King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 202
Final Exam
222
May 22, 2023

EXAM COVER

Number of versions: 4
Number of questions: 20

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Department of Mathematics

Math 202

Final Exam

222

May 22, 2023

Net Time Allowed: 180 Minutes

MASTER VERSION

1. If c is constant, then the solution of the differential equation

$$y' + \frac{1}{x+1}\, y = \frac{\ln x}{x+1}$$

is given by

(a) $y = \dfrac{x}{x+1}\ln x - \dfrac{x}{x+1} + \dfrac{c}{x+1}$ ________________ (correct)

(b) $y = \dfrac{x}{x+1}\ln x + \dfrac{c}{x+1}$

(c) $y = \dfrac{x+1}{x}\ln x + \dfrac{x+1}{x} + c(x+1)$

(d) $y = \dfrac{x}{x+1}\ln x + \dfrac{x}{x+1} + \dfrac{c}{x+1}$

(e) $y = \dfrac{x+1}{x}\ln x - \dfrac{x+1}{x} + c(x+1)$

2. If c is constant, then the solution of the exact differential equation

$$(y^2 + y\sin x)\, dx + \left(2xy - \cos x - \frac{1}{1+y^2}\right) dy = 0$$

is given by

(a) $xy^2 - y\cos x - \tan^{-1} y = c$ ________________ (correct)

(b) $xy^2 + y\sin x + \tan^{-1} y = c$

(c) $xy^2 - 2y\cos x + \tan^{-1} y = c$

(d) $xy^2 - y\cos x + \tan^{-1} y = c$

(e) $xy^2 - y\cos x + 2\tan^{-1} y = c$

3. If c is constant, then the solution of the homogeneous differential equation

$$(x+3y)\,dx-(3x+y)\,dy=0$$

is given by

(a) $(y-x)^2=c(y+x)$ ____(correct)

(b) $y-x=c(y+x)$

(c) $y-x=c(y+x)^2$

(d) $(y+x)^2=c(y-x)$

(e) $(y+x)^2=c(y-2x)$

4. If $y_1=1$ is a solution of the differential equation $(1-x^2)y''-2xy'=0$, then by using reduction of order, a second solution $y_2=$

(a) $\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$ ____(correct)

(b) $\frac{1}{2}\ln\left|\frac{2+x}{1-x}\right|$

(c) $\frac{1}{2}\ln\left|\frac{1+x}{2-x}\right|$

(d) $\frac{1}{3}\ln\left|\frac{1-x}{2+x}\right|$

(e) $\frac{1}{3}\ln\left|\frac{1-x}{1+x}\right|$

5. A homogeneous linear differential equation with constant coefficients whose general solution is

$$y = c_1 \cos x + c_2 \sin x + c_3 \cos 2x + c_4 \sin 2x$$

is given by

(a) $y^{(4)} + 5y'' + 4y = 0$ ____(correct)

(b) $y^{(4)} + 4y'' + 5y = 0$

(c) $y^{(4)} + 5y'' - 4y = 0$

(d) $y^{(4)} - 5y'' + 4y = 0$

(e) $y^{(4)} + 5y'' + 6y = 0$

6. Using the undetermined coefficients, a form of a particular solution for the differential equation

$$y'' + 3y' = 4x - 5$$

is given by

(a) $y_p = Ax^2 + Bx$ ____(correct)

(b) $y_p = Ax^3 + Bx$

(c) $y_p = Ax^2 + Bx^3$

(d) $y_p = Ax^2 + Bx^4$

(e) $y_p = Ax + B$

7. By using variation of parameters method, a particular solution of the differential equation

$$y'' - 9y = \frac{9x}{e^{3x}} \text{ is given by}$$

(a) $y_p = \left(-\frac{1}{24} - \frac{1}{4}x - \frac{3}{4}x^2\right) e^{-3x}$ ______________________________(correct)

(b) $y_p = \left(-\frac{1}{24} + x + \frac{3}{4}x^2\right) e^{-3x}$

(c) $y_p = \left(\frac{1}{24} + 3x - x^2\right) e^{-3x}$

(d) $y_p = \left(1 - \frac{1}{4}xe - \frac{3}{4}x^2\right) e^{-3x}$

(e) $y_p = \left(\frac{1}{4} - x - \frac{3}{4}x^2\right) e^{-3x}$

8. The general solution of the differential equation $x^3y''' - 6y = 0$ is given by

(a) $y = c_1x^3 + c_2 \cos(\sqrt{2} \ln x) + c_3 \sin(\sqrt{2} \ln x)$ ______________________________(correct)

(b) $y = c_1x^2 + c_2 \cos(\sqrt{2} \ln x) + c_3 \sin(\sqrt{2} \ln x)$

(c) $y = c_1x^3 + c_2 \cos(2 \ln x) + c_3 \sin(2 \ln x)$

(d) $y = c_1x^2 + c_2 \cos(2 \ln x) + c_3 \sin(2 \ln x)$

(e) $y = c_1x^3 + c_2 \cos(3 \ln x) + c_3 \sin(3 \ln x)$

9. The minimum radius of convergence of a power series solution of the second order differential equation $(x^2 - 2x + 5)y'' + xy' - y = 0$ about the ordinary point $x = -1$ is equal to

(a) $2\sqrt{2}$ ____ (correct)

(b) 5

(c) $3\sqrt{2}$

(d) $2\sqrt{3}$

(e) 1

10. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2 + 1)y'' - 6y = 0$, then the recurrence relation is given by

(a) $c_2 = 3c_0,\ c_3 = c_1,\ c_{k+2} = \dfrac{3-k}{k+1}c_k,\ k = 2, 3, \ldots$ ____________________ (correct)

(b) $c_2 = c_0,\ c_3 = c_1,\ c_{k+2} = \dfrac{k-3}{k+1}c_k,\ k = 2, 3, \ldots$

(c) $c_2 = 2c_0,\ c_3 = 2c_1,\ c_{k+2} = \dfrac{4-k}{k+1}c_k,\ k = 2, 3, \ldots$

(d) $c_2 = 3c_0,\ c_3 = 2c_1,\ c_{k+2} = \dfrac{3+k}{k+1}c_k,\ k = 2, 3, \ldots$

(e) $c_2 = 4c_0,\ c_3 = c_1,\ c_{k+2} = \dfrac{3-k}{k+1}c_k,\ k = 2, 3, \ldots$

11. The number of regular singular points of the differential equation

$$x^3(x^2-25)(x-2)^2y''+3x(x-2)y'+7(x+5)y=0$$

is

(a) 3 ____ (correct)

(b) 2

(c) 1

(d) 0

(e) 5

12. If $y=\sum_{n=0}^{\infty} c_n x^{n+r}$ is a series solution for the differential equation $2xy''-y'+2y=0$ about $x=0$, then the non-integer indicial root is equal to

(a) $\frac{3}{2}$ ____ (correct)

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{3}{4}$

(e) $\frac{4}{3}$

13. If $c_0 \neq 0, c_1 = 0,\ c_k = -\frac{c_{k-2}}{k(2k-1)},\ k = 2, 3, 4, \ldots$ is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of the differential equation $2x^2y'' - xy' + (x^2+1)y = 0$ about $x = 0$, then the solution is given by

