King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 202

Final Exam

213

August 11, 2022

Net Time Allowed: 180 Minutes

USE THIS AS A TEMPLATE

Write your questions, once you are satisfied upload this file.

1. If the differential equation

$$(6xy^3 + \cos y)\,dx + (2kx^2y^2 - x\sin y)\,dy = 0$$

is exact, then k is equal to

(a) $\frac{9}{2}$
(b) $\frac{7}{2}$
(c) $\frac{11}{2}$
(d) $\frac{9}{4}$
(e) $\frac{7}{4}$

2. If c is a constant, solving $e^x y\,\frac{dy}{dx} = e^{-y} + 3e^{-2x-y}$, gives

(a) $ye^y - e^y + e^{-x} + e^{-3x} = c$
(b) $ye^y + e^y + e^{-x} - e^{-3x} = c$
(c) $ye^y - e^y + e^{-x} + 2e^{-3x} = c$
(d) $ye^y - e^y + 2e^{-x} - e^{-3x} = c$
(e) $ye^y + e^y + 2e^{-x} - 2e^{-3x} = c$

3. If c is a constant, solving $ydx - 4(x + y^6)\,dy = 0$, gives

   (a) $x = 2y^6 + cy^4$
   (b) $x = 2y^4 + cy^2$
   (c) $x = y^6 - cy^4$
   (d) $y = 2x^6 + cx^4$
   (e) $x = 3y^6 + y^4$

4. If a, b, c and d are the roots of the auxiliary equation of the differential equation $y^{(4)} - 7y'' - 18y = 0$, then $a + b + c + d =$

   (a) 0
   (b) 6
   (c) -6
   (d) 2
   (e) -2

5. If $D^4 + aD^3 + bD^2 + cD + 20$ annihilates $\dfrac{2x - 3\cos x}{e^{2x}}$, where $D = \dfrac{d}{dx}$, then $a + b + c =$

(a) 69
(b) 68
(c) 67
(d) 70
(e) 71

6. If $y_p = Ax^2 + Bx$ is a particular solution of the differential equation $2x^2y'' + 5xy' + y = x^2 - x$, then $15A + 6B =$

(a) 0
(b) 1
(c) 2
(d) -1
(e) -2

7. The general solution of the differential equation

$$y''' + 2y'' + y' = 10$$

is $y =$

(a) $c_1 + c_2e^{-x} + c_3xe^{-x} + 10x$

(b) $c_1 + c_2e^{-x} + 10x$

(c) $c_1 + c_2e^{-x} + c_3xe^{-x} + x$

(d) $c_1 + c_2e^{-x} + c_3xe^{x} + 5x$

(e) $c_1 + c_2e^{-x} + c_3e^{x} + 10x$

8. If $y = y(x)$ is the solution of the initial-value problem $x^2y'' - xy' + y = 0$, $y(1) = 3$, $y'(1) = -1$, then $y(e) =$

(a) $-e$

(b) $2e$

(c) $3e$

(d) $-2e$

(e) $-3e$

9. The general solution of the differential equation

   $4y'' + 36y = \csc 3x$ is $y =$

   (a) $c_1 \cos 3x + c_2 \sin 3x - \frac{1}{12} x \cos 3x + \frac{1}{36}(\sin 3x)\ \ln\, |\sin 3x|$
   (b) $c_1 \cos 3x + c_2 \sin 3x + \frac{1}{4} x \sin 3x + \frac{1}{36}(\cos x)\ \ln\, |\cos x|$
   (c) $c_1 \cos 3x + c_2 \sin 3x - x \cos 3x + (\sin 3x)\ \ln\, |\sin 3x|$
   (d) $c_1 \cos 3x + c_2 \sin 3x - x \sin 3x + (\cos x)\ \ln |\cos x|$
   (e) $c_1 \cos 3x + c_2 \sin 3x - x \cos 3x + (\cos 3x)\ \ln |\cos 3x|$

10. If c is a constant, solving this homogeneous differential equation $(x-y)\,dx + x dy = 0$, gives

   (a) $y = cx - x \ln |x|$
   (b) $y = cx^2 - x \ln |x|$
   (c) $y = cx + x^2 \ln |x|$
   (d) $y = cx - 2x \ln |x|$
   (e) $y = cx + 2 \ln |x|$

11. The minimum radius of convergence of the power series solutions for the differential equation $(x^2 - 2x + 10)y'' + xy' - 4y = 0$ about the ordinary point $x = 1$, is $R =$

(a) 3
(b) 2
(c) 1
(d) 4
(e) 5

12. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution about the ordinary point $x_0 = 0$ of the differential equation $y'' + xy = 0$, then the coefficients c_n satisfy

(a) $c_{n+2} = \dfrac{-c_{n-1}}{(n+2)(n+1)}$, $n \geq 1$
(b) $c_{n+2} = \dfrac{-c_n}{(n+2)(n+1)}$, $n \geq 1$
(c) $c_n = \dfrac{-c_{n+2}}{(n+2)(n+1)}$, $n \geq 1$
(d) $c_{n+2} = \dfrac{-1}{(n+2)(n+1)}$, $n \geq 1$
(e) $c_{n+2} = \dfrac{-c_{n-1}}{(n+1)(n+3)}$, $n \geq 1$

