King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 202
Final Exam
221
December 25, 2022

EXAM COVER

Number of versions: 4
Number of questions: 20

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Net Time Allowed: 180 Minutes

MASTER VERSION

1. Consider the following functions
   (a) $y = 0$ (b) $y = 2$ (c) $y = 2x$ (d) $y = 2x^2$
   Which of these functions are solutions of the differential equation $xy'' - y' = 0$?

   (a) a,b,d only ________________________________ (correct)
   (b) c only
   (c) a,b only
   (d) a,b,c,d
   (e) a,d only

2. The solution of the linear differential equation $y\dfrac{dx}{dy} - x = 2y^2$
   with $y(1) = 5$ is $x =$

   (a) $2y^2 - \dfrac{49}{5}y$ ________________________________ (correct)
   (b) $y^2 - \dfrac{24}{5}y$
   (c) $y - 4$
   (d) $2y^2 + \dfrac{1}{5}y$
   (e) $y^2 - \dfrac{1}{5}y$

3. If y is a solution of the differential equation $x\dfrac{dy}{dx} + y = \dfrac{1}{y^2}$, then there exists a constant c such that

(a) $y^3 = 1 + cx^{-3}$ ________________________________ (correct)

(b) $y^2 = 1 + cx^{-2}$

(c) $y^3 = 1 + cx^{-2}$

(d) $y^2 = 1 + cx^{-3}$

(e) $y = 1 + cx^{-1}$

4. If y is a solution of the exact differential equation $\dfrac{dx}{dy} = -\dfrac{4y^2 + 6xy}{3y^2 + 2x}$, then there exists a constant c such that $c =$

(a) $3xy^2 + x^2 + \dfrac{4}{3}y^3$ ________________________________ (correct)

(b) $xy^2 + x^2 - \dfrac{4}{3}y^3$

(c) $3xy^2 + x + \dfrac{4}{3}y$

(d) $3xy^2 + x^3 + \dfrac{4}{3}y^2$

(e) $xy^2 + x + \dfrac{3}{4}y^3$

5. The linear differential operator with least order that annihilates the function $f(x) = 8x - \sin x + 10\cos 5x$ is:

   (a) $D^6 + 26D^4 + 25D^2$ ________ (correct)

   (b) $D^6 + 26D^3 + 25D$

   (c) $D^4 + D^2$

   (d) $D^4 + 25D^2$

   (e) $D^6 + 25D^4 + 26D^2$

6. Given that $y = \sin x$ is a solution of the differential equation

   $$y^{(4)} + 2y''' + 11y'' + 2y' + 10y = 0.$$

   The general solution of that differential equation is $y =$

   (a) $c_1 \cos x + c_2 \sin x + e^{-x}(c_3 \cos 3x + c_4 \sin 3x)$ ________ (correct)

   (b) $c_1 \cos x + c_2 \sin x + e^{x}(c_3 \cos 3x + c_4 \sin 3x)$

   (c) $c_1 \cos x + c_2 \sin x + e^{-x}(c_3 \cos x + c_4 \sin x)$

   (d) $c_1 \cos x + c_2 \sin x + e^{x}(c_3 \cos 2x + c_4 \sin 2x)$

   (e) $c_1 \cos x + c_2 \sin x + c_3 \cos 3x + c_4 \sin 3x$

7. If $y_p = u_1 e^x - 1 + e^{-x}\tan^{-1} e^x$ is a particular solution of the differential equation $y'' - y = \dfrac{2e^x}{e^x + e^{-x}}$, then $u_1(0) =$

(a) $\dfrac{\pi}{4}$ ______________________________ (correct)
(b) 0
(c) $\dfrac{\pi}{2}$
(d) $-\dfrac{\pi}{4}$
(e) $-\dfrac{\pi}{2}$

8. If $m_1 = i$ is a root of the auxiliary equation of a homogeneous second-order Cauchy-Euler equation with real coefficients, then one possible equation is:

(a) $x^2y'' + xy' + y = 0$ ______________________________ (correct)
(b) $x^2y'' - xy' - y = 0$
(c) $2x^2y'' + xy' + 2y = 0$
(d) $x^2y'' + 2xy' + y = 0$
(e) $2x^2y'' + 2xy' + y = 0$

9. The minimum radius of convergence of power series solution for the differential equation

$$(x^2 - 2x + 10)y'' + xy' - 4y = 0$$

about the ordinary point $x = 1$ is:

(a) 3 ________________________________ (correct)
(b) 4
(c) 5
(d) 2
(e) 6

10. If we solve the differential equation

$$2xy'' + (1 + x)y' + y = 0$$

about the regular singular point $x = 0$, by considering $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ then we will have the recurrence relation ($k \geq 0$)

(a) $c_{k+1} = -\dfrac{1}{2k + 2r + 1} c_k$ ________________________________ (correct)
(b) $c_{k+1} = \dfrac{1}{k + r + 1} c_k$
(c) $c_{k+1} = -\dfrac{1}{k + r + 1} c_k$
(d) $c_{k+1} = \dfrac{1}{2k + 2r + 1} c_k$
(e) $c_{k+1} = \dfrac{-1}{2k + r + 1} c_k$

11. Which one of the following statements is **TRUE** about the differential equation

$$x^3(x^2-25)(x-2)^2y'' + 3x(x-2)y' + 7(x+5)y = 0?$$

(a) $x = 0$ is an irregular singular point ________________ (correct)