(a) $y = x^{\frac{1}{2}}\left[1 - \dfrac{1}{6}x^2 + \dfrac{1}{168}x^4 - \ldots\right]$ ______ (correct)

(b) $y = x^{\frac{1}{2}}\left[1 - \dfrac{1}{6}x^2 - \dfrac{1}{168}x^4 + \ldots\right]$

(c) $y = x^{\frac{1}{2}}\left[1 + \dfrac{1}{6}x^2 + \dfrac{1}{168}x^4 - \ldots\right]$

(d) $y = x^{\frac{1}{2}}\left[1 + \dfrac{1}{3}x^2 + \dfrac{1}{68}x^4 - \ldots\right]$

(e) $y = x^{\frac{1}{2}}\left[1 - \dfrac{1}{3}x^2 + \dfrac{1}{68}x^4 + \ldots\right]$

14. If $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$ and $X_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$ are two solution vectors of a homogeneous linear system $X' = AX$, then the Wronskian $W(X_1, X_2) =$

(a) $8e^{4t}$ ______ (correct)

(b) $6e^{4t}$

(c) $8e^{8t}$

(d) $6e^{8t}$

(e) $8e^{6t}$

15. If the general solution of the system $X' = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} X$ is given by

$$X = c_1 \begin{pmatrix} -2 \\ m \end{pmatrix} e^{nt} + c_2 \begin{pmatrix} 1 \\ k \end{pmatrix} e^{4t},$$

then $m + n + k =$

(a) 3 ________ (correct)
(b) 4
(c) 2
(d) -2
(e) -4

16. If $X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t}$ is a solution of the linear system $X' = \begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix} X$ that corresponds to the only eigenvalue $\lambda = -1$, then a second linearly independent solution of the system is given by

(a) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} e^{-t}$ ________ (correct)
(b) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + t \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} e^{-t}$
(c) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-t}$
(d) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{-t}$
(e) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + t \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{-t}$

17. Given that $K = \begin{pmatrix} 1 \\ 1-2i \end{pmatrix}$ is an eigenvector that corresponds to the eigenvalue $\lambda = 5+2i$ of the matrix $A = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix}$. If $X(t)$ is the solution of the initial value problem $X' = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} X$, $X(0) = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$ then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{pmatrix} 2 \\ -8 \end{pmatrix} e^{\frac{5\pi}{2}}$ ____(correct)

(b) $\begin{pmatrix} 1 \\ 4 \end{pmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{pmatrix} 1 \\ -8 \end{pmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{\frac{5\pi}{2}}$

18. If $X_c = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t}$ is the general solution of the homogeneous linear system $X' = AX$, then using the variation of parameters method, a particular solution X_p of the non-homogeneous system $X' = AX + \begin{pmatrix} 0 \\ 4t \end{pmatrix}$ is given by

(a) $X_p = \begin{pmatrix} 4t \\ 8t-4 \end{pmatrix}$ ____(correct)

(b) $X_p = \begin{pmatrix} t \\ 8t-4 \end{pmatrix}$

(c) $X_p = \begin{pmatrix} 4t \\ t-1 \end{pmatrix}$

(d) $X_p = \begin{pmatrix} t \\ t-1 \end{pmatrix}$

(e) $X_p = \begin{pmatrix} 4t \\ t+1 \end{pmatrix}$

19. Using the exponential of a matrix method, if the general solution of the system $X' = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 5 & 1 & 0 \end{pmatrix} X$ is given by $X = \begin{pmatrix} 1 & 0 & 0 \\ g(t) & 1 & 0 \\ h(t) & f(t) & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$,
then $g(2) + h(2) + f(2) =$

(a) 24 ________________ (correct)
(b) 22
(c) 20
(d) 18
(e) 26

20. The eigenvalues of the matrix

$$A = \begin{pmatrix} 5 & -1 & 0 \\ 0 & -5 & 9 \\ 5 & -1 & 0 \end{pmatrix}$$

are

(a) $\lambda = 0$, $\lambda = 4$ and $\lambda = -4$ ________________ (correct)
(b) $\lambda = 0$, $\lambda = 3$ and $\lambda = -3$
(c) $\lambda = 1$, $\lambda = 4$ and $\lambda = -4$
(d) $\lambda = 1$, $\lambda = 3$ and $\lambda = -3$
(e) $\lambda = 1$, $\lambda = 1 \pm 2i$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE01 **CODE01**

Math 202
Final Exam
222
May 22, 2023
Net Time Allowed: 180 Minutes

Name			
ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. A homogeneous linear differential equation with constant coefficients whose general solution is

$$y = c_1 \cos x + c_2 \sin x + c_3 \cos 2x + c_4 \sin 2x$$

is given by

(a) $y^{(4)} + 5y'' - 4y = 0$
(b) $y^{(4)} - 5y'' + 4y = 0$
(c) $y^{(4)} + 4y'' + 5y = 0$
(d) $y^{(4)} + 5y'' + 6y = 0$
(e) $y^{(4)} + 5y'' + 4y = 0$

2. If $y_1 = 1$ is a solution of the differential equation $(1 - x^2)y'' - 2xy' = 0$, then by using reduction of order, a second solution $y_2 =$

(a) $\frac{1}{2} \ln \left| \frac{1+x}{2-x} \right|$
(b) $\frac{1}{2} \ln \left| \frac{2+x}{1-x} \right|$
(c) $\frac{1}{2} \ln \left| \frac{1+x}{1-x} \right|$
(d) $\frac{1}{3} \ln \left| \frac{1-x}{1+x} \right|$
(e) $\frac{1}{3} \ln \left| \frac{1-x}{2+x} \right|$

3. If c is constant, then the solution of the homogeneous differential equation

$$(x + 3y)\, dx - (3x + y)\, dy = 0$$

is given by

(a) $y - x = c(y + x)$
(b) $y - x = c(y + x)^2$
(c) $(y + x)^2 = c(y - 2x)$
(d) $(y + x)^2 = c(y - x)$
(e) $(y - x)^2 = c(y + x)$

4. If c is constant, then the solution of the exact differential equation

$$(y^2 + y \sin x)\, dx + \left(2xy - \cos x - \frac{1}{1 + y^2}\right) dy = 0$$

is given by

(a) $xy^2 - y \cos x - \tan^{-1} y = c$
(b) $xy^2 - y \cos x + 2 \tan^{-1} y = c$
(c) $xy^2 + y \sin x + \tan^{-1} y = c$
(d) $xy^2 - 2y \cos x + \tan^{-1} y = c$
(e) $xy^2 - y \cos x + \tan^{-1} y = c$

5. If c is constant, then the solution of the differential equation

$$y' + \frac{1}{x+1}\,y = \frac{\ln x}{x+1}$$

is given by

(a) $y = \dfrac{x+1}{x}\ln x - \dfrac{x+1}{x} + c(x+1)$

(b) $y = \dfrac{x+1}{x}\ln x + \dfrac{x+1}{x} + c(x+1)$

(c) $y = \dfrac{x}{x+1}\ln x + \dfrac{c}{x+1}$

(d) $y = \dfrac{x}{x+1}\ln x - \dfrac{x}{x+1} + \dfrac{c}{x+1}$

(e) $y = \dfrac{x}{x+1}\ln x + \dfrac{x}{x+1} + \dfrac{c}{x+1}$

6. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2+1)y'' - 6y = 0$, then the recurrence relation is given by

