King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208
Final Exam
221
December 24, 2022

EXAM COVER

Number of versions: 4
Number of questions: 20

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221

December 24, 2022

Net Time Allowed: 180 Minutes

MASTER VERSION

1. If $y(x)$ is the solution of the initial value problem

$$\frac{dy}{dx} = \frac{2x+1}{2y},\ y(-2) = -1, \text{ then } y(2) =$$

(a) $-\sqrt{5}$ ______________________________ (correct)
(b) $\sqrt{5}$
(c) $-\sqrt{3}$
(d) $\sqrt{3}$
(e) 0

2. The general solution of the differential equation

$$x\frac{dy}{dx} - 3y = x^3$$

is

(a) $y = x^3 \ln x + cx^3$ ______________________________ (correct)
(b) $y = x^2 \ln x + cx^2$
(c) $y = x^3 \ln x + cx^2$
(d) $y = x^3 \ln x + cx$
(e) $y = x^2 \ln x + cx^3$

3. The general solution of the exact differential equation

$$(3x^2 + 2y^2)\,dx + (4xy + 6y^2)\,dy = 0$$

is

(a) $x^3 + 2xy^2 + 2y^3 = c$ ________________ (correct)
(b) $x^3 - 2xy^2 + 2y^3 = c$
(c) $x^3 - 2xy^2 - 2y^3 = c$
(d) $x^3 - 2x^2y - 2y^3 = c$
(e) $x^3 - 2x^2y + 2y^3 = c$

4. If $(x, y, z) = (a, b, c)$ is the solution of the system

$$\begin{cases} 2x + 8y + 3z = 2 \\ x + 3y + 2z = 5 \\ 2x + 7y + 4z = 8 \end{cases}$$

then $a + b + c =$

(a) 5 ________________ (correct)
(b) 6
(c) 7
(d) 3
(e) 0

5. Let S be a subspace of $\mathbb{R}^4$ defined by $S = \{(a,b,c,d)|a = b+c+d\}$. A basis for the subspace is

(a) $\{(1,1,0,0),\ (1,0,1,0),\ (1,0,0,1)\}$ ________________ (correct)
(b) $\{(1,1,0,0),\ (1,0,1,0)\}$
(c) $\{(1,0,1,0),\ (1,0,0,1)\}$
(d) $\{(1,1,1,0),\ (1,0,1,0),\ (1,0,1,1)\}$
(e) $\{(1,1,0,0),\ (1,0,1,0),\ (2,0,1,2)\}$

6. If $y(x)$ is the solution of the initial-value problem

$$y'' + 4y = 2x;\ y(0) = 1,\ y'(0) = 2$$

then $y(\pi) =$

(a) $1 + \frac{\pi}{2}$ ________________ (correct)
(b) $1 - \frac{\pi}{2}$
(c) $1 + \frac{\pi}{4}$
(d) $1 - \frac{\pi}{4}$
(e) $1 + \frac{\pi}{3}$

7. By using the method of variation of parameters, a particular solution y_p of the differential equation

$$y'' - 9y = \frac{9x}{e^{3x}}$$

is

(a) $y_p = -\frac{3}{4}x^2e^{-3x} - \frac{1}{4}xe^{-3x} - \frac{1}{24}\,e^{-3x}$ ______________________________ (correct)

(b) $y_p = \frac{1}{4}x^3e^{-3x} + \frac{1}{4}xe^{-3x} - \frac{1}{24}\,e^{-3x}$

(c) $y_p = \frac{3}{4}x^2e^{-3x} + \frac{1}{2}xe^{-3x} - \frac{1}{24}\,e^{-3x}$

(d) $y_p = -\frac{3}{4}xe^{-3x} + \frac{1}{4}xe^{-3x} - \frac{1}{24}\,e^{-3x}$

(e) $y_p = \frac{1}{2}x^2e^{-3x} + \frac{1}{4}x^3e^{-3x} - \frac{1}{24}\,e^{-3x}$

8. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & -2 & 2 \end{bmatrix}$. Using Cayley-Hamilton Theorem,
$A^4 = aA^3 + bA^2 + cA$.
$a + b + c =$

(a) 1 ______________________________ (correct)

(b) 0

(c) 2

(d) 3

(e) 4

9. The general solution of the first order homogeneous system $X' = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix} X$ is given by

$$X = c_1 \begin{bmatrix} a \\ b \\ -2 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} \alpha \\ \beta \\ 1 \end{bmatrix} e^{\lambda t} + c_3 \begin{bmatrix} e \\ f \\ 13 \end{bmatrix}$$

then $a \cdot b \cdot \lambda =$

(a) -24 ________________________________ (correct)

