Original

Q12/page 12 (Sec 1.1)

1. If $y = a - \frac{6}{5}e^{-20t}$ is a solution of the differential equation $\frac{dy}{dt} + 20y = 24$, then $a =$

(a) $\frac{6}{5}$ ______________________________ (correct)

(b) $\frac{5}{6}$

(c) $\frac{-6}{5}$

(d) $\frac{-5}{6}$

(e) 1

Q10/page 19 (Sec 1.2)

2. If $x = c_1 \cos t + c_2 \sin t$ is a two-parameter family of solutions of the initial-value problem

$$x'' + x = 0$$
$$x\left(\frac{\pi}{4}\right) = \sqrt{2},\ x'\left(\frac{\pi}{4}\right) = 2\sqrt{2},$$

then $c_1 + c_2 =$

(a) 2 ______________________________ (correct)

(b) -2

(c) 1

(d) -1

(e) 0

Q37/page 53 (sec 2-2)

3. If $y = c$ is a singular solution of the differential equation $\dfrac{dy}{dx} = x\sqrt{1-y^2}$, then the sum of all $c's$ is:

   (a) 0 ________________ (correct)

   (b) 1

   (c) -1

   (d) 2

   (e) -2

Q5/page 51 (sec 2-2)

4. The solution of the initial-value problem

   $$\frac{dy}{dx} = e^{-x^2},\ y(3) = 5 \text{ is } y(x) =$$

   (a) $5 + \displaystyle\int_3^x e^{-t^2}\,dt$ ________________ (correct)

   (b) $3 + \displaystyle\int_3^x e^{-t^2}\,dt$

   (c) $5 + \displaystyle\int_0^x e^{-t^2}\,dt$

   (d) $3 + \displaystyle\int_0^x e^{-t^2}\,dt$

   (e) $5 + e^{-x^2}$

Q 19 / page 62 (sec 2.3)

5. If y is a solution of the differential equation

$$(x+1)\frac{dy}{dx} + (x+2)y = 2xe^{-x}$$

where $x > -1$, then there exists a constant c such that $y =$

(a) $\dfrac{x^2}{x+1}e^{-x} + \dfrac{c}{x+1}e^{-x}$ ______________________________ (correct)

(b) $\dfrac{x}{x+1}e^{x} + \dfrac{c}{x+1}e^{-x}$

(c) $\dfrac{x^2}{x+1}e^{-x} + \dfrac{c}{x-1}e^{x}$

(d) $\dfrac{x^2}{x+1}e^{-x} + \dfrac{c}{x+1}e^{x}$

(e) $\dfrac{cx^2}{x+1}e^{-x}$

Q 29 / page 70 / (sec 2.4)

6. If y is a solution of the exact differential equation obtained by multiplying both sides of the differential equation

$$(-xy\sin x + 2y\cos x)\,dx + 2x\cos x dy = 0 \text{ by } \mu(x,y) = xy,$$

then there exists a constant c such that

(a) $x^2y^2\cos x = c$ ______________________________ (correct)

(b) $x^2y\cos x = c$

(c) $x^2y^2\sin x = c$

(d) $xy^2\cos x = c$

(e) $xy\sin x = c$

Q2 / page 74 (sec 2.5)

7. If y is a nonzero solution of the differential equation $x\dfrac{dy}{dx} + y = x^2y^2$, then there exists a constant c such that $y =$

(a) $\dfrac{1}{-x^2 + cx}$ ______________________________ (correct)

(b) $\dfrac{1}{x^2 + x}$

(c) $\dfrac{c}{x^2 + cx}$

(d) $\dfrac{1}{x^2 + cx}$

(e) $\dfrac{1}{(c+1)x}$

Q6 / page 75 (sec 2.5)

8. If y is a nonzero solution of the differential equation $(y^2 + yx)\,dx - x^2\,dy = 0$, then there exists a constant c such that

(a) $y \ln |x| + x = cy$ ______________________________ (correct)

(b) $\ln |x| + x = cy$

(c) $y \ln |x| + y = cy$

(d) $y \ln |x| + x = cx$

(e) $x \ln |y| + x = cy$

Q 18(j) / page 82 (Ch 2 Review)

9. The differential equation $2xyy' + y^2 = 2x^2$ can be classified as:

(a) only homogeneous, exact and Bernoulli differential equation ________(correct)

(b) only seperable, exact and homogeneous differential equation

(c) only linear, exact and Bernoulli differential equation

(d) only seperable, linear and homogeneous differential equation

(e) only homogeneous and exact differential equation

Q17/ page 92 (sec 3.1)

10. A thermometer reading $20°C$ is placed in an oven preheated to a constant temperature. Through a glass of window in the oven door, an observer records that the thermometer reads $60°$ after $\frac{1}{2}$ minute and $90°C$ after 1 minute. What is the temperature of the oven?

(a) $180°$ ________(correct)

(b) $170°$

(c) $160°$

(d) $190°$

(e) $200°$

Q 23/page 20 (sec 1.2)

11. (5 points) Determine a region of the xy-plane for which the differential equation $(x^2 + y^2)\, y' = y^2$ would have a unique solution whose graph passes through a point (x_0, y_0) in the region.

$$y' = \frac{y^2}{x^2+y^2} \Rightarrow f(x,y) = \frac{y^2}{x^2+y^2}$$

$$\Rightarrow \frac{\partial f}{\partial y} = \frac{2x^2 y}{(x^2+y^2)^2}$$ 3 pts

The differential equation will have a unique solution in any region not containing $(0,0)$.