(a) $c_2 = 3c_0,\ c_3 = 2c_1,\ c_{k+2} = \dfrac{3+k}{k+1}c_k,\ k = 2, 3, \ldots$

(b) $c_2 = 3c_0,\ c_3 = c_1,\ c_{k+2} = \dfrac{3-k}{k+1}c_k,\ k = 2, 3, \ldots$

(c) $c_2 = 2c_0,\ c_3 = 2c_1,\ c_{k+2} = \dfrac{4-k}{k+1}c_k,\ k = 2, 3, \ldots$

(d) $c_2 = 4c_0,\ c_3 = c_1,\ c_{k+2} = \dfrac{3-k}{k+1}c_k,\ k = 2, 3, \ldots$

(e) $c_2 = c_0,\ c_3 = c_1,\ c_{k+2} = \dfrac{k-3}{k+1}c_k,\ k = 2, 3, \ldots$

7. The general solution of the differential equation $x^3y''' - 6y = 0$ is given by

(a) $y = c_1x^2 + c_2 \cos(2 \ln x) + c_3 \sin(2 \ln x)$
(b) $y = c_1x^3 + c_2 \cos(3 \ln x) + c_3 \sin(3 \ln x)$
(c) $y = c_1x^3 + c_2 \cos(2 \ln x) + c_3 \sin(2 \ln x)$
(d) $y = c_1x^2 + c_2 \cos(\sqrt{2} \ln x) + c_3 \sin(\sqrt{2} \ln x)$
(e) $y = c_1x^3 + c_2 \cos(\sqrt{2} \ln x) + c_3 \sin(\sqrt{2} \ln x)$

8. Using the undetermined coefficients, a form of a particular solution for the differential equation

$$y'' + 3y' = 4x - 5$$

is given by

(a) $y_p = Ax^3 + Bx$
(b) $y_p = Ax^2 + Bx^4$
(c) $y_p = Ax + B$
(d) $y_p = Ax^2 + Bx^3$
(e) $y_p = Ax^2 + Bx$

9. The minimum radius of convergence of a power series solution of the second order differential equation $(x^2 - 2x + 5)y'' + xy' - y = 0$ about the ordinary point $x = -1$ is equal to

(a) 5
(b) 1
(c) $2\sqrt{2}$
(d) $3\sqrt{2}$
(e) $2\sqrt{3}$

10. By using variation of parameters method, a particular solution of the differential equation

$y'' - 9y = \dfrac{9x}{e^{3x}}$ is given by

(a) $y_p = \left(-\dfrac{1}{24} - \dfrac{1}{4}x - \dfrac{3}{4}x^2\right) e^{-3x}$
(b) $y_p = \left(\dfrac{1}{4} - x - \dfrac{3}{4}x^2\right) e^{-3x}$
(c) $y_p = \left(-\dfrac{1}{24} + x + \dfrac{3}{4}x^2\right) e^{-3x}$
(d) $y_p = \left(1 - \dfrac{1}{4}xe - \dfrac{3}{4}x^2\right) e^{-3x}$
(e) $y_p = \left(\dfrac{1}{24} + 3x - x^2\right) e^{-3x}$

11. If the general solution of the system $X' = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} X$ is given by

$$X = c_1 \begin{pmatrix} -2 \\ m \end{pmatrix} e^{nt} + c_2 \begin{pmatrix} 1 \\ k \end{pmatrix} e^{4t},$$

then $m + n + k =$

(a) 2
(b) -2
(c) 4
(d) 3
(e) -4

12. If $c_0 \neq 0, c_1 = 0,\ c_k = -\frac{c_{k-2}}{k(2k-1)},\ k = 2, 3, 4, \ldots$ is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of the differential equation $2x^2y'' - xy' + (x^2 + 1)y = 0$ about $x = 0$, then the solution is given by

(a) $y = x^{\frac{1}{2}} \left[1 + \frac{1}{3}x^2 + \frac{1}{68}x^4 - \ldots \right]$
(b) $y = x^{\frac{1}{2}} \left[1 - \frac{1}{6}x^2 - \frac{1}{168}x^4 + \ldots \right]$
(c) $y = x^{\frac{1}{2}} \left[1 - \frac{1}{6}x^2 + \frac{1}{168}x^4 - \ldots \right]$
(d) $y = x^{\frac{1}{2}} \left[1 - \frac{1}{3}x^2 + \frac{1}{68}x^4 + \ldots \right]$
(e) $y = x^{\frac{1}{2}} \left[1 + \frac{1}{6}x^2 + \frac{1}{168}x^4 - \ldots \right]$

13. If $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ is a series solution for the differential equation $2xy'' - y' + 2y = 0$ about $x = 0$, then the non-integer indicial root is equal to

(a) $\frac{3}{4}$
(b) $\frac{4}{3}$
(c) $\frac{3}{2}$
(d) $\frac{1}{2}$
(e) $\frac{2}{3}$

14. If $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$ and $X_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$ are two solution vectors of a homogeneous linear system $X' = AX$, then the Wronskian $W(X_1, X_2) =$

(a) $8e^{4t}$
(b) $6e^{8t}$
(c) $8e^{8t}$
(d) $8e^{6t}$
(e) $6e^{4t}$

15. The number of regular singular points of the differential equation

$$x^3(x^2-25)(x-2)^2y'' + 3x(x-2)y' + 7(x+5)y = 0$$

is

(a) 5
(b) 0
(c) 3
(d) 1
(e) 2

16. The eigenvalues of the matrix

$$A = \begin{pmatrix} 5 & -1 & 0 \\ 0 & -5 & 9 \\ 5 & -1 & 0 \end{pmatrix}$$

are

(a) $\lambda = 1$, $\lambda = 1 \pm 2i$
(b) $\lambda = 0$, $\lambda = 3$ and $\lambda = -3$
(c) $\lambda = 1$, $\lambda = 4$ and $\lambda = -4$
(d) $\lambda = 0$, $\lambda = 4$ and $\lambda = -4$
(e) $\lambda = 1$, $\lambda = 3$ and $\lambda = -3$

17. If $X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t}$ is a solution of the linear system $X' = \begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix} X$ that corresponds to the only eigenvalue $\lambda = -1$, then a second linearly independent solution of the system is given by

(a) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-t}$

(b) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + t \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{-t}$

(c) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} e^{-t}$

(d) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + t \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} e^{-t}$

(e) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{-t}$

18. Using the exponential of a matrix method, if the general solution of the system $X' = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 5 & 1 & 0 \end{pmatrix} X$ is given by $X = \begin{pmatrix} 1 & 0 & 0 \\ g(t) & 1 & 0 \\ h(t) & f(t) & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$, then $g(2) + h(2) + f(2) =$