(b) 12

(c) 24

(d) -12

(e) 0

10. Let $A = \begin{bmatrix} 3 & 5 & -2 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$. An eigenvector corresponding to the eigenvalue $\lambda = 2$ of A is

(a) $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ ________________________________ (correct)

(b) $\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$

(e) $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

11. Let $A = \begin{bmatrix} 9 & -8 \\ 6 & -5 \end{bmatrix}$. A diagonalization matrix P, such that $P^{-1}AP$ is diagonal is

(a) $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$ ______ (correct)

(b) $P = \begin{bmatrix} -1 & 4 \\ 1 & -3 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 2 \\ 1 & 5 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

12. The differential equation

$$t^3x''' - 2t^2x'' + 3tx' + 5x = \ln t$$

is equivalent to the system of first-order equations.

(a) $x_1' = x_2,\ x_2' = x_3,\ t^3\,x_3' = -5x_1 - 3tx_2 + 2t^2x_3 + \ln t$ ______ (correct)

(b) $x_1' = x_2,\ x_2' = x_3,\ x_3' = -5x_1 - 3tx_2 - 2x_3 + \ln t$

(c) $x_1' = x_1,\ x_2' = x_2,\ x_3' = -5x_1 - 3tx_2 + 2t^2x_3 + \ln t$

(d) $x_1' = x_2,\ x_2' = x_3,\ x_3' = -5x_1 - 3tx_2 + 2t^2x_3 - \ln t$

(e) $x_1' = x_2',\ x_2' = x_1,\ t^3\,x_3' = 5x_1 + 3tx_2 + 2t^2x_3 + \ln t$

13. If $X = c_1 \begin{bmatrix} 5 \\ -6 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{4t}$ is the solution of the initial value problem

$$X' = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} X,\ X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

then $c_2^2 - c_1^2 =$

(a) 35 ____(correct)

(b) 36

(c) 37

(d) 34

(e) 32

14. The solution of $X' = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} X,\ X(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ at $t = \dfrac{\pi}{4}$ equals

(a) $\begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{\frac{\pi}{4}}$ ____(correct)

(b) $\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{\frac{\pi}{4}}$

(c) $\begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{\frac{\pi}{4}}$

(d) $\begin{pmatrix} -2 \\ 0 \end{pmatrix} e^{\frac{\pi}{4}}$

(e) $\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-\frac{\pi}{4}}$

15. Let

$$A = \begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

If $e^{At} = \begin{bmatrix} 1 & f(t) & h(t) \\ 0 & 1 & 3t \\ 0 & 0 & 1 \end{bmatrix}$, then $f(2) + h(2) =$

(a) 32 ________ (correct)

(b) 30

(c) 28

(d) 26

(e) 24

16. A possible fundamental matrix for the system $X' = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} X$ is

(a) $\Phi(t) = \begin{bmatrix} e^{-2t} & 2e^{5t} \\ -3e^{-2t} & e^{5t} \end{bmatrix}$ ________ (correct)

(b) $\Phi(t) = \begin{bmatrix} e^{-2t} & 2e^{5t} \\ 0 & e^{5t} \end{bmatrix}$

(c) $\Phi(t) = \begin{bmatrix} e^{-2t} & e^{5t} \\ e^{-2t} & 2e^{5t} \end{bmatrix}$

(d) $\Phi(t) = \begin{bmatrix} 3e^{-2t} & 0 \\ 5e^{-2t} & e^{5t} \end{bmatrix}$

(e) $\Phi(t) = \begin{bmatrix} e^{-2t} & e^{5t} \\ -3e^{-2t} & 0 \end{bmatrix}$

17. Using variation of parameters to find a particular solution X_p of the nonhomogeneous system $X' = AX + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t$ where $X_c = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$ form a general solution of the associated homogeneous system, then $X_p(1) =$

(a) $\begin{bmatrix} 7e \\ 5e \end{bmatrix}$ ______ (correct)

(b) $\begin{bmatrix} 3e \\ 2e \end{bmatrix}$

(c) $\begin{bmatrix} 7e \\ e \end{bmatrix}$

(d) $\begin{bmatrix} e \\ 5e \end{bmatrix}$

(e) $\begin{bmatrix} 0 \\ 3e \end{bmatrix}$

18. The general solution of the differential equation

$$y''' + 3y'' - 4y = 0$$

is

(a) $y(x) = c_1 e^{x} + c_2 e^{-2x} + c_3 x e^{-2x}$ ______ (correct)

