King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 202

Exam 1

222

February 20, 2023

Net Time Allowed: 120 Minutes

USE THIS AS A TEMPLATE

Write your questions, once you are satisfied upload this file.

Q12/P.13 (Section 1.1)

1. If $y = A + Be^{-20x}$ is a solution of the differential equation $\frac{dy}{dx} + 20y = 24$, then $A =$

   (a) $\frac{6}{5}$

   (b) $\frac{3}{2}$

   (c) $-\frac{3}{2}$

   (d) $-\frac{6}{5}$

   (e) $\frac{7}{6}$

Similar to Q25-28/P.20 (Section 1.2)

2. Using the existance and uniqueness theorem, a value of β so that the initial-value problem

$$\frac{dy}{dx} = \frac{\sqrt{y-3x}}{\sqrt{9-x^2}}, \; y(2) = \beta,$$

   has a unique solution is

   (a) 7

   (b) 6

   (c) 5

   (d) 4

   (e) 3

Q19/P.52 (Section 2.2)

3. The solution of the differential equation

$$\frac{dy}{dx} = \frac{xy + 3x - y - 3}{xy - 2x + 4y - 8}$$

is given by

(a) $(y - x) + 5\ln\left|\frac{x+4}{y+3}\right| = c$

(b) $(y - x) + 5\ln\left|\frac{y+3}{x+4}\right| = c$

(c) $(y + x) + 5\ln\left|\frac{x+4}{y+3}\right| = c$

(d) $(y + x) + 5\ln\left|\frac{y+3}{x+4}\right| = c$

(e) $(y - x) + 3\ln\left|\frac{x+4}{y+3}\right| = c$

Q43/P. 21 (Section 1.2)

4. Given that $y = c_1\cos(2x) + c_2\sin(2x)$ is the general solution of the differential equation $y'' + 4y = 0$. The boundary-value problem

$$y'' + 4y = 0$$
$$y(0) = 0,\ y(\pi) = 2$$

(a) has no solution

(b) has infinitly many solution

(c) has a unique solution $y = 2\cos(2x)$

(d) has a unique solution $y = 2\sin(2x)$

(e) has a unique solution $y = 2\cos(2x) + 2\sin(2x)$

Q33/P.63 (Section 2.3)

5. The solution of the initial-value problem

$$(x+1)\frac{dy}{dx} + y = \ln x, \;\; y(1) = 10$$

is given by

(a) $(x+1)y = x\ln x - x + 21$

(b) $(x+1)y = \ln x - x + 21$

(c) $(x+1)y = x^2 \ln x - x + 21$

(d) $(x+1)y = x\ln x + x + 19$

(e) $(x+1)y = x^2 \ln x + x + 19$

Similar to Q27-28/P. 70 (Section 2.4)

6. The function $k(x)$ with $k(0) = 0$ that makes

$$\frac{dy}{dx} = \frac{y\cos x + 2xe^y + 3}{k(x) - x^2 e^y + 2x}$$

an exact differential equation is

(a) $k(x) = -\sin x - 2x$

(b) $k(x) = \sin x + 3x$

(c) $k(x) = 2\sin x + x$

(d) $k(x) = 3\sin x - x$

(e) $k(x) = -\sin x + x$

Q12/P.70 (Section 2.4)

7. The solution of the exact differential equation

$$(3x^2y + e^y)\,dx + (x^3 + xe^y - 2y)\,dy = 0$$

is

(a) $x^3y + xe^y - y^2 = c$

(b) $x^3y - xe^y + y^2 = c$

(c) $x^3y + 2xe^y + y^2 = c$

(d) $x^3y + 2xe^y - y^2 = c$

(e) $x^3y - xe^y + 2y^2 = c$

Q36/P. 71 (Section 2.4)

8. An integrating factor that can by used to make the differential equation

$$(y^2 + xy^3)\,dx + (5y^2 - xy + y^3 \sin y)\,dy = 0$$

exact is given by

(a) $\dfrac{1}{y^3}$

(b) $\dfrac{1}{y^2}$

(c) $\dfrac{1}{y}$

(d) y^3

(e) y^2

Q10/ P. 75 (Section 2.5)

9. The solution of the homogeneous differential equation

$$x\frac{dy}{dx} = y + \sqrt{x^2 - y^2},\ x > 0$$

is given by

(a) $y = x\sin(\ln x + c)$

(b) $y = x^2 \sin(\ln x + c)$

(c) $y = 2x\ \sin(\ln x + c)$

(d) $y = 2x^2\ \sin(\ln x + c)$

(e) $y = 3x\sin(2\ln x + c)$

Q18 / P. 75 (Section 2.5)

10. By using an appropriate substitution, the differential equation $x\frac{dy}{dx} - (1+x)y = xy^2$ can be transformed to one of the following linear differential equations

(a) $\frac{du}{dx} + \left(1 + \frac{1}{x}\right) u = -1$

(b) $\frac{du}{dx} + \left(1 + \frac{1}{x}\right) u = 1$

(c) $\frac{du}{dx} + \left(1 + \frac{1}{x}\right) u = 2$

(d) $\frac{du}{dx} + \left(1 + \frac{1}{x}\right) u = -2$

(e) $\frac{du}{dx} + \left(1 - \frac{1}{x}\right) u = 1$

Q27 / P. 75 (Section 2.5)

11. The solution of the differential equation

$$\frac{dy}{dx} = 2 + \sqrt{y - 2x + 3}$$

is given by

(a) $4(y - 2x + 3) = (x + c)^2$

(b) $2(y - 2x + 3) = (x + c)^2$

(c) $4(y + 2x - 3) = (x + c)^2$

(d) $2(y - 2x + 3) = (x + c)^3$

(e) $4(y - 2x + 3) = (x + c)^3$

Q4 / P. 91 (Section 3.1)

12. The population of bacteria in a culture grows at a rate proportional to the number of bacteria present at time t. After 3 hours it is observed that 400 bacteria are present. After 10 hours 2000 bacteria are present. What was the initial number of bacteria?

(a) $\dfrac{400}{\sqrt[7]{125}}$

(b) $\dfrac{400}{\sqrt[5]{125}}$

(c) $\dfrac{400}{\sqrt[7]{250}}$

(d) $\dfrac{400}{\sqrt[5]{250}}$

(e) $\dfrac{400}{\sqrt[3]{250}}$

Similar to Q1-8 / P. 12 (Section 1.1)

13. Which one of the following is a fourth-order linear differential equation

(a) $\sin x \dfrac{d^4y}{dx^4} + 3x^2\dfrac{d^2y}{dx^2} + e^x y = \ln x$

(b) $x^4\dfrac{d^4y}{dx^4} + x^3\dfrac{d^2y}{dx^3} + y = e^y$

(c) $x^3\dfrac{d^3y}{dx^3} + x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = x$

(d) $x^4\left(\dfrac{dy}{dx}\right)^4 + x^3\dfrac{d^3y}{dx^3} + x\dfrac{dy}{dx} + y = e^x$

(e) $x^4\dfrac{d^4y}{dx^4} + y^3\dfrac{d^3y}{dx^3} + x^2\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = x$

Example 3/P.49 (Section 2.2)

14. Given the differential equation $\dfrac{dy}{dx} = y^2 - 4$, then the sum of all constant solutions is

(a) 0

(b) 2

(c) -2

(d) 4

(e) -4