(a) 18
(b) 20
(c) 26
(d) 22
(e) 24

19. Given that $K = \begin{pmatrix} 1 \\ 1-2i \end{pmatrix}$ is an eigenvector that corresponds to the eigenvalue $\lambda = 5+2i$ of the matrix $A = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix}$. If $X(t)$ is the solution of the initial value problem $X' = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} X$, $X(0) = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$ then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{\frac{5\pi}{2}}$

(b) $\begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{pmatrix} 2 \\ -8 \end{pmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{pmatrix} 1 \\ 4 \end{pmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{pmatrix} 1 \\ -8 \end{pmatrix} e^{\frac{5\pi}{2}}$

20. If $X_c = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t}$ is the general solution of the homogeneous linear system $X' = AX$, then using the variation of parameters method, a particular solution X_p of the non-homogeneous system $X' = AX + \begin{pmatrix} 0 \\ 4t \end{pmatrix}$ is given by

(a) $X_p = \begin{pmatrix} t \\ t-1 \end{pmatrix}$

(b) $X_p = \begin{pmatrix} 4t \\ 8t-4 \end{pmatrix}$

(c) $X_p = \begin{pmatrix} 4t \\ t-1 \end{pmatrix}$

(d) $X_p = \begin{pmatrix} 4t \\ t+1 \end{pmatrix}$

(e) $X_p = \begin{pmatrix} t \\ 8t-4 \end{pmatrix}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE02 **CODE02**

Math 202
Final Exam
222
May 22, 2023
Net Time Allowed: 180 Minutes

Name	
ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If c is constant, then the solution of the exact differential equation

$$(y^2 + y\sin x)\,dx + \left(2xy - \cos x - \frac{1}{1+y^2}\right)\,dy = 0$$

is given by

(a) $xy^2 - y\cos x - \tan^{-1}\, y = c$
(b) $xy^2 - 2y\cos x + \tan^{-1}\, y = c$
(c) $xy^2 - y\cos x + 2\tan^{-1}\, y = c$
(d) $xy^2 - y\cos x + \tan^{-1}\, y = c$
(e) $xy^2 + y\sin x + \tan^{-1}\, y = c$

2. If c is constant, then the solution of the homogeneous differential equation

$$(x + 3y)\,dx - (3x + y)\,dy = 0$$

is given by

(a) $(y + x)^2 = c(y - 2x)$
(b) $y - x = c(y + x)$
(c) $(y - x)^2 = c(y + x)$
(d) $(y + x)^2 = c(y - x)$
(e) $y - x = c(y + x)^2$

3. If $y_1 = 1$ is a solution of the differential equation $(1-x^2)y'' - 2xy' = 0$, then by using reduction of order, a second solution $y_2 =$

(a) $\frac{1}{3} \ln \left| \frac{1-x}{1+x} \right|$

(b) $\frac{1}{2} \ln \left| \frac{1+x}{1-x} \right|$

(c) $\frac{1}{2} \ln \left| \frac{1+x}{2-x} \right|$

(d) $\frac{1}{2} \ln \left| \frac{2+x}{1-x} \right|$

(e) $\frac{1}{3} \ln \left| \frac{1-x}{2+x} \right|$

4. If c is constant, then the solution of the differential equation

$$y' + \frac{1}{x+1} y = \frac{\ln x}{x+1}$$

is given by

(a) $y = \frac{x+1}{x} \ln x + \frac{x+1}{x} + c(x+1)$

(b) $y = \frac{x}{x+1} \ln x - \frac{x}{x+1} + \frac{c}{x+1}$

(c) $y = \frac{x}{x+1} \ln x + \frac{x}{x+1} + \frac{c}{x+1}$

(d) $y = \frac{x+1}{x} \ln x - \frac{x+1}{x} + c(x+1)$

(e) $y = \frac{x}{x+1} \ln x + \frac{c}{x+1}$

5. A homogeneous linear differential equation with constant coefficients whose general solution is

$$y = c_1 \cos x + c_2 \sin x + c_3 \cos 2x + c_4 \sin 2x$$

is given by

(a) $y^{(4)} + 4y'' + 5y = 0$
(b) $y^{(4)} + 5y'' + 6y = 0$
(c) $y^{(4)} + 5y'' - 4y = 0$
(d) $y^{(4)} - 5y'' + 4y = 0$
(e) $y^{(4)} + 5y'' + 4y = 0$

6. By using variation of parameters method, a particular solution of the differential equation

$$y'' - 9y = \frac{9x}{e^{3x}} \text{ is given by}$$

(a) $y_p = \left(1 - \frac{1}{4}xe - \frac{3}{4}x^2\right) e^{-3x}$
(b) $y_p = \left(-\frac{1}{24} + x + \frac{3}{4}x^2\right) e^{-3x}$
(c) $y_p = \left(\frac{1}{4} - x - \frac{3}{4}x^2\right) e^{-3x}$
(d) $y_p = \left(\frac{1}{24} + 3x - x^2\right) e^{-3x}$
(e) $y_p = \left(-\frac{1}{24} - \frac{1}{4}x - \frac{3}{4}x^2\right) e^{-3x}$

7. The minimum radius of convergence of a power series solution of the second order differential equation $(x^2-2x+5)y''+xy'-y=0$ about the ordinary point $x=-1$ is equal to

(a) 5
(b) $2\sqrt{3}$
(c) $2\sqrt{2}$
(d) $3\sqrt{2}$
(e) 1

8. If $y=\sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2+1)y''-6y=0$, then the recurrence relation is given by

(a) $c_2=4c_0,\ c_3=c_1,\ c_{k+2}=\dfrac{3-k}{k+1}c_k,\ k=2,3,\ldots$
(b) $c_2=3c_0,\ c_3=2c_1,\ c_{k+2}=\dfrac{3+k}{k+1}c_k,\ k=2,3,\ldots$
(c) $c_2=2c_0,\ c_3=2c_1,\ c_{k+2}=\dfrac{4-k}{k+1}c_k,\ k=2,3,\ldots$
(d) $c_2=3c_0,\ c_3=c_1,\ c_{k+2}=\dfrac{3-k}{k+1}c_k,\ k=2,3,\ldots$
(e) $c_2=c_0,\ c_3=c_1,\ c_{k+2}=\dfrac{k-3}{k+1}c_k,\ k=2,3,\ldots$

9. Using the undetermined coefficients, a form of a particular solution for the differential equation

$$y'' + 3y' = 4x - 5$$

is given by

(a) $y_p = Ax^2 + Bx$

(b) $y_p = Ax^2 + Bx^3$

(c) $y_p = Ax^3 + Bx$

(d) $y_p = Ax^2 + Bx^4$

(e) $y_p = Ax + B$

10. The general solution of the differential equation $x^3 y''' - 6y = 0$ is given by

(a) $y = c_1 x^2 + c_2 \cos(2 \ln x) + c_3 \sin(2 \ln x)$

(b) $y = c_1 x^3 + c_2 \cos(\sqrt{2} \ln x) + c_3 \sin(\sqrt{2} \ln x)$

(c) $y = c_1 x^3 + c_2 \cos(2 \ln x) + c_3 \sin(2 \ln x)$

(d) $y = c_1 x^2 + c_2 \cos(\sqrt{2} \ln x) + c_3 \sin(\sqrt{2} \ln x)$

(e) $y = c_1 x^3 + c_2 \cos(3 \ln x) + c_3 \sin(3 \ln x)$

11. If the general solution of the system $X' = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} X$ is given by

$$X = c_1 \begin{pmatrix} -2 \\ m \end{pmatrix} e^{nt} + c_2 \begin{pmatrix} 1 \\ k \end{pmatrix} e^{4t},$$

then $m + n + k =$

(a) -2
(b) 3
(c) -4
(d) 4
(e) 2

12. If $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ is a series solution for the differential equation $2xy'' - y' + 2y = 0$ about $x = 0$, then the non-integer indicial root is equal to