(b) $y(x) = c_1 e^{-x} + c_2 e^{-2x} + c_3 x e^{-2x}$

(c) $y(x) = c_1 e^{x} + c_2 e^{-2x} + c_3 e^{3x}$

(d) $y(x) = c_1 e^{x} + c_2 e^{-2x} + c_3 e^{4x}$

(e) $y(x) = c_1 e^{-x} + c_2 e^{3x} + c_3 x e^{3x}$

19. The characteristic equation of a matrix A is $(\lambda+1)(\lambda-5)^3=0$, where we have only two linearly independent eigenvectors corresponding to $\lambda=5$. The Jordan normal form of A is

(a) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$ ________________ (correct)

(b) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(e) $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

20. The matrix $A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$ has only one eigenvalue $\lambda = -1$ which is defective of defect 2. If we choose $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A+I)^3\, v_3 = 0$, and $(A+I)^2 v_3 \neq 0$, then the general solution of $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} + 1 \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$ ______________(correct)

(b) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 + \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(d) $X = \left(c_1 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ -t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} + 1 \\ t^2 \\ t \end{bmatrix} \right) e^{-t}$

(e) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 - \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

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Department of Mathematics

CODE01 **CODE01**

Math 208
Final Exam
221
December 24, 2022
Net Time Allowed: 180 Minutes

Name			
ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The differential equation

$$t^3x''' - 2t^2x'' + 3tx' + 5x = \ln t$$

is equivalent to the system of first-order equations.

(a) $x_1' = x_2,\ x_2' = x_3,\ x_3' = -5x_1 - 3tx_2 + 2t^2x_3 - \ln t$

(b) $x_1' = x_2',\ x_2' = x_1,\ t^3\,x_3' = 5x_1 + 3tx_2 + 2t^2x_3 + \ln t$

(c) $x_1' = x_1,\ x_2' = x_2,\ x_3' = -5x_1 - 3tx_2 + 2t^2x_3 + \ln t$

(d) $x_1' = x_2,\ x_2' = x_3,\ x_3' = -5x_1 - 3tx_2 - 2x_3 + \ln t$

(e) $x_1' = x_2,\ x_2' = x_3,\ t^3\,x_3' = -5x_1 - 3tx_2 + 2t^2x_3 + \ln t$

2. The solution of $X' = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} X,\ X(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ at $t = \dfrac{\pi}{4}$ equals

(a) $\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{\frac{\pi}{4}}$

(b) $\begin{pmatrix} -2 \\ 0 \end{pmatrix} e^{\frac{\pi}{4}}$

(c) $\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-\frac{\pi}{4}}$

(d) $\begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{\frac{\pi}{4}}$

(e) $\begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{\frac{\pi}{4}}$

3. Let $A = \begin{bmatrix} 3 & 5 & -2 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$. An eigenvector corresponding to the eigenvalue $\lambda = 2$ of A is

(a) $\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

(e) $\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

4. By using the method of variation of parameters, a particular solution y_p of the differential equation

$$y'' - 9y = \frac{9x}{e^{3x}}$$

is

(a) $y_p = \frac{1}{4}x^3e^{-3x} + \frac{1}{4}xe^{-3x} - \frac{1}{24}e^{-3x}$

(b) $y_p = -\frac{3}{4}xe^{-3x} + \frac{1}{4}xe^{-3x} - \frac{1}{24}e^{-3x}$

(c) $y_p = -\frac{3}{4}x^2e^{-3x} - \frac{1}{4}xe^{-3x} - \frac{1}{24}e^{-3x}$

(d) $y_p = \frac{1}{2}x^2e^{-3x} + \frac{1}{4}x^3e^{-3x} - \frac{1}{24}e^{-3x}$

(e) $y_p = \frac{3}{4}x^2e^{-3x} + \frac{1}{2}xe^{-3x} - \frac{1}{24}e^{-3x}$

5. Let $A = \begin{bmatrix} 9 & -8 \\ 6 & -5 \end{bmatrix}$. A diagonalization matrix P, such that $P^{-1}AP$ is diagonal is

(a) $P = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 2 \\ 1 & 5 \end{bmatrix}$

(c) $P = \begin{bmatrix} -1 & 4 \\ 1 & -3 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$

