}) dx - x dy = 0$ , then which one of the following is a solution (here  $c$  is a constant)

(a)  $\sqrt{x^2 + y^2} + y = c$

(correct)

(b)  $\sqrt{x^2 + y^2} + y = c$

(c)  $\sqrt{x + y} + y = c$

(d)  $\sqrt{x^2 - y} + y = c$

(e)  $\sqrt{x^2 + y} + y = c$

4. If we solve the differential equation  $\frac{dy}{dx} + \frac{y}{x} = x^2$ , then which one of the following is a solution (here  $c$  is a constant)

(a)  $xy = \frac{x^4}{4} + c$

(correct)

(b)  $xy^2 = \frac{x^3}{4} + c$

(c)  $xy^2 = \frac{x^4}{4} + c$

(d)  $y = \frac{x^3}{4} + c$

(e)  $y = \frac{x^4}{4} + c$

5. If  $y = \sin x$  is an integrating factor of the linear differential equation  $\frac{dy}{dx} + p(x)y = \sin 2x$ , then  $p(x)$  can be

(a)  $\cot x$

(correct)

(b)  $\sin x$

(c)  $\ln \sin x$

(d)  $\ln \cos x$

(e)  $\tan x$

6. If  $y = y(x)$  is the solution of the initial value problem

$$\left(\frac{2 + \sin x}{y + 1}\right) \frac{dy}{dx} = -\cos x, \quad y(0) = 1, \quad \text{then } y\left(\frac{\pi}{2}\right) \text{ equals}$$

(a)  $\frac{1}{3}$

(correct)

(b)  $\frac{-1}{3}$

(c)  $\frac{-2}{3}$

(d)  $\frac{2}{3}$

(e)  $\frac{-4}{3}$

7. The sum of all values of  $m$  for which  $y = x^m$  is a solution of the differential equation  $x^2y'' - 7xy' + 15y = 0$  is

(a) 8

(correct)

(b)  $-8$

(c) 2

(d)  $-2$

(e) 0

8. The sum of all values of  $c$  for which  $y = c$  is a constant solution of the differential equation  $y' = y^2 + 2y - 3$  is

(a)  $-2$

(correct)

(b) 2

(c)  $-1$

(d) 1

(e) 0

9. Using the Existence and Uniqueness Theorem, the initial value problem  $(y - x)y' = y + x$ ,  $y(a) = b$  has a unique solution if

(a)  $a = 1, b = -1$

(correct)

(b)  $a = 1, b = 1$

(c)  $a = 0, b = 0$

(d)  $a = -1, b = -1$

(e)  $a = 2, b = 2$

10. **(15 points)** By using an appropriate integrating factor, transform the differential equation  $(y^2 + xy^3) dx + (5y^2 - xy + y^3 \sin y) dy = 0$  into an exact equation, then solve it.

11. **(13 points)** Solve the initial value problem  $y^{\frac{1}{2}} \frac{dy}{dx} + y^{\frac{3}{2}} = 1$ ,  $y(0) = 4$ .

12. **(8 points)** Solve the differential equation  $\frac{dy}{dx} = 2 + \sqrt{y - 2x + 3}$



13. **(10 points)** A small metal bar, whose initial temperature was  $20^{\circ}C$ , is dropped into a large container of boiling water. How long will it take the bar to reach  $90^{\circ}C$  if it is known that its temperature increases  $2^{\circ}C$  in one second?

(Note that the temperature of boiling water is  $100^{\circ}c$ )