(a) $\frac{1}{2}$
(b) $\frac{4}{3}$
(c) $\frac{2}{3}$
(d) $\frac{3}{2}$
(e) $\frac{3}{4}$

13. If $c_0 \neq 0, c_1 = 0,\ c_k = -\frac{c_{k-2}}{k(2k-1)},\ k = 2, 3, 4, \ldots$ is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of the differential equation $2x^2y'' - xy' + (x^2 + 1)y = 0$ about $x = 0$, then the solution is given by

(a) $y = x^{\frac{1}{2}} \left[1 - \dfrac{1}{6}x^2 + \dfrac{1}{168}x^4 - \ldots\right]$

(b) $y = x^{\frac{1}{2}} \left[1 - \dfrac{1}{3}x^2 + \dfrac{1}{68}x^4 + \ldots\right]$

(c) $y = x^{\frac{1}{2}} \left[1 + \dfrac{1}{6}x^2 + \dfrac{1}{168}x^4 - \ldots\right]$

(d) $y = x^{\frac{1}{2}} \left[1 - \dfrac{1}{6}x^2 - \dfrac{1}{168}x^4 + \ldots\right]$

(e) $y = x^{\frac{1}{2}} \left[1 + \dfrac{1}{3}x^2 + \dfrac{1}{68}x^4 - \ldots\right]$

14. The number of regular singular points of the differential equation

$$x^3(x^2 - 25)(x - 2)^2y'' + 3x(x - 2)y' + 7(x + 5)y = 0$$

is

(a) 0

(b) 5

(c) 2

(d) 1

(e) 3

15. If $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$ and $X_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$ are two solution vectors of a homogeneous linear system $X' = AX$, then the Wronskian $W(X_1, X_2) =$

(a) $8e^{6t}$
(b) $6e^{8t}$
(c) $6e^{4t}$
(d) $8e^{8t}$
(e) $8e^{4t}$

16. Using the exponential of a matrix method, if the general solution of the system $X' = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 5 & 1 & 0 \end{pmatrix} X$ is given by $X = \begin{pmatrix} 1 & 0 & 0 \\ g(t) & 1 & 0 \\ h(t) & f(t) & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$, then $g(2) + h(2) + f(2) =$

(a) 22
(b) 18
(c) 24
(d) 26
(e) 20

17. If $X_c = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t}$ is the general solution of the homogeneous linear system $X' = AX$, then using the variation of parameters method, a particular solution X_p of the non-homogeneous system $X' = AX + \begin{pmatrix} 0 \\ 4t \end{pmatrix}$ is given by

(a) $X_p = \begin{pmatrix} 4t \\ t-1 \end{pmatrix}$

(b) $X_p = \begin{pmatrix} 4t \\ 8t-4 \end{pmatrix}$

(c) $X_p = \begin{pmatrix} 4t \\ t+1 \end{pmatrix}$

(d) $X_p = \begin{pmatrix} t \\ 8t-4 \end{pmatrix}$

(e) $X_p = \begin{pmatrix} t \\ t-1 \end{pmatrix}$

18. Given that $K = \begin{pmatrix} 1 \\ 1-2i \end{pmatrix}$ is an eigenvector that corresponds to the eigenvalue $\lambda = 5 + 2i$ of the matrix $A = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix}$. If $X(t)$ is the solution of the initial value problem $X' = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} X$, $X(0) = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$ then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{\frac{5\pi}{2}}$

(b) $\begin{pmatrix} 2 \\ -8 \end{pmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{pmatrix} 1 \\ 4 \end{pmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{pmatrix} 1 \\ -8 \end{pmatrix} e^{\frac{5\pi}{2}}$

19. If $X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t}$ is a solution of the linear system $X' = \begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix} X$ that corresponds to the only eigenvalue $\lambda = -1$, then a second linearly independent solution of the system is given by

(a) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + t \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{-t}$

(b) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + t \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} e^{-t}$

(c) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-t}$

(d) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{-t}$

(e) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} e^{-t}$

20. The eigenvalues of the matrix

$$A = \begin{pmatrix} 5 & -1 & 0 \\ 0 & -5 & 9 \\ 5 & -1 & 0 \end{pmatrix}$$

are

(a) $\lambda = 1$, $\lambda = 1 \pm 2i$

(b) $\lambda = 1$, $\lambda = 4$ and $\lambda = -4$

(c) $\lambda = 1$, $\lambda = 3$ and $\lambda = -3$

(d) $\lambda = 0$, $\lambda = 3$ and $\lambda = -3$

(e) $\lambda = 0$, $\lambda = 4$ and $\lambda = -4$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE03 **CODE03**

Math 202
Final Exam
222
May 22, 2023
Net Time Allowed: 180 Minutes

Name			
ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
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6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. A homogeneous linear differential equation with constant coefficients whose general solution is

$$y = c_1 \cos x + c_2 \sin x + c_3 \cos 2x + c_4 \sin 2x$$

is given by

(a) $y^{(4)} + 5y'' + 4y = 0$
(b) $y^{(4)} + 5y'' + 6y = 0$
(c) $y^{(4)} + 4y'' + 5y = 0$
(d) $y^{(4)} + 5y'' - 4y = 0$
(e) $y^{(4)} - 5y'' + 4y = 0$

2. If c is constant, then the solution of the exact differential equation

$$(y^2 + y \sin x)\, dx + \left(2xy - \cos x - \frac{1}{1+y^2}\right) dy = 0$$

is given by

(a) $xy^2 - y \cos x + \tan^{-1} y = c$
(b) $xy^2 + y \sin x + \tan^{-1} y = c$
(c) $xy^2 - 2y \cos x + \tan^{-1} y = c$
(d) $xy^2 - y \cos x - \tan^{-1} y = c$
(e) $xy^2 - y \cos x + 2 \tan^{-1} y = c$

3. If c is constant, then the solution of the homogeneous differential equation

$$(x+3y)\,dx-(3x+y)\,dy=0$$

is given by

(a) $(y+x)^2=c(y-x)$
(b) $(y+x)^2=c(y-2x)$
(c) $y-x=c(y+x)^2$
(d) $(y-x)^2=c(y+x)$
(e) $y-x=c(y+x)$