6. The general solution of the differential equation

$$y''' + 3y'' - 4y = 0$$

is

(a) $y(x) = c_1e^{-x} + c_2e^{3x} + c_3xe^{3x}$

(b) $y(x) = c_1e^{-x} + c_2e^{-2x} + c_3xe^{-2x}$

(c) $y(x) = c_1e^{x} + c_2e^{-2x} + c_3e^{3x}$

(d) $y(x) = c_1e^{x} + c_2e^{-2x} + c_3e^{4x}$

(e) $y(x) = c_1e^{x} + c_2e^{-2x} + c_3xe^{-2x}$

7. Let S be a subspace of $\mathbb{R}^4$ defined by $S = \{(a,b,c,d) | a = b+c+d\}$. A basis for the subspace is

(a) $\{(1,1,1,0),\ (1,0,1,0),\ (1,0,1,1)\}$
(b) $\{(1,1,0,0),\ (1,0,1,0),\ (2,0,1,2)\}$
(c) $\{(1,1,0,0),\ (1,0,1,0)\}$
(d) $\{(1,1,0,0),\ (1,0,1,0),\ (1,0,0,1)\}$
(e) $\{(1,0,1,0),\ (1,0,0,1)\}$

8. If $y(x)$ is the solution of the initial-value problem

$$y'' + 4y = 2x;\ y(0) = 1,\ y'(0) = 2$$

then $y(\pi) =$

(a) $1 - \frac{\pi}{4}$
(b) $1 + \frac{\pi}{4}$
(c) $1 + \frac{\pi}{3}$
(d) $1 - \frac{\pi}{2}$
(e) $1 + \frac{\pi}{2}$

9. If $(x, y, z) = (a, b, c)$ is the solution of the system

$$\begin{cases} 2x + 8y + 3z = 2 \\ x + 3y + 2z = 5 \\ 2x + 7y + 4z = 8 \end{cases}$$

then $a + b + c =$

(a) 3
(b) 5
(c) 0
(d) 7
(e) 6

10. The general solution of the exact differential equation

$$(3x^2 + 2y^2)\,dx + (4xy + 6y^2)\,dy = 0$$

is

(a) $x^3 + 2xy^2 + 2y^3 = c$
(b) $x^3 - 2xy^2 - 2y^3 = c$
(c) $x^3 - 2x^2y + 2y^3 = c$
(d) $x^3 - 2x^2y - 2y^3 = c$
(e) $x^3 - 2xy^2 + 2y^3 = c$

11. If $X = c_1 \begin{bmatrix} 5 \\ -6 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{4t}$ is the solution of the initial value problem

$$X' = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} X, \; X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

then $c_2^2 - c_1^2 =$

(a) 32
(b) 36
(c) 34
(d) 37
(e) 35

12. Let

$$A = \begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

If $e^{At} = \begin{bmatrix} 1 & f(t) & h(t) \\ 0 & 1 & 3t \\ 0 & 0 & 1 \end{bmatrix}$, then $f(2) + h(2) =$

(a) 26
(b) 32
(c) 28
(d) 30
(e) 24

13. If $y(x)$ is the solution of the initial value problem

$$\frac{dy}{dx} = \frac{2x+1}{2y},\ y(-2) = -1,\ \text{then}\ y(2) =$$

(a) $\sqrt{3}$
(b) $-\sqrt{5}$
(c) 0
(d) $-\sqrt{3}$
(e) $\sqrt{5}$

14. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & -2 & 2 \end{bmatrix}$. Using Cayley-Hamilton Theorem,
$A^4 = aA^3 + bA^2 + cA.$
$a + b + c =$

(a) 1
(b) 3
(c) 0
(d) 4
(e) 2

15. A possible fundamental matrix for the system $X' = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} X$ is

(a) $\Phi(t) = \begin{bmatrix} 3e^{-2t} & 0 \\ 5e^{-2t} & e^{5t} \end{bmatrix}$

(b) $\Phi(t) = \begin{bmatrix} e^{-2t} & 2e^{5t} \\ -3e^{-2t} & e^{5t} \end{bmatrix}$

(c) $\Phi(t) = \begin{bmatrix} e^{-2t} & 2e^{5t} \\ 0 & e^{5t} \end{bmatrix}$

(d) $\Phi(t) = \begin{bmatrix} e^{-2t} & e^{5t} \\ e^{-2t} & 2e^{5t} \end{bmatrix}$

(e) $\Phi(t) = \begin{bmatrix} e^{-2t} & e^{5t} \\ -3e^{-2t} & 0 \end{bmatrix}$

16. The general solution of the differential equation

$$x\frac{dy}{dx} - 3y = x^3$$

is

(a) $y = x^2 \ln x + cx^2$

(b) $y = x^2 \ln x + cx^3$

(c) $y = x^3 \ln x + cx^3$

(d) $y = x^3 \ln x + cx$

(e) $y = x^3 \ln x + cx^2$

17. Using variation of parameters to find a particular solution X_p of the nonhomogeneous system $X' = AX + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t$ where $X_c = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$ form a general solution of the associated homogeneous system, then $X_p(1) =$