(b) $x = 2$ is an irregular singular point

(c) $x = 5$ is an irregular singular point

(d) $x = -5$ is an irregular singular point

(e) $x = 0$ is an ordinary point

12. The sum of the roots of the indicial equation of the differential equation $x^2y'' + \left(\frac{5}{3}x + x^2\right)y' - \frac{1}{3}y = 0$ is

(a) $\frac{-2}{3}$ ________________ (correct)

(b) $\frac{2}{3}$

(c) $\frac{5}{3}$

(d) $\frac{-5}{3}$

(e) $\frac{4}{3}$

13. If $K = \begin{pmatrix} 1 \\ a \\ -13 \end{pmatrix}$ is an eigenvector with eigenvalue $\lambda = 0$ of $A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$, then $a =$

(a) 6 ______ (correct)
(b) 5
(c) 4
(d) 3
(e) 7

14. If $X = \begin{pmatrix} a \\ 2 \end{pmatrix} e^{-3t/2}$ is a solution of the system $X' = \begin{pmatrix} -1 & \frac{1}{4} \\ 1 & -1 \end{pmatrix} X$ then $a =$

(a) -1 ______ (correct)
(b) 1
(c) 0
(d) -2
(e) 2

15. If $X = c_1 \begin{pmatrix} 1 \\ a \\ 0 \end{pmatrix} e^t + c_2 \begin{pmatrix} 2 \\ b \\ 1 \end{pmatrix} e^{2t} + c_3 \begin{pmatrix} 1 \\ 0 \\ c \end{pmatrix} e^{dt}$ is the general solution of $X' = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 1 & -1 \end{pmatrix} X,$ then $a + b + c + d =$

(a) 4 ——————————————————————————(correct)
(b) 5
(c) 6
(d) 3
(e) 7

16. The general solution of the system

$$\begin{aligned} \frac{dx}{dt} &= 3y - x \\ \frac{dy}{dt} &= 5y - 3x \end{aligned} \quad \text{is } X = \begin{pmatrix} x \\ y \end{pmatrix} =$$

(a) $c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{2t} + \begin{pmatrix} -1/3 \\ 0 \end{pmatrix} e^{2t} \right]$ ——————————(correct)
(b) $c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} te^{2t}$
(c) $c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{2t} + \begin{pmatrix} 0 \\ -1/3 \end{pmatrix} e^{2t} \right]$
(d) $c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \left[\begin{pmatrix} -1/3 \\ 0 \end{pmatrix} te^{2t} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} \right]$
(e) $c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \left[\begin{pmatrix} 2 \\ 2 \end{pmatrix} te^{2t} + \begin{pmatrix} 1/3 \\ 0 \end{pmatrix} e^{2t} \right]$

17. The solution of the initial value problem

$$X' = \begin{pmatrix} 4 & -5 \\ 5 & -4 \end{pmatrix} X,\ X(0) = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \text{ is } X =$$

(a) $\begin{pmatrix} -5\sin 3t \\ 3\cos 3t - 4\sin 3t \end{pmatrix}$ ——————————(correct)

(b) $\begin{pmatrix} 5\sin 3t \\ 3\cos 3t + \sin 3t \end{pmatrix}$

(c) $\begin{pmatrix} \sin 3t \\ 3\cos 3t + 4\sin 3t \end{pmatrix}$

(d) $\begin{pmatrix} 5\sin 3t \\ 3\sin 3t + 3\cos 3t \end{pmatrix}$

(e) $\begin{pmatrix} 3\sin 3t \\ \sin 3t + \cos 3t \end{pmatrix}$

18. A particular solution X_p of the nonhomogeneous system

$$\begin{aligned} \frac{dx}{dt} &= 3x - 3y + 4 \\ \frac{dy}{dt} &= 2x - 2y - 1 \end{aligned} \quad \text{is } X_p =$$

(a) $\begin{pmatrix} -11 \\ -11 \end{pmatrix} t + \begin{pmatrix} -15 \\ -10 \end{pmatrix}$ ——————————(correct)

(b) $\begin{pmatrix} 11 \\ 11 \end{pmatrix} t + \begin{pmatrix} -15 \\ 10 \end{pmatrix}$

(c) $\begin{pmatrix} -11 \\ -11 \end{pmatrix} t + \begin{pmatrix} -10 \\ 15 \end{pmatrix}$

(d) $\begin{pmatrix} 1 \\ 1 \end{pmatrix} t + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(e) $\begin{pmatrix} -1 \\ -1 \end{pmatrix} t + \begin{pmatrix} -15 \\ -10 \end{pmatrix}$

19. Given $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix}$. If $e^{At} = \begin{pmatrix} t+1 & t & ct \\ t & t+a & t \\ -2t & -2t & bt+1 \end{pmatrix}$ then $a+b+c=$

(a) 0 ______ (correct)
(b) 1
(c) 2
(d) -1
(e) -2

20. If $e^{At} = \begin{pmatrix} \cosh t & \sinh t \\ \sinh t & \cosh t \end{pmatrix}$ is the solution of the system $X' = AX$ for some 2×2 matrix A. A particular solution of the system $X' = AX + \begin{pmatrix} \cosh t \\ \sinh t \end{pmatrix}$ is

(a) $t \begin{pmatrix} \cosh t \\ \sinh t \end{pmatrix}$ ______ (correct)
(b) $t \begin{pmatrix} \sinh t \\ \cosh t \end{pmatrix}$
(c) $\begin{pmatrix} \cosh t \\ \sinh t \end{pmatrix}$
(d) $\begin{pmatrix} \sinh t \\ \cosh t \end{pmatrix}$
(e) $t \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$