4. If c is constant, then the solution of the differential equation

$$y'+\frac{1}{x+1}\,y=\frac{\ln x}{x+1}$$

is given by

(a) $y=\dfrac{x+1}{x}\ln x+\dfrac{x+1}{x}+c(x+1)$
(b) $y=\dfrac{x+1}{x}\ln x-\dfrac{x+1}{x}+c(x+1)$
(c) $y=\dfrac{x}{x+1}\ln x-\dfrac{x}{x+1}+\dfrac{c}{x+1}$
(d) $y=\dfrac{x}{x+1}\ln x+\dfrac{x}{x+1}+\dfrac{c}{x+1}$
(e) $y=\dfrac{x}{x+1}\ln x+\dfrac{c}{x+1}$

5. If $y_1 = 1$ is a solution of the differential equation $(1 - x^2)y'' - 2xy' = 0$, then by using reduction of order, a second solution $y_2 =$

(a) $\frac{1}{3}\ln\left|\frac{1-x}{1+x}\right|$

(b) $\frac{1}{3}\ln\left|\frac{1-x}{2+x}\right|$

(c) $\frac{1}{2}\ln\left|\frac{2+x}{1-x}\right|$

(d) $\frac{1}{2}\ln\left|\frac{1+x}{2-x}\right|$

(e) $\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$

6. By using variation of parameters method, a particular solution of the differential equation

$$y'' - 9y = \frac{9x}{e^{3x}} \text{ is given by}$$

(a) $y_p = \left(-\frac{1}{24} + x + \frac{3}{4}x^2\right)e^{-3x}$

(b) $y_p = \left(\frac{1}{24} + 3x - x^2\right)e^{-3x}$

(c) $y_p = \left(1 - \frac{1}{4}xe - \frac{3}{4}x^2\right)e^{-3x}$

(d) $y_p = \left(\frac{1}{4} - x - \frac{3}{4}x^2\right)e^{-3x}$

(e) $y_p = \left(-\frac{1}{24} - \frac{1}{4}x - \frac{3}{4}x^2\right)e^{-3x}$

7. Using the undetermined coefficients, a form of a particular solution for the differential equation

$$y'' + 3y' = 4x - 5$$

is given by

(a) $y_p = Ax^2 + Bx^4$
(b) $y_p = Ax + B$
(c) $y_p = Ax^3 + Bx$
(d) $y_p = Ax^2 + Bx^3$
(e) $y_p = Ax^2 + Bx$

8. The general solution of the differential equation $x^3y''' - 6y = 0$ is given by

(a) $y = c_1x^3 + c_2\cos(3\ln x) + c_3\sin(3\ln x)$
(b) $y = c_1x^3 + c_2\cos(2\ln x) + c_3\sin(2\ln x)$
(c) $y = c_1x^3 + c_2\cos(\sqrt{2}\ln x) + c_3\sin(\sqrt{2}\ln x)$
(d) $y = c_1x^2 + c_2\cos(2\ln x) + c_3\sin(2\ln x)$
(e) $y = c_1x^2 + c_2\cos(\sqrt{2}\ln x) + c_3\sin(\sqrt{2}\ln x)$

9. The minimum radius of convergence of a power series solution of the second order differential equation $(x^2 - 2x + 5)y'' + xy' - y = 0$ about the ordinary point $x = -1$ is equal to

(a) 5
(b) $3\sqrt{2}$
(c) $2\sqrt{2}$
(d) 1
(e) $2\sqrt{3}$

10. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2 + 1)y'' - 6y = 0$, then the recurrence relation is given by

(a) $c_2 = 4c_0,\ c_3 = c_1,\ c_{k+2} = \dfrac{3-k}{k+1}c_k,\ k = 2, 3, \ldots$
(b) $c_2 = 3c_0,\ c_3 = c_1,\ c_{k+2} = \dfrac{3-k}{k+1}c_k,\ k = 2, 3, \ldots$
(c) $c_2 = c_0,\ c_3 = c_1,\ c_{k+2} = \dfrac{k-3}{k+1}c_k,\ k = 2, 3, \ldots$
(d) $c_2 = 2c_0,\ c_3 = 2c_1,\ c_{k+2} = \dfrac{4-k}{k+1}c_k,\ k = 2, 3, \ldots$
(e) $c_2 = 3c_0,\ c_3 = 2c_1,\ c_{k+2} = \dfrac{3+k}{k+1}c_k,\ k = 2, 3, \ldots$

11. If the general solution of the system $X' = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} X$ is given by

$$X = c_1 \begin{pmatrix} -2 \\ m \end{pmatrix} e^{nt} + c_2 \begin{pmatrix} 1 \\ k \end{pmatrix} e^{4t},$$

then $m + n + k =$

(a) 2
(b) 3
(c) -2
(d) 4
(e) -4

12. If $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ is a series solution for the differential equation $2xy'' - y' + 2y = 0$ about $x = 0$, then the non-integer indicial root is equal to

(a) $\frac{2}{3}$
(b) $\frac{3}{4}$
(c) $\frac{4}{3}$
(d) $\frac{1}{2}$
(e) $\frac{3}{2}$

13. If $c_0 \neq 0, c_1 = 0,\ c_k = -\frac{c_{k-2}}{k(2k-1)},\ k = 2, 3, 4, \ldots$ is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of the differential equation $2x^2y'' - xy' + (x^2+1)y = 0$ about $x = 0$, then the solution is given by

(a) $y = x^{\frac{1}{2}}\left[1 - \dfrac{1}{3}x^2 + \dfrac{1}{68}x^4 + \ldots\right]$

(b) $y = x^{\frac{1}{2}}\left[1 + \dfrac{1}{3}x^2 + \dfrac{1}{68}x^4 - \ldots\right]$

(c) $y = x^{\frac{1}{2}}\left[1 + \dfrac{1}{6}x^2 + \dfrac{1}{168}x^4 - \ldots\right]$

(d) $y = x^{\frac{1}{2}}\left[1 - \dfrac{1}{6}x^2 - \dfrac{1}{168}x^4 + \ldots\right]$

(e) $y = x^{\frac{1}{2}}\left[1 - \dfrac{1}{6}x^2 + \dfrac{1}{168}x^4 - \ldots\right]$

14. The number of regular singular points of the differential equation

$$x^3(x^2-25)(x-2)^2y'' + 3x(x-2)y' + 7(x+5)y = 0$$

is

(a) 0

(b) 2

(c) 3

(d) 5

(e) 1

15. If $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$ and $X_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$ are two solution vectors of a homogeneous linear system $X' = AX$, then the Wronskian $W(X_1, X_2) =$

(a) $8e^{4t}$

(b) $6e^{4t}$

(c) $6e^{8t}$

(d) $8e^{6t}$

(e) $8e^{8t}$

16. If $X_c = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t}$ is the general solution of the homogeneous linear system $X' = AX$, then using the variation of parameters method, a particular solution X_p of the non-homogeneous system $X' = AX + \begin{pmatrix} 0 \\ 4t \end{pmatrix}$ is given by