(a) $\begin{bmatrix} 0 \\ 3e \end{bmatrix}$

(b) $\begin{bmatrix} 7e \\ 5e \end{bmatrix}$

(c) $\begin{bmatrix} 7e \\ e \end{bmatrix}$

(d) $\begin{bmatrix} 3e \\ 2e \end{bmatrix}$

(e) $\begin{bmatrix} e \\ 5e \end{bmatrix}$

18. The general solution of the first order homogeneous system $X' = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix} X$ is given by

$$X = c_1 \begin{bmatrix} a \\ b \\ -2 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} \alpha \\ \beta \\ 1 \end{bmatrix} e^{\lambda t} + c_3 \begin{bmatrix} e \\ f \\ 13 \end{bmatrix}$$

then $a \cdot b \cdot \lambda =$

(a) -24

(b) 12

(c) 0

(d) -12

(e) 24

19. The matrix $A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$ has only one eigenvalue $\lambda = -1$ which is defective of defect 2. If we choose $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A+I)^3 v_3 = 0$, and $(A+I)^2 v_3 \neq 0$, then the general solution of $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 + \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(b) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 - \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(d) $X = \left(c_1 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ -t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} + 1 \\ t^2 \\ t \end{bmatrix} \right) e^{-t}$

(e) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} + 1 \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

20. The characteristic equation of a matrix A is $(\lambda+1)(\lambda-5)^3=0$, where we have only two linearly independent eigenvectors corresponding to $\lambda=5$. The Jordan normal form of A is

(a) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(e) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE02 **CODE02**

Math 208
Final Exam
221
December 24, 2022
Net Time Allowed: 180 Minutes

Name	
ID	

Sec	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & -2 & 2 \end{bmatrix}$. Using Cayley-Hamilton Theorem,
$A^4 = aA^3 + bA^2 + cA.$
$a + b + c =$

(a) 4
(b) 3
(c) 1
(d) 0
(e) 2

2. The solution of $X' = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} X$, $X(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ at $t = \frac{\pi}{4}$ equals

(a) $\begin{pmatrix} -2 \\ 0 \end{pmatrix} e^{\frac{\pi}{4}}$
(b) $\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-\frac{\pi}{4}}$
(c) $\begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{\frac{\pi}{4}}$
(d) $\begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{\frac{\pi}{4}}$
(e) $\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{\frac{\pi}{4}}$

3. By using the method of variation of parameters, a particular solution y_p of the differential equation

$$y'' - 9y = \frac{9x}{e^{3x}}$$

is

(a) $y_p = \frac{3}{4}x^2e^{-3x} + \frac{1}{2}xe^{-3x} - \frac{1}{24}\,e^{-3x}$

(b) $y_p = -\frac{3}{4}xe^{-3x} + \frac{1}{4}xe^{-3x} - \frac{1}{24}\,e^{-3x}$

(c) $y_p = -\frac{3}{4}x^2e^{-3x} - \frac{1}{4}xe^{-3x} - \frac{1}{24}\,e^{-3x}$

(d) $y_p = \frac{1}{2}x^2e^{-3x} + \frac{1}{4}x^3e^{-3x} - \frac{1}{24}\,e^{-3x}$

(e) $y_p = \frac{1}{4}x^3e^{-3x} + \frac{1}{4}xe^{-3x} - \frac{1}{24}\,e^{-3x}$

4. The differential equation

$$t^3x''' - 2t^2x'' + 3tx' + 5x = \ln t$$

is equivalent to the system of first-order equations.

(a) $x_1' = x_1,\ x_2' = x_2,\ x_3' = -5x_1 - 3tx_2 + 2t^2x_3 + \ln t$

(b) $x_1' = x_2',\ x_2' = x_1,\ t^3\,x_3' = 5x_1 + 3tx_2 + 2t^2x_3 + \ln t$

(c) $x_1' = x_2,\ x_2' = x_3,\ t^3\,x_3' = -5x_1 - 3tx_2 + 2t^2x_3 + \ln t$

(d) $x_1' = x_2,\ x_2' = x_3,\ x_3' = -5x_1 - 3tx_2 - 2x_3 + \ln t$

(e) $x_1' = x_2,\ x_2' = x_3,\ x_3' = -5x_1 - 3tx_2 + 2t^2x_3 - \ln t$

5. A possible fundamental matrix for the system $X' = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} X$ is

(a) $\Phi(t) = \begin{bmatrix} e^{-2t} & 2e^{5t} \\ -3e^{-2t} & e^{5t} \end{bmatrix}$

