1. Given that $y = c_1 e^x \cos x + c_2 e^x \sin x$ is the general solution of a second-order differential equation with the following boundary-value conditions: $y(0) = a,\ y(\pi) = b$. This boundary value problem, with $a \neq 0$ has infinitely many solutions if $b =$

(a) $-ae^{\pi}$ (correct)

(b) ae^{π}

(c) $-a$

(d) a

(e) 0

2. Which one of the following set of solutions of a given third-order linear differential equation form a fundamental set of solutions?

(a) $\{2 + x, 2 + |x|, e^x\}$ (correct)

(b) $\{x, x^2, 4x - 3x^2\}$

(c) $\{e^x, e^{2x}, 0\}$

(d) $\{e^x, e^{2x}\}$

(e) $\{\cos 2x, 5, \cos^2 x\}$

3. Given that $y_{p_1} = 3e^{2x}$ and $y_{p_2} = x^2 + 3x$, are respectively, particular solutions of the differential equations $L(y) = -9e^{2x}$ and $L(y) = 5x^2 + 3x - 16$ where L is a second-order linear differential operator. A particular solution of the differential equation $\frac{1}{3}L(y) = -10x^2 - 6x + 32 + e^{2x}$ is

(a) $-6x^2 - 18x - e^{2x}$ (correct)

(b) $6x^2 + 18x - e^{2x}$

(c) $-\frac{2}{3}x^2 - 2x - \frac{1}{9}e^{2x}$

(d) $\frac{2}{3}x^2 + 2x - \frac{1}{9}e^{2x}$

(e) $6x^2 - 18x - e^{2x}$

4. Given that $y_1(x) = e^{2x}$ is a solution of the differential equation $(3x-1)y'' - (3x+2)y' - (6x-8)y = 0$. By using the reduction of order formula, a second solution $y_2(x)$ is

(a) $3xe^{-x}$ (correct)

(b) xe^{x}

(c) xe^{2x}

(d) x^2e^{-x}

(e) $3e^{-x}$

5. The solution of the initial-value problem $y''' + 12y'' + 36y' = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = -12$ is

(a) xe^{-6x} (correct)

(b) $1 + e^{-6x} + xe^{-6x}$

(c) $-1 + e^{-6x} + xe^{-6x}$

(d) $-1 - \frac{1}{6}e^{-6x}$

(e) $6xe^{-6x}$

6. If $y^{(4)} + ay''' + by'' + cy' + dy = 0$ is a homogeneous linear differential equation with real constant coefficients whose fundamental set of solutions contains the functions xe^{-10x} and $e^{-x}\sin x$, then $a + b =$

(a) 164 (correct)

(b) 144

(c) 160

(d) 150

(e) 170

7. Using the substitution $x = e^t$, we can transform the differential equation $x^3y''' - 3x^2y'' + 6xy' - 6y = 3 + \ln x^3$ into the following differential equation with constant coefficients

(a) $y''' - 6y'' + 11y' - 6y = 3 + 3t$ (correct)

(b) $y''' - 3y'' + 6y' - 6y = 3 + 3t$

(c) $y''' - 6y'' + 11y' - 6y = 3t$

(d) $y''' - 6y'' + 6y' - 6y = 3 + 3t$

(e) $y''' - 3y'' + 6y' - 6y = 3t$

8. The linear differential operator with least order that annihilates the function $(2 - e^x)^2 \left(1 - \frac{1}{4}e^{2x}\right)$ is

(a) $D(D-1)(D-3)(D-4)$ (correct)

(b) $D(D-1)(D-2)(D-3)(D-4)$

(c) $D(D-1)^2(D-2)$

(d) $D(D-1)(D-2)$

(e) $D(D-1)^2(D-3)$

9. By using the undetermined-coefficients method in solving the differential equation $y'' + 4y = \cos^2 x$, the most suitable form of the particular solution (where A, B, C, D, and E are constants) is

(a) $A + Bx\cos 2x + Cx\sin 2x$ (correct)

(b) $A + B\cos 2x + Cx\sin 2x$

(c) $A + B\cos 2x + C\sin 2x$

(d) $A + B\cos 2x + C\sin 2x + Dx\cos 2x + Ex\sin 2x$

(e) $A + Bx\cos 2x + C\sin 2x$

10. **(9 points)** Solve $\dfrac{d^3x}{dt^3} - \dfrac{d^2x}{dt^2} - 4x = 0.$

11. **(10 points)** Solve $y'' + 3y' = 4x - 5$. By using the undetermined-coefficients method (Annihilator Approach).

12. **(13 points)** Solve $y'' - 2y' + y = \dfrac{e^x}{1 + x^2}$.

13. **(14 points)** Solve $xy'' + y' = x$, $y(1) = 1$, $y'(1) = \dfrac{-1}{2}$.