King Fahd University of Petroleum and Minerals Department of Mathematics and Statistics

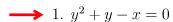
MATH 208 - Major Exam I - Term 211

Duration: 90 minutes

1. The solution of the initial value problem:

$$\frac{dy}{dx} = \frac{y}{x+y^2} \quad y(2) = 1$$

is equal to



$$2. \ y^2 + y + x = 4$$

3.
$$y^2 - y - x = -2$$

4.
$$y^2 + y + 2x = 6$$

5.
$$y^2 - y - 2x = -4$$

2. Consider the differential equation

$$xy' + \frac{y-1}{3} = \frac{1}{3(y-1)^2}$$

Given that its general solution is:

$$\ln|1 - (y - 1)^3| = \ln|\frac{1}{x}| + C$$

Then: (choose the true statement)

- \longrightarrow 1. the singular solution is y=2 only.
 - 2. the singular solution is y = -1 only.
 - 3. the singular solutions are y = 1 and y = -1.
 - $4. \ \,$ there is no singular solutions.
 - 5. the singular solution is $y = \frac{1}{3}$.

3. In a certain culture of bacteria, the number of bacteria was initially 1000. After two hours, the population became 1500. Assuming that the population of bacteria grows at a rate proportional to the number of bacteria presented at time t, then the time t when the bacteria population will be 2250 equals to

(calculational tip: $(15)^2 = 225$)

- \longrightarrow 1. 4 hours.
 - 2. 3.5 hours.
 - 3. 3 hours.
 - 4. 4.25 hours.
 - 5. 2.5 hours.

4. Let A and B be two constants in the differential equation

$$[Axy + (A + B)y\sin 2x]dx + [x^2 + B\cos^2 x]dy = 0.$$

Then the values of A and B that make the differential equation **exact** are

$$\longrightarrow$$
 1. $A = 2, B = -1.$

- 2. A = 1, B = 1.
- 3. A = -1, B = -1.
- 4. A = 1, B = -2.
- 5. $A = 1, B = \frac{-1}{2}$.

5. Let c_1 and c_2 be arbitrary constants. Verify that $y = c_1 x + c_2 x \ln x$ is a solution of the differential equation

$$x^2y'' - xy' + y = 0.$$

$$y = c_1 x + c_2 x h x$$

$$y' = c_1 + c_2 h x + c_2$$

$$y'' = c_2^2$$

$$LHS = X^{2}y'' - Xy' + y = X^{2}(\frac{c_{2}}{x}) - X(\frac{c_{1} + c_{2} h_{x} + c_{2}}{x})$$

$$+ c_{1}x + c_{2}x h_{x}$$

$$= c_{2}x - c_{1}x - c_{2}x h_{x} - c_{2}x$$

$$+ c_{1}x + c_{2}x h_{x}$$

6. Solve the differential equation:

$$xy' - 2y = x^3 e^x, \quad (x > 0)$$

$$y' - \frac{2}{x}y = x^{2}e^{x}$$

$$\int_{-\frac{2}{x}}^{-\frac{2}{x}} dx = \frac{1}{x^{2}}$$

$$e = \frac{1}{x^{2}}$$

$$od(y, \frac{1}{x^{2}}) = e^{x}dx$$

$$= y, \frac{1}{x^{2}} = e^{x} + c$$

$$(oR: y = x^{2}e^{x} + cx^{2})$$

7. Find the one-parameter family of solutions of the following differential equation:

$$y \sec x \, \frac{dy}{dx} = xy^2 - x - 1 + y^2$$

y secx
$$\frac{dy}{dx} = x(y^2-1) + (y^2-1)$$

 $y = x(y^2-1) + (y^2-1)$
 $y = x(y^2-1) + (y^2-1)$
 $y = x(y^2-1) + (y^2-1)$

$$\frac{y}{y^2-1}dy=(x+1)\cos x\,dx$$

$$\Rightarrow \frac{1}{2}\ln|y^2-1| = (x+1)\sin x + \cos x + C \qquad \frac{x+1}{\cos x}$$

8. Solve the **exact** differential equation:

$$(2xy + y - \tan y) dx + (x^2 - x \tan^2 y + \sec^2 y) dy = 0.$$

(Do NOT verify that the differential equation is exact.)

•
$$f_x = M \Rightarrow F = \int M dx$$

$$= F = \int (2xy + y - \tan y) dx$$

$$\Rightarrow F = x^2y + yx - (\tan y) x + g(y)$$

$$\Rightarrow F_y = N \Rightarrow x^2 + x - x \sec^2 y + g'(y) = x^2 - x \tan^2 y$$

$$+ \sec^2 y$$

$$\Rightarrow x(1 - \sec^2 y) + g'(y) = -x \tan^2 y + \sec^2 y$$

$$\Rightarrow g'(y) = \sec^2 y$$

$$\Rightarrow g'(y) = \tan y$$

$$\exists \text{ The Cs.s.} \quad \text{is } \left[\frac{x^2y + 5x - (\tan y) \times + \tan y}{x^2 + 5x - (\tan y) \times + \tan y} \right] = C$$