(b) $\Phi(t) = \begin{bmatrix} e^{-2t} & 2e^{5t} \\ 0 & e^{5t} \end{bmatrix}$

(c) $\Phi(t) = \begin{bmatrix} 3e^{-2t} & 0 \\ 5e^{-2t} & e^{5t} \end{bmatrix}$

(d) $\Phi(t) = \begin{bmatrix} e^{-2t} & e^{5t} \\ e^{-2t} & 2e^{5t} \end{bmatrix}$

(e) $\Phi(t) = \begin{bmatrix} e^{-2t} & e^{5t} \\ -3e^{-2t} & 0 \end{bmatrix}$

6. Let

$$A = \begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

If $e^{At} = \begin{bmatrix} 1 & f(t) & h(t) \\ 0 & 1 & 3t \\ 0 & 0 & 1 \end{bmatrix}$, then $f(2) + h(2) =$

(a) 32

(b) 28

(c) 30

(d) 24

(e) 26

7. Let $A = \begin{bmatrix} 3 & 5 & -2 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$. An eigenvector corresponding to the eigenvalue $\lambda = 2$ of A is

(a) $\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

(e) $\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$

8. The general solution of the first order homogeneous system $X' = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix} X$ is given by

$$X = c_1 \begin{bmatrix} a \\ b \\ -2 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} \alpha \\ \beta \\ 1 \end{bmatrix} e^{\lambda t} + c_3 \begin{bmatrix} e \\ f \\ 13 \end{bmatrix}$$

then $a \cdot b \cdot \lambda =$

(a) 12

(b) 0

(c) 24

(d) -12

(e) -24

9. If $y(x)$ is the solution of the initial value problem

$$\frac{dy}{dx} = \frac{2x+1}{2y},\ y(-2) = -1, \text{ then } y(2) =$$

(a) $-\sqrt{5}$
(b) $-\sqrt{3}$
(c) $\sqrt{3}$
(d) 0
(e) $\sqrt{5}$

10. If $y(x)$ is the solution of the initial-value problem

$$y'' + 4y = 2x;\ y(0) = 1,\ y'(0) = 2$$

then $y(\pi) =$

(a) $1 + \frac{\pi}{4}$
(b) $1 + \frac{\pi}{2}$
(c) $1 - \frac{\pi}{2}$
(d) $1 - \frac{\pi}{4}$
(e) $1 + \frac{\pi}{3}$

11. If $X = c_1 \begin{bmatrix} 5 \\ -6 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{4t}$ is the solution of the initial value problem

$$X' = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} X,\ X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

then $c_2^2 - c_1^2 =$

(a) 37
(b) 35
(c) 34
(d) 36
(e) 32

12. Let S be a subspace of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d) | a = b + c + d\}$. A basis for the subspace is

(a) $\{(1, 1, 0, 0),\ (1, 0, 1, 0),\ (2, 0, 1, 2)\}$
(b) $\{(1, 1, 0, 0),\ (1, 0, 1, 0),\ (1, 0, 0, 1)\}$
(c) $\{(1, 1, 0, 0),\ (1, 0, 1, 0)\}$
(d) $\{(1, 0, 1, 0),\ (1, 0, 0, 1)\}$
(e) $\{(1, 1, 1, 0),\ (1, 0, 1, 0),\ (1, 0, 1, 1)\}$

13. The general solution of the differential equation

$$x\frac{dy}{dx} - 3y = x^3$$

is

(a) $y = x^2 \ln x + cx^2$

(b) $y = x^2 \ln x + cx^3$

(c) $y = x^3 \ln x + cx^2$

(d) $y = x^3 \ln x + cx^3$

(e) $y = x^3 \ln x + cx$

14. Let $A = \begin{bmatrix} 9 & -8 \\ 6 & -5 \end{bmatrix}$. A diagonalization matrix P, such that $P^{-1}AP$ is diagonal is

(a) $P = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

(d) $P = \begin{bmatrix} -1 & 4 \\ 1 & -3 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 2 \\ 1 & 5 \end{bmatrix}$

