Q14 (page 130)(sec 4.1)

1. Given that the two-parameter family $y = c_1x^2+c_2x^4+3$ is a solution of the differential equation $x^2y'' - 5xy' + 8y = 24$ on $(-\infty, \infty)$. Then the differential equation has a solution satisfying the boundary conditions:

   (a) $y(1) = 3,\ y(2) = 15$ ____________________(correct)

   (b) $y(0) = 0,\ y(1) = 2$

   (c) $y(1) = 3,\ y'(0) = 1$

   (d) $y(-1) = 0,\ y(1) = 4$

   (e) $y(-2) = 0,\ y(2) = 2$

Similar to Q15-22 (page 130-131) (sec 4.1)

2. Which one of the following sets form a fundamental set of solutions of a third-order linear differential equation?

   (a) $\{\sinh x,\ \cosh x, 1\}$ ____________________(correct)

   (b) $\{\sinh x,\ \cosh x, e^x\}$

   (c) $\{\sinh x,\ \cosh x\}$

   (d) $\{\cos 2x,\ \cos^2 x\}$

   (e) $\{e^x, e^{-x}, \sinh x\}$

Ex 11 (page 129) (sec 4.1)

3. Given that $y_{p_1} = 4x^2$, $y_{p_2} = e^{2x}$, and $y_{p_3} = xe^x$ are particular solutions of the differential equations

$$L(y) = -16x^2 + 24x - 8,\ L(y) = 2e^{2x},\ \text{and}\ L(y) = 2xe^x - e^x,$$

respectively where L is a linear differential operator. Which of the following is a particular solution of the differential equation $L(y) = -16x^2 + 24x - 8 + 2e^{2x} + 2xe^x - e^x$?

(a) $4x^2 + e^{2x} + xe^x$ ____(correct)

(b) $4x^2 - e^{2x} - xe^x$

(c) $-4x^2 - e^{2x} + xe^x$

(d) $-4x^2 + e^{2x} - xe^x$

(e) $-4x^2 + e^{2x} + xe^x$

Similar to Q1-22 (page 134) (sec 4.2)

4. If $y_1 = e^x$ is a solution of the differential equation $xy'' - (2x+1)y' + (x+1)y = 0$, then which of the following is a second solution y_2 that is linearly independent with y_1?

(a) x^2e^x ____(correct)

(b) x^2e^{2x}

(c) xe^{2x}

(d) xe^x

(e) x^3e^{-x}

Q28 (page 140) (sec 4.3)

5. If $y = c_1 + c_2x + c_3e^{-\frac{1}{2}x} + c_4e^{bx}\cos cx + c_5e^{bx}\sin cx$ is the general solution of the differential equation $2y^{(5)} - 7y^{(4)} + 12y''' + 8y'' = 0$, then $b^2 + c^2 =$
(Hint: $y = e^{-\frac{1}{2}x}$ is a solution of the differential equation)

(a) 8 ____(correct)

(b) 6

(c) 10

(d) 12

(e) 4

Q22, 26 (page 159) (sec 4.5)

6. A linear differential operator that annihilates the function

$$8x - \sin x + e^{-x}\sin x$$

is:

(a) $D^6 + 2D^5 + 3D^4 + 2D^3 + 2D^2$ ____(correct)

(b) $D^6 - 2D^5 - 3D^4 - 2D^3 - 2D^2$

(c) $D^5 - 2D^4 + 3D^3 - 2D^2 + 2D$

(d) $D^6 - 2D^5 + 3D^4 - 2D^3 + 2D^2$

(e) $D^6 - 2D^5 - 3D^4 + 2D^3 - 2D^2$

Q43(page 159) (sec 4.5)

7. The general solution of the differential equation

$$y'' - y' - 12y = e^{4x}$$

is $y =$

(a) $c_1e^{4x} + c_2e^{-3x} + \frac{1}{7}xe^{4x}$ ______ (correct)

(b) $c_1e^{4x} + c_2e^{-3x} + \frac{1}{7}e^{4x}$

(c) $c_1e^{4x} + c_2e^{3x} + \frac{1}{6}xe^{4x}$

(d) $c_1e^{-4x} + c_2e^{3x} + \frac{1}{7}e^{4x}$

(e) $c_1e^{4x} + c_2e^{-3x} + xe^{4x}$

Q13.(page 165) (sec 4.6)

8. The general solution of the differential equation

$$y'' + 3y' + 2y = \sin e^{x}$$

is $y =$

(a) $c_1e^{-x} + c_2e^{-2x} - e^{-2x}\sin e^{x}$ ______ (correct)

(b) $c_1e^{-x} + c_2e^{-2x} + 2e^{-2x}\sin e^{x}$

(c) $c_1e^{-x} + c_2e^{-2x} - e^{-x}\cos e^{x}$

(d) $c_1e^{-x} + c_2e^{-2x} + e^{-x}\cos e^{x}$

(e) $c_1e^{x} + c_2e^{2x} + e^{-2x}\sin e^{x}$

Q 36 (page 165) (Sec 4.6)

9. Given that $y_1 = x^2$ is a solution of the associated homogeneous equation of the differential equation $x^4y'' + x^3y' - 4x^2y = 1$. Which of the following is a particular solution of the differential equation?