2pts

Q34 (page 71) (Sec 2-4)

12. (11 points) Transform the differential equation

$$\cos x dx + \left(1 + \frac{2}{y}\right) \sin x \, dy = 0$$

into an exact equation by finding an appropriate integrating factor. Show that the new differential equation is an exact, **but do not solve it**.

$M(x, y) = \cos x$ & $N(x, y) = \left(1 + \frac{2}{y}\right) \sin x$ (1pt)

$\Rightarrow \frac{\partial M}{\partial y} = 0$ & $\frac{\partial N}{\partial x} = \left(1 + \frac{2}{y}\right) \cos x$ — (2pts)

$\Rightarrow$ The differential equation is not exact. (2pts)

Now, $\frac{M_y - N_x}{N} = -\cot x$ (function of x only)

$I.F. = e^{-\int \cot x \, dx} = \csc x$ (3pts) (1pt)

The new differential equation is $\underbrace{\cot x}_{M'} dx + \underbrace{\left(1 + \frac{2}{y}\right)}_{N'} dy = 0$

It is an exact equation since $\frac{\partial M'}{\partial y} = 0 = \frac{\partial N'}{\partial x}$

(2pts)

Q3/page 74 (sec 2.5)

13. (14 points) Solve the initial-value problem

$$\frac{dy}{dx} = (-2x+y)^2 - 7,\ y(0) = 0,$$

by finding an explicit solution.

1pt

Let $u = -2x + y \Rightarrow \frac{du}{dx} = -2 + \frac{dy}{dx}$ 1pt

Substitute to get $\frac{du}{dx} + 2 = u^2 - 7$ or

$$\frac{du}{dx} = u^2 - 9 \Rightarrow \frac{du}{u^2-9} = dx$$ 1pt

The new equation is seperable and so, we have

2pts

1pt

$$\frac{du}{(u-3)(u+3)} = dx \text{ or } \frac{1}{6}\left[\frac{1}{u-3} - \frac{1}{u+3}\right] du = dx$$

If we integrate, we get $\frac{1}{6} \ln\left|\frac{u-3}{u+3}\right| = x + c_1$ 2pts

or $\frac{u-3}{u+3} = e^{6x+6c_1} = ce^{6x}$ 1pt

2pts

If we solve for u, we get $u = \frac{3(1+ce^{6x})}{1-ce^{6x}}$ or $y = 2x + \frac{3(1+ce^{6x})}{1-ce^{6x}}$

2pts

Since $y(0) = 0$, we get $c = -1$ and so our solution in

an explicit way is

$$y = 2x + \frac{3(1-e^{6x})}{1+e^{6x}}$$ 1pt

MASTER

#37/page 63 (sec 2.3)

14. (10 points) Solve the initial-value problem

$$\frac{dy}{dx} + 2y = f(x),\ y(0) = 0,$$

where

$$f(x) = \begin{cases} 1, & 0 \le x \le 3 \\ 0, & x > 3 \end{cases}$$

$y' + 2y = f(x)$ linear

I.F. $= e^{\int 2\,dx} = e^{2x}$ 1pt

2pts

$$\Rightarrow \frac{d}{dx}(y e^{2x}) = e^{2x} f(x) = \begin{cases} e^{2x}, & 0 \le x \le 3 \\ 0, & x > 3 \end{cases}$$

$$\Rightarrow y e^{2x} = \begin{cases} \frac{1}{2} e^{2x} + c_1, & 0 \le x \le 3 \\ c_2, & x > 3 \end{cases}$$ 2pts

$y(0) = 0 \Rightarrow 0 = \frac{1}{2} + c_1 \Rightarrow c_1 = -\frac{1}{2}$ 1pt $\Rightarrow y e^{2x} = \begin{cases} \frac{1}{2} e^{2x} - \frac{1}{2}, & 0 \le x \le 3 \\ c_2, & x > 3 \end{cases}$

For continuity, we must have $\frac{1}{2} e^{6} - \frac{1}{2} = c_2$ 2pts

$$\Rightarrow y e^{2x} = \begin{cases} \frac{1}{2} e^{2x} - \frac{1}{2}, & 0 \le x \le 3 \\ \frac{1}{2} e^{6} - \frac{1}{2}, & x > 3 \end{cases}$$

$$\Rightarrow y = \begin{cases} \frac{1}{2}(1 - e^{-2x}), & 0 \le x \le 3 \\ \frac{1}{2}(e^{6} - 1) e^{-2x}, & x > 3 \end{cases}$$ 2pts

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Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	A $_{10}$	C $_{5}$	E $_{6}$	A $_{8}$
2	A	D $_{1}$	E $_{2}$	E $_{7}$	E $_{2}$
3	A	C $_{7}$	A $_{8}$	A $_{4}$	E $_{10}$
4	A	E $_{9}$	C $_{3}$	B $_{5}$	D $_{7}$
5	A	E $_{5}$	A $_{7}$	E $_{1}$	A $_{5}$
6	A	C $_{2}$	C $_{6}$	A $_{9}$	A $_{6}$
7	A	E $_{8}$	A $_{9}$	A $_{3}$	D $_{3}$
8	A	A $_{6}$	E $_{1}$	B $_{2}$	D $_{1}$
9	A	E $_{4}$	B $_{4}$	A $_{8}$	B $_{4}$
10	A	E $_{3}$	D $_{10}$	B $_{10}$	D $_{9}$
11		$_{12}$	$_{14}$	$_{11}$	$_{14}$
12		$_{11}$	$_{11}$	$_{12}$	$_{12}$
13		$_{13}$	$_{13}$	$_{13}$	$_{11}$
14		$_{14}$	$_{12}$	$_{14}$	$_{13}$