13. If we solve the differential equation $3xy'' + y' - y = 0$ about the regular singular point $x = 0$, by considering $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ then we will have the recurrence relation

$$c_{k+1} = \frac{c_k}{(k+r+1)(3k+3r+1)}, \ k \geq 0$$

where r are the roots of the indicial equation. The first three nonzero terms in the series solution that corresponds to the smaller indicial root evaluated at $x = 1$ could be

(a) $1, 1, \frac{1}{8}$
(b) $2, 3, \frac{1}{6}$
(c) $2, 2, \frac{2}{5}$
(d) $1, 1, \frac{1}{6}$
(e) $2, 2, \frac{1}{8}$

14. Which one of the following statements is TRUE about the differential equation

$$(x^3 - 2x^2 - 3x)^2 y'' + x(x-3)^2 y' - (x+1)y = 0?$$

(a) $x = -1$ is an irregular singular point
(b) $x = 1$ is an irregular singular point
(c) $x = 0$ is an irregular singular point
(d) $x = 3$ is an irregular singular point
(e) $x = 3$ is an ordinary point

15. If $X = \begin{pmatrix} 1 \\ b \end{pmatrix} e^{-5t}$ is a solution of the system

$$\frac{dx}{dt} = 3x - 4y,$$

$$\frac{dy}{dt} = 4x - 7y,$$

then $b =$

(a) 2
(b) 0
(c) -1
(d) -2
(e) 1

16. If the general solution of the system

$$\frac{dx}{dt} = x + y - z$$

$$\frac{dy}{dt} = 2y$$

$$\frac{dz}{dt} = y - z$$

is given by

$$X = c_1 \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} e^{\lambda_1 t} + c_2 \begin{pmatrix} c \\ d \\ 1 \end{pmatrix} e^{\lambda_2 t} + c_3 \begin{pmatrix} e \\ f \\ 2 \end{pmatrix} e^{-t},\ \lambda_1 \neq \lambda_2$$

Then $a \cdot b + c \cdot d + e \cdot f =$

(a) 6
(b) 5
(c) 4
(d) 7
(e) 8

17. Consider the following initial value problem

$$X' = AX, \ X(0) = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

where $A = \begin{bmatrix} 2 & 4 \\ -1 & 6 \end{bmatrix}$. Then $X(1) =$

(a) $\begin{bmatrix} 25e^4 \\ 19e^4 \end{bmatrix}$

(b) $\begin{bmatrix} 19e^4 \\ 25e^4 \end{bmatrix}$

(c) $\begin{bmatrix} 10e^4 \\ 15e^4 \end{bmatrix}$

(d) $\begin{bmatrix} 25e^4 \\ 10e^4 \end{bmatrix}$

(e) $\begin{bmatrix} 15e^4 \\ 19e^4 \end{bmatrix}$

18. The solution of $X' = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} X$, $X(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ at $t = \frac{\pi}{4}$ equals

(a) $\begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{\frac{\pi}{4}}$

(b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{\frac{\pi}{4}}$

(c) $\begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{\frac{\pi}{4}}$

(d) $\begin{bmatrix} -2 \\ 0 \end{bmatrix} e^{\frac{\pi}{4}}$

(e) $\begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-\frac{\pi}{4}}$

19. Consider the non homogeneous system

$$X' = AX + \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^t$$

then the particular solution $X_p(-1) =$

(a) $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

(c) $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

(d) $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

(e) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

20. If the exponential matrix of

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix}$$

is given by

$$e^{At} = \begin{pmatrix} t+1 & t & t \\ h_1(t) & t+1 & t \\ h_2(t) & h_3(t) & -2t+1 \end{pmatrix}$$

then $h_1(1) + h_2(2) + h_3(3) =$

(a) -9

(b) 9

(c) -10

(d) 10

(e) -8

21. The first order differential equation

$$\frac{dr}{d\theta} = r\theta + r + \theta + 1$$

is not seperable
(Answer True or False by filling in the OMR sheet)

(a) False
(b) True

22. If $y_1 = e^x$ and $y_2 = e^{-x}$ are solutions of a homogeneous linear differential equation (DE), then $y = -5e^{-x} + 10e^x$ is also a solution of the DE.
(Answer True or False by filling in the OMR sheet)

(a) True
(b) False

23. The linear differential equation, $y' + k_1 y = k_2$, where k_1 and k_2 are non zero constants, possesses a constant solution
(Answer True or False by filling in the OMR sheet)

(a) True
(b) False

24. If the set consisting of two functions f_1 and f_2 is linearly independent on an interval I, then the Wronskian $W(f_1, f_2) \neq 0$ for all x in I.
(Answer True or False by filling in the OMR sheet)

(a) False
(b) True

25. $k_1 = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$ is an eigenvector of the coefficient matrix

$$\begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}.$$

(Answer True or False by filling in the OMR sheet)

(a) True
(b) False