(a) $\dfrac{-1}{16x^2} - \dfrac{1}{4x^2}\ln x$ ________________ (correct)

(b) $\dfrac{1}{16x^2} + \dfrac{1}{4x}\ln x$

(c) $\dfrac{1}{16x} - \dfrac{1}{4x^2}\ln x$

(d) $\dfrac{-1}{16x^2} - \dfrac{1}{2x^2}\ln x$

(e) $\dfrac{-1}{8x} - \dfrac{1}{4x^2}\ln x$

Q 17 (page 171) (Sec 4.7)

10. The general solution of the differential equation $xy^{(4)} + 6y''' = 0$ is $y =$

(a) $c_1 + c_2x + c_3x^2 + c_4x^{-3}$ ________________ (correct)

(b) $c_1 + c_2x + c_3x^2 + c_4x^3$

(c) $c_1 + c_2x^2 + c_3x^{-3} + c_4x^{-4}$

(d) $c_1x + c_2x^2 + c_3x^{-3} + c_4x^{-4}$

(e) $c_1 + c_2x + c_3x^{-3} + c_4x^{-4}$

Q36 (page 172) (Sec 4.7)

11. By using the substitution $x = e^t$, if we transform the differential equation

$$x^3y''' - 3x^2y'' + 6xy' - 6y = 3 + \ln x^3$$

into the differential equation

$$\frac{d^3y}{dt^3} + A\frac{d^2y}{dt^2} + B\frac{dy}{dt} + Cy = D + Et,$$

then $A + B + C + D + E =$

(a) 5 ________________________________(correct)

(b) 6

(c) 7

(d) 4

(e) 3

Ex 7 (page 157) (Sec 4.5)

12. Which of the following is the form of a particular solution for the differential equation

$$y''' - 4y'' + 4y' = 5x^2 - 6x + 4x^2e^{2x} + 3e^{5x}?$$

(a) $Ax + Bx^2 + Cx^3 + Ex^2e^{2x} + Fx^3e^{2x} + Gx^4e^{2x} + He^{5x}$ ________________(correct)

(b) $A + Bx + Cx^2 + Ee^{2x} + Fxe^{2x} + Gx^2e^{2x} + He^{5x}$

(c) $Ax + Bx^2 + Cx^3 + Ex^2e^{x} + Fe^{5x}$

(d) $Ax^2 + Bx^2e^{2x} + Cx^3e^{2x} + Ex^4e^{2x} + Fe^{5x}$

(e) $Ax + Bx^2 + Cx^3 + Ex^2e^{2x} + Fx^3e^{2x} + Ge^{5x}$

Q8 (page 252) (Sec 6.2)

13. If $y = \sum_{k=0}^{\infty} c_k x^k$ is a power series solution about the ordinary point $x_0 = 0$ of the differential equation $y'' + x^2 y = 0$, then the coefficients c_k satisfy

(a) $c_{k+2} = \dfrac{-1}{(k+2)(k+1)} c_{k-2}$, $k \geq 2$ ——————————(correct)

(b) $c_{k+2} = \dfrac{1}{k(k+1)} c_{k-1}$, $k \geq 3$

(c) $c_{k+2} = \dfrac{1}{(k+2)(k+1)} c_{k-1}$, $k \geq 2$

(d) $c_{k+2} = \dfrac{-1}{k+2} c_{k-2}$, $k \geq 2$

(e) $c_{k+2} = \dfrac{-1}{k(k+1)(k+2)} c_{k-2}$, $k \geq 2$

Ex 7 (page 249) (Sec 6.2)

14. If we solve the differential equation

$$y'' - (1+x)y = 0$$

about the ordinary point $x = 0$, by considering $y = \sum_{n=0}^{\infty} c_n x^n$, then we will have the three-term recurrence relation

$$c_{k+2} = \frac{c_k + c_{k-1}}{(k+1)(k+2)}, \ k \geq 1 \text{ and } c_2 = \frac{1}{2} c_0$$

If we assume $c_0 = 1$, and $c_1 = 0$, then the first three non zero terms of one series solution evaluated at $x = 1$ could be

(a) $1, \frac{1}{2}, \frac{1}{6}$ ——————————(correct)

(b) $2, \frac{1}{4}, \frac{1}{12}$

(c) $1, \frac{1}{2}, \frac{1}{8}$

(d) $1, \frac{1}{4}, \frac{1}{6}$

(e) $2, 1, \frac{1}{6}$