1. The general solution of the differential equation $xy^{(4)} + 6y''' = 0$ is given by

   (a) $y = c_1 + c_2x + c_3x^2 + c_4x^{-3}$ ________________________________ (correct)
   (b) $y = c_1 + c_2x^2 + c_3x^{-1} + c_4x^{-2}$
   (c) $y = c_1 + c_2x + c_3x^{-1} + c_4x^{-3}$
   (d) $y = c_1 + c_2x^2 + c_3x^{-2} + c_4x^{-3}$
   (e) $y = c_1 + c_2x + c_3x^{-2} + c_4x^2$

2. Using the substitution $x = e^t$, the Cauchy-Euler equation $x^2y'' - 3xy' + 13y = 4 + 3x$ is transformed to

   (a) $y''(t) - 4y'(t) + 13y(t) = 4 + 3e^t$ ________________________________ (correct)
   (b) $y''(t) + 4y'(t) - 13y(t) = 4 + 3e^t$
   (c) $y''(t) - 4y'(t) + 13y(t) = 4 + 3t$
   (d) $y''(t) + 4y'(t) - 13y(t) = 4 + 3t$
   (e) $y''(t) - 3y'(t) + 13y(t) = 4 + 3e^t$

3. The general solution of the differential equation $y'' + 2y' + y = e^{-t} \ln t$ is given by

(a) $y = c_1e^{-t} + c_2te^{-t} + \frac{1}{2}t^2e^{-t} \ln t - \frac{3}{4}t^2e^{-t}$ ________________________(correct)

(b) $y = c_1e^{-t} + c_2te^{-t} - \frac{1}{2}t^2e^{-t} \ln t - \frac{3}{4}t^2e^{-t}$

(c) $y = c_1e^{-t} + c_2te^{-t} - \frac{1}{2}t^2e^{-t} \ln t + \frac{3}{4}t^2e^{-t}$

(d) $y = c_1e^{-t} + c_2te^{-t} + \frac{1}{2}t^2e^{-t} \ln t + \frac{3}{4}t^2e^{-t}$

(e) $y = c_1e^{t} + c_2te^{t} + \frac{1}{2}t^2e^{-t} \ln t - \frac{3}{4}t^2e^{-t}$

4. A linear differential operator that annihilates the function $(2 - e^x)^2$ is given by

(a) $D^3 - 3D^2 + 2D$ ________________________(correct)

(b) $D^3 + 3D^2 + 2D$

(c) $D^3 - 3D^2 - 2D$

(d) $D^3 - 3D^2 + 2$

(e) $D^3 + 3D^2 - 2$

5. The most suitable form of a particular solution for the differential equation $(D^2 - 4)y = e^{2x} - x^2 e^{-2x}$ is given by

   (a) $Axe^{2x} + (Bx + Cx^2 + Ex^3)\, e^{-2x}$ ______ (correct)
   (b) $Ax^2e^{2x} + (Bx + Cx^2 + Ex^3)\, e^{-2x}$
   (c) $Axe^{2x} + (B + Cx + Ex^2)\, e^{-2x}$
   (d) $Ax^2e^{2x} + (B + Cx + Ex^2)\, e^{-2x}$
   (e) $Axe^{2x} + (Bx + Ex^2)\, e^{-2x}$

6. If $y_p = Ax + B$ is a particular solution of the differential equation $y'' + 7y' + 12y = 12x + 19$, then $2A + 3B =$

   (a) 5 ______ (correct)
   (b) 0
   (c) -5
   (d) 6
   (e) -6

7. If $y(x)$ is the solution of the initial-value problem $y''+16y=0,\ y(0)=2,\ y'(0)=-2$, then $y\left(\frac{\pi}{2}\right)=$

(a) 2 ______________________________ (correct)

(b) -2

(c) 0

(d) $\frac{1}{2}$

(e) $-\frac{1}{2}$

8. The general solution of the differential equation $y'''+3y''+3y'+y=0$ is given by

(a) $y=c_1e^{-x}+c_2xe^{-x}+c_3x^2e^{-x}$ ______________________________ (correct)

(b) $y=c_1e^{-x}+c_2xe^{-x}+c_3e^{-x}$

(c) $y=c_1e^{x}+c_2xe^{x}+c_3x^2e^{x}$

(d) $y=c_1e^{x}+c_2e^{2x}+c_3e^{3x}$

(e) $y=c_1e^{x}+c_2xe^{-x}+c_3x^2e^{-x}$

9. A homogeneous linear differential equation with constant coefficients whose general solution is given by $y = c_1 e^{-x} \cos x + c_2 e^{-x} \sin x$ is

(a) $y'' + 2y' + 2y = 0$ ________________ (correct)

(b) $y'' - 2y' + 2y = 0$

(c) $y'' + 2y' - 2y = 0$

(d) $y'' - 2y' - 2y = 0$

(e) $y'' + 2y' - 3y = 0$

10. If $y_1 = \dfrac{\cos x}{\sqrt{x}}$ is a solution of the differential equation

$$x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0 \text{ on } \left(0, \frac{\pi}{2}\right),$$

then a second linearly independent solution is

(a) $\dfrac{\sin x}{\sqrt{x}}$ ________________ (correct)

(b) $\sin(\sqrt{x})$

(c) $\sin(2\sqrt{x})$

(d) $\dfrac{\sin x}{x}$

(e) $\dfrac{\sin x}{x^2}$

11. Which one of the following set of functions is linearly dependent

(a) $\left\{e^{x+2},\, e^{x-3}\right\}$ ________________ (correct)
(b) $\left\{e^{x},\, e^{2x}\right\}$
(c) $\{1, e^{x},\, xe^{x}\}$
(d) $\left\{1, xe^{x},\, xe^{2x}\right\}$
(e) $\left\{x, x^2,\, 4x - x^3\right\}$

12. Let L be a linear differential operator. Assume that $y_1 = x^2$, $y_2 = 3\sin x$ are particular solutions of the differential equation

$$L(y) = 3x^2,\;\; L(y) = \cos x, \text{ respectively.}$$

A particular solution of the differential equation $L(y) = \dfrac{x^2}{3} - 2\,\cos x$ is

(a) $\dfrac{x^2}{9} - 6\sin x$ ________________ (correct)
(b) $3x^2 - 6\sin x$
(c) $\dfrac{x^2}{9} - \dfrac{3}{2}\sin x$
(d) $\dfrac{x^2}{9} + \dfrac{3}{2}\sin x$
(e) $9x^2 - \dfrac{3}{2}\sin x$

13. Which one of the following cannot form a fundamental set of solutions for a 3rd order homogeneous linear differential equation?

(a) $\{e^x, e^{-x}, \sinh x\}$ ____(correct)

(b) $\{e^x, e^{-x}, e^{2x}\}$

(c) $\{1, x, x^2\}$

(d) $\{x, x^2, x + x^3\}$

(e) $\{e^{-x}, e^{2x}, \cosh(2x)\}$

14. Given $y = c_1 e^x \cos x + c_2 e^x \sin x$ is a two-parameter family of solutions of the differential equation $y'' - 2y' + 2y = 0$. A member of the family that satisfies $y(0) = 1$, $y'(\pi) = 0$ is given by

(a) $y = e^x \cos x - e^x \sin x$ ____(correct)

(b) $y = e^x \cos x + e^x \sin x$

(c) $y = e^x \cos x + 3e^x \sin x$

(d) $y = e^x \cos x - 2e^x \sin x$

(e) $y = e^x \cos x + 4e^x \sin x$