15. The general solution of the differential equation

$$y''' + 3y'' - 4y = 0$$

is

(a) $y(x) = c_1e^{-x} + c_2e^{-2x} + c_3xe^{-2x}$

(b) $y(x) = c_1e^{-x} + c_2e^{3x} + c_3xe^{3x}$

(c) $y(x) = c_1e^{x} + c_2e^{-2x} + c_3e^{3x}$

(d) $y(x) = c_1e^{x} + c_2e^{-2x} + c_3xe^{-2x}$

(e) $y(x) = c_1e^{x} + c_2e^{-2x} + c_3e^{4x}$

16. Using variation of parameters to find a particular solution X_p of the nonhomogeneous system $X' = AX + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t$ where $X_c = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$ form a general solution of the associated homogeneous system, then $X_p(1) =$

(a) $\begin{bmatrix} 3e \\ 2e \end{bmatrix}$

(b) $\begin{bmatrix} 7e \\ 5e \end{bmatrix}$

(c) $\begin{bmatrix} e \\ 5e \end{bmatrix}$

(d) $\begin{bmatrix} 0 \\ 3e \end{bmatrix}$

(e) $\begin{bmatrix} 7e \\ e \end{bmatrix}$

17. The general solution of the exact differential equation

$$(3x^2 + 2y^2)\,dx + (4xy + 6y^2)\,dy = 0$$

is

(a) $x^3 - 2x^2y + 2y^3 = c$
(b) $x^3 + 2xy^2 + 2y^3 = c$
(c) $x^3 - 2xy^2 + 2y^3 = c$
(d) $x^3 - 2x^2y - 2y^3 = c$
(e) $x^3 - 2xy^2 - 2y^3 = c$

18. If $(x, y, z) = (a, b, c)$ is the solution of the system

$$\begin{cases} 2x + 8y + 3z = 2 \\ x + 3y + 2z = 5 \\ 2x + 7y + 4z = 8 \end{cases}$$

then $a + b + c =$

(a) 6
(b) 5
(c) 3
(d) 0
(e) 7

19. The characteristic equation of a matrix A is $(\lambda+1)(\lambda-5)^3=0$, where we have only two linearly independent eigenvectors corresponding to $\lambda=5$. The Jordan normal form of A is

(a) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(e) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

20. The matrix $A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$ has only one eigenvalue $\lambda = -1$ which is defective of defect 2. If we choose $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A+I)^3\, v_3 = 0$, and $(A+I)^2 v_3 \neq 0$, then the general solution of $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ -t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2}+1 \\ t^2 \\ t \end{bmatrix} \right) e^{-t}$

(b) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1-\frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(d) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1+\frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(e) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2}+1 \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE03 **CODE03**

Math 208
Final Exam
221
December 24, 2022
Net Time Allowed: 180 Minutes

Name	

ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $(x, y, z) = (a, b, c)$ is the solution of the system

$$\begin{cases} 2x + 8y + 3z = 2 \\ x + 3y + 2z = 5 \\ 2x + 7y + 4z = 8 \end{cases}$$

then $a + b + c =$

(a) 3

(b) 5

(c) 0

(d) 6

(e) 7

2. If $y(x)$ is the solution of the initial-value problem

$$y'' + 4y = 2x;\ y(0) = 1,\ y'(0) = 2$$

then $y(\pi) =$

(a) $1 + \frac{\pi}{3}$

(b) $1 - \frac{\pi}{4}$

(c) $1 + \frac{\pi}{4}$

(d) $1 + \frac{\pi}{2}$

(e) $1 - \frac{\pi}{2}$

3. The differential equation

$$t^3x''' - 2t^2x'' + 3tx' + 5x = \ln t$$

is equivalent to the system of first-order equations.

(a) $x_1' = x_2,\ x_2' = x_3,\ x_3' = -5x_1 - 3tx_2 - 2x_3 + \ln t$
(b) $x_1' = x_2',\ x_2' = x_1,\ t^3\, x_3' = 5x_1 + 3tx_2 + 2t^2x_3 + \ln t$
(c) $x_1' = x_2,\ x_2' = x_3,\ x_3' = -5x_1 - 3tx_2 + 2t^2x_3 - \ln t$
(d) $x_1' = x_2,\ x_2' = x_3,\ t^3\, x_3' = -5x_1 - 3tx_2 + 2t^2x_3 + \ln t$
(e) $x_1' = x_1,\ x_2' = x_2,\ x_3' = -5x_1 - 3tx_2 + 2t^2x_3 + \ln t$

4. Let $A = \begin{bmatrix} 3 & 5 & -2 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$. An eigenvector corresponding to the eigenvalue $\lambda = 2$ of A is

(a) $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

(e) $\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$

5. Using variation of parameters to find a particular solution X_p of the nonhomogeneous system $X' = AX + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t$ where $X_c = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$ form a general solution of the associated homogeneous system, then $X_p(1) =$