(a) $X_p = \begin{pmatrix} 4t \\ t+1 \end{pmatrix}$

(b) $X_p = \begin{pmatrix} 4t \\ 8t-4 \end{pmatrix}$

(c) $X_p = \begin{pmatrix} t \\ 8t-4 \end{pmatrix}$

(d) $X_p = \begin{pmatrix} 4t \\ t-1 \end{pmatrix}$

(e) $X_p = \begin{pmatrix} t \\ t-1 \end{pmatrix}$

17. The eigenvalues of the matrix

$$A = \begin{pmatrix} 5 & -1 & 0 \\ 0 & -5 & 9 \\ 5 & -1 & 0 \end{pmatrix}$$

are

(a) $\lambda = 1$, $\lambda = 3$ and $\lambda = -3$

(b) $\lambda = 1$, $\lambda = 1 \pm 2i$

(c) $\lambda = 1$, $\lambda = 4$ and $\lambda = -4$

(d) $\lambda = 0$, $\lambda = 3$ and $\lambda = -3$

(e) $\lambda = 0$, $\lambda = 4$ and $\lambda = -4$

18. If $X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t}$ is a solution of the linear system $X' = \begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix} X$ that corresponds to the only eigenvalue $\lambda = -1$, then a second linearly independent solution of the system is given by

(a) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-t}$

(b) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + t \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{-t}$

(c) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} e^{-t}$

(d) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{-t}$

(e) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + t \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} e^{-t}$

19. Given that $K = \begin{pmatrix} 1 \\ 1-2i \end{pmatrix}$ is an eigenvector that corresponds to the eigenvalue $\lambda = 5 + 2i$ of the matrix $A = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix}$. If $X(t)$ is the solution of the initial value problem $X' = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} X$, $X(0) = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$ then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{pmatrix} 2 \\ -8 \end{pmatrix} e^{\frac{5\pi}{2}}$

(b) $\begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{pmatrix} 1 \\ 4 \end{pmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{pmatrix} 1 \\ -8 \end{pmatrix} e^{\frac{5\pi}{2}}$

20. Using the exponential of a matrix method, if the general solution of the system $X' = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 5 & 1 & 0 \end{pmatrix} X$ is given by $X = \begin{pmatrix} 1 & 0 & 0 \\ g(t) & 1 & 0 \\ h(t) & f(t) & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$,
then $g(2) + h(2) + f(2) =$

(a) 26
(b) 22
(c) 20
(d) 24
(e) 18

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE04 **CODE04**

Math 202
Final Exam
222
May 22, 2023
Net Time Allowed: 180 Minutes

Name	

ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $y_1 = 1$ is a solution of the differential equation $(1-x^2)y'' - 2xy' = 0$, then by using reduction of order, a second solution $y_2 =$

(a) $\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$

(b) $\frac{1}{3}\ln\left|\frac{1-x}{2+x}\right|$

(c) $\frac{1}{3}\ln\left|\frac{1-x}{1+x}\right|$

(d) $\frac{1}{2}\ln\left|\frac{2+x}{1-x}\right|$

(e) $\frac{1}{2}\ln\left|\frac{1+x}{2-x}\right|$

2. If c is constant, then the solution of the exact differential equation

$$(y^2 + y\sin x)\,dx + \left(2xy - \cos x - \frac{1}{1+y^2}\right)\,dy = 0$$

is given by

(a) $xy^2 + y\sin x + \tan^{-1}\, y = c$

(b) $xy^2 - y\cos x + 2\tan^{-1}\, y = c$

(c) $xy^2 - y\cos x + \tan^{-1}\, y = c$

(d) $xy^2 - y\cos x - \tan^{-1}\, y = c$

(e) $xy^2 - 2y\cos x + \tan^{-1}\, y = c$

3. If c is constant, then the solution of the differential equation

$$y' + \frac{1}{x+1}\,y = \frac{\ln x}{x+1}$$

is given by

(a) $y = \dfrac{x+1}{x}\ln x + \dfrac{x+1}{x} + c(x+1)$

(b) $y = \dfrac{x}{x+1}\ln x - \dfrac{x}{x+1} + \dfrac{c}{x+1}$

(c) $y = \dfrac{x+1}{x}\ln x - \dfrac{x+1}{x} + c(x+1)$

(d) $y = \dfrac{x}{x+1}\ln x + \dfrac{c}{x+1}$

(e) $y = \dfrac{x}{x+1}\ln x + \dfrac{x}{x+1} + \dfrac{c}{x+1}$

4. A homogeneous linear differential equation with constant coefficients whose general solution is

$$y = c_1\cos x + c_2\sin x + c_3\cos 2x + c_4\sin 2x$$

is given by

(a) $y^{(4)} - 5y'' + 4y = 0$

(b) $y^{(4)} + 5y'' + 6y = 0$

(c) $y^{(4)} + 5y'' - 4y = 0$

(d) $y^{(4)} + 4y'' + 5y = 0$

(e) $y^{(4)} + 5y'' + 4y = 0$

5. If c is constant, then the solution of the homogeneous differential equation

$$(x + 3y)\,dx - (3x + y)\,dy = 0$$

is given by

(a) $(y - x)^2 = c(y + x)$
(b) $y - x = c(y + x)^2$
(c) $(y + x)^2 = c(y - 2x)$
(d) $y - x = c(y + x)$
(e) $(y + x)^2 = c(y - x)$

6. The general solution of the differential equation $x^3y''' - 6y = 0$ is given by

(a) $y = c_1x^3 + c_2 \cos(\sqrt{2}\,\ln x) + c_3 \sin(\sqrt{2}\,\ln x)$
(b) $y = c_1x^2 + c_2 \cos(\sqrt{2}\,\ln x) + c_3 \sin(\sqrt{2}\,\ln x)$
(c) $y = c_1x^2 + c_2 \cos(2\,\ln x) + c_3 \sin(2\,\ln x)$
(d) $y = c_1x^3 + c_2 \cos(2\,\ln x) + c_3 \sin(2\,\ln x)$
(e) $y = c_1x^3 + c_2 \cos(3\,\ln x) + c_3 \sin(3\,\ln x)$

7. By using variation of parameters method, a particular solution of the differential equation

$$y'' - 9y = \frac{9x}{e^{3x}} \text{ is given by}$$

(a) $y_p = \left(\frac{1}{4} - x - \frac{3}{4}x^2\right) e^{-3x}$

(b) $y_p = \left(\frac{1}{24} + 3x - x^2\right) e^{-3x}$

(c) $y_p = \left(-\frac{1}{24} + x + \frac{3}{4}x^2\right) e^{-3x}$

(d) $y_p = \left(1 - \frac{1}{4}xe - \frac{3}{4}x^2\right) e^{-3x}$

(e) $y_p = \left(-\frac{1}{24} - \frac{1}{4}x - \frac{3}{4}x^2\right) e^{-3x}$

8. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2+1)y'' - 6y = 0$, then the recurrence relation is given by

(a) $c_2 = 2c_0,\ c_3 = 2c_1,\ c_{k+2} = \frac{4-k}{k+1}c_k,\ k = 2, 3, \ldots$

(b) $c_2 = 3c_0,\ c_3 = 2c_1,\ c_{k+2} = \frac{3+k}{k+1}c_k,\ k = 2, 3, \ldots$

(c) $c_2 = c_0,\ c_3 = c_1,\ c_{k+2} = \frac{k-3}{k+1}c_k,\ k = 2, 3, \ldots$

(d) $c_2 = 4c_0,\ c_3 = c_1,\ c_{k+2} = \frac{3-k}{k+1}c_k,\ k = 2, 3, \ldots$

(e) $c_2 = 3c_0,\ c_3 = c_1,\ c_{k+2} = \frac{3-k}{k+1}c_k,\ k = 2, 3, \ldots$

9. Using the undetermined coefficients, a form of a particular solution for the differential equation

$$y'' + 3y' = 4x - 5$$

is given by

(a) $y_p = Ax^3 + Bx$
(b) $y_p = Ax^2 + Bx^4$
(c) $y_p = Ax + B$
(d) $y_p = Ax^2 + Bx$
(e) $y_p = Ax^2 + Bx^3$

10. The minimum radius of convergence of a power series solution of the second order differential equation $(x^2 - 2x + 5)y'' + xy' - y = 0$ about the ordinary point $x = -1$ is equal to

(a) $3\sqrt{2}$
(b) 1
(c) $2\sqrt{2}$
(d) $2\sqrt{3}$
(e) 5

11. If $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ is a series solution for the differential equation $2xy'' - y' + 2y = 0$ about $x = 0$, then the non-integer indicial root is equal to

(a) $\frac{4}{3}$
(b) $\frac{2}{3}$
(c) $\frac{3}{4}$
(d) $\frac{1}{2}$
(e) $\frac{3}{2}$

12. If the general solution of the system $X' = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} X$ is given by

$$X = c_1 \begin{pmatrix} -2 \\ m \end{pmatrix} e^{nt} + c_2 \begin{pmatrix} 1 \\ k \end{pmatrix} e^{4t},$$

then $m + n + k =$

(a) -2
(b) -4
(c) 3
(d) 2
(e) 4

13. If $c_0 \neq 0, c_1 = 0,\ c_k = -\frac{c_{k-2}}{k(2k-1)}$, $k = 2, 3, 4, \ldots$ is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of the differential equation $2x^2y'' - xy' + (x^2 + 1)y = 0$ about $x = 0$, then the solution is given by

(a) $y = x^{\frac{1}{2}}\left[1 - \dfrac{1}{6}x^2 + \dfrac{1}{168}x^4 - \ldots\right]$

(b) $y = x^{\frac{1}{2}}\left[1 + \dfrac{1}{3}x^2 + \dfrac{1}{68}x^4 - \ldots\right]$

(c) $y = x^{\frac{1}{2}}\left[1 - \dfrac{1}{3}x^2 + \dfrac{1}{68}x^4 + \ldots\right]$

(d) $y = x^{\frac{1}{2}}\left[1 + \dfrac{1}{6}x^2 + \dfrac{1}{168}x^4 - \ldots\right]$

(e) $y = x^{\frac{1}{2}}\left[1 - \dfrac{1}{6}x^2 - \dfrac{1}{168}x^4 + \ldots\right]$

14. If $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$ and $X_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$ are two solution vectors of a homogeneous linear system $X' = AX$, then the Wronskian $W(X_1, X_2) =$

(a) $6e^{4t}$

(b) $8e^{6t}$

(c) $8e^{4t}$

(d) $6e^{8t}$

(e) $8e^{8t}$

15. The number of regular singular points of the differential equation

$$x^3(x^2-25)(x-2)^2y'' + 3x(x-2)y' + 7(x+5)y = 0$$

is

(a) 5
(b) 2
(c) 0
(d) 1
(e) 3

16. Using the exponential of a matrix method, if the general solution of the system $X' = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 5 & 1 & 0 \end{pmatrix} X$ is given by $X = \begin{pmatrix} 1 & 0 & 0 \\ g(t) & 1 & 0 \\ h(t) & f(t) & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$, then $g(2) + h(2) + f(2) =$

(a) 18
(b) 20
(c) 24
(d) 26
(e) 22

17. If $X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t}$ is a solution of the linear system $X' = \begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix} X$ that corresponds to the only eigenvalue $\lambda = -1$, then a second linearly independent solution of the system is given by

(a) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-t}$

(b) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + t \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} e^{-t}$

(c) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + t \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{-t}$

(d) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} e^{-t}$

(e) $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{-t}$

18. Given that $K = \begin{pmatrix} 1 \\ 1 - 2i \end{pmatrix}$ is an eigenvector that corresponds to the eigenvalue $\lambda = 5 + 2i$ of the matrix $A = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix}$. If $X(t)$ is the solution of the initial value problem $X' = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} X$, $X(0) = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$ then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{\frac{5\pi}{2}}$

(b) $\begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{pmatrix} 2 \\ -8 \end{pmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{pmatrix} 1 \\ -8 \end{pmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{pmatrix} 1 \\ 4 \end{pmatrix} e^{\frac{5\pi}{2}}$

19. The eigenvalues of the matrix

$$A = \begin{pmatrix} 5 & -1 & 0 \\ 0 & -5 & 9 \\ 5 & -1 & 0 \end{pmatrix}$$

are

(a) $\lambda = 1$, $\lambda = 4$ and $\lambda = -4$

(b) $\lambda = 0$, $\lambda = 3$ and $\lambda = -3$

(c) $\lambda = 0$, $\lambda = 4$ and $\lambda = -4$

(d) $\lambda = 1$, $\lambda = 3$ and $\lambda = -3$

(e) $\lambda = 1$, $\lambda = 1 \pm 2i$

20. If $X_c = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t}$ is the general solution of the homogeneous linear system $X' = AX$, then using the variation of parameters method, a particular solution X_p of the non-homogeneous system $X' = AX + \begin{pmatrix} 0 \\ 4t \end{pmatrix}$ is given by

(a) $X_p = \begin{pmatrix} 4t \\ t-1 \end{pmatrix}$

(b) $X_p = \begin{pmatrix} 4t \\ 8t-4 \end{pmatrix}$

(c) $X_p = \begin{pmatrix} t \\ t-1 \end{pmatrix}$

(d) $X_p = \begin{pmatrix} t \\ 8t-4 \end{pmatrix}$

(e) $X_p = \begin{pmatrix} 4t \\ t+1 \end{pmatrix}$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	E 5	A 2	A 5	A 4
2	A	C 4	C 3	D 2	D 2
3	A	E 3	B 4	D 3	B 1
4	A	A 2	B 1	C 1	E 5
5	A	D 1	E 5	E 4	A 3
6	A	B 10	E 7	E 7	A 8
7	A	E 8	C 9	E 6	E 7
8	A	E 6	D 10	C 8	E 10
9	A	C 9	A 6	C 9	D 6
10	A	A 7	B 8	B 10	C 9
11	A	D 15	B 15	B 15	E 12
12	A	C 13	D 12	E 12	C 15
13	A	C 12	A 13	E 13	A 13
14	A	A 14	E 11	C 11	C 14
15	A	C 11	E 14	A 14	E 11
16	A	D 20	C 19	B 18	C 19
17	A	C 16	B 18	E 20	D 16
18	A	E 19	B 17	C 16	C 17
19	A	C 17	E 16	A 17	C 20
20	A	B 18	E 20	D 19	B 18

Answer Counts

V	A	B	C	D	E
1	3	2	7	3	5
2	3	6	3	2	6
3	3	3	5	3	6
4	4	2	6	3	5