(a) $\begin{bmatrix} 7e \\ e \end{bmatrix}$

(b) $\begin{bmatrix} 7e \\ 5e \end{bmatrix}$

(c) $\begin{bmatrix} e \\ 5e \end{bmatrix}$

(d) $\begin{bmatrix} 3e \\ 2e \end{bmatrix}$

(e) $\begin{bmatrix} 0 \\ 3e \end{bmatrix}$

6. A possible fundamental matrix for the system $X' = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} X$ is

(a) $\Phi(t) = \begin{bmatrix} 3e^{-2t} & 0 \\ 5e^{-2t} & e^{5t} \end{bmatrix}$

(b) $\Phi(t) = \begin{bmatrix} e^{-2t} & e^{5t} \\ -3e^{-2t} & 0 \end{bmatrix}$

(c) $\Phi(t) = \begin{bmatrix} e^{-2t} & 2e^{5t} \\ 0 & e^{5t} \end{bmatrix}$

(d) $\Phi(t) = \begin{bmatrix} e^{-2t} & e^{5t} \\ e^{-2t} & 2e^{5t} \end{bmatrix}$

(e) $\Phi(t) = \begin{bmatrix} e^{-2t} & 2e^{5t} \\ -3e^{-2t} & e^{5t} \end{bmatrix}$

7. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & -2 & 2 \end{bmatrix}$. Using Cayley-Hamilton Theorem,
$A^4 = aA^3 + bA^2 + cA$.
$a + b + c =$

(a) 1
(b) 3
(c) 0
(d) 4
(e) 2

8. The general solution of the exact differential equation

$$(3x^2 + 2y^2)\,dx + (4xy + 6y^2)\,dy = 0$$

is

(a) $x^3 - 2xy^2 + 2y^3 = c$
(b) $x^3 - 2x^2y - 2y^3 = c$
(c) $x^3 - 2xy^2 - 2y^3 = c$
(d) $x^3 + 2xy^2 + 2y^3 = c$
(e) $x^3 - 2x^2y + 2y^3 = c$

9. Let $A = \begin{bmatrix} 9 & -8 \\ 6 & -5 \end{bmatrix}$. A diagonalization matrix P, such that $P^{-1}AP$ is diagonal is

(a) $P = \begin{bmatrix} 1 & 2 \\ 1 & 5 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$

(d) $P = \begin{bmatrix} -1 & 4 \\ 1 & -3 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

10. The solution of $X' = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} X$, $X(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ at $t = \frac{\pi}{4}$ equals

(a) $\begin{pmatrix} -2 \\ 0 \end{pmatrix} e^{\frac{\pi}{4}}$

(b) $\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{\frac{\pi}{4}}$

(c) $\begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{\frac{\pi}{4}}$

(d) $\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-\frac{\pi}{4}}$

(e) $\begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{\frac{\pi}{4}}$

11. By using the method of variation of parameters, a particular solution y_p of the differential equation

$$y'' - 9y = \frac{9x}{e^{3x}}$$

is

(a) $y_p = \frac{3}{4}x^2e^{-3x} + \frac{1}{2}xe^{-3x} - \frac{1}{24}\,e^{-3x}$
(b) $y_p = -\frac{3}{4}x^2e^{-3x} - \frac{1}{4}xe^{-3x} - \frac{1}{24}\,e^{-3x}$
(c) $y_p = -\frac{3}{4}xe^{-3x} + \frac{1}{4}xe^{-3x} - \frac{1}{24}\,e^{-3x}$
(d) $y_p = \frac{1}{2}x^2e^{-3x} + \frac{1}{4}x^3e^{-3x} - \frac{1}{24}\,e^{-3x}$
(e) $y_p = \frac{1}{4}x^3e^{-3x} + \frac{1}{4}xe^{-3x} - \frac{1}{24}\,e^{-3x}$

12. Let S be a subspace of $\mathbb{R}^4$ defined by $S = \{(a,b,c,d)|a = b+c+d\}$. A basis for the subspace is

(a) $\{(1,1,0,0),\ (1,0,1,0)\}$
(b) $\{(1,0,1,0),\ (1,0,0,1)\}$
(c) $\{(1,1,1,0),\ (1,0,1,0),\ (1,0,1,1)\}$
(d) $\{(1,1,0,0),\ (1,0,1,0),\ (1,0,0,1)\}$
(e) $\{(1,1,0,0),\ (1,0,1,0),\ (2,0,1,2)\}$

13. The general solution of the differential equation

$$x\frac{dy}{dx} - 3y = x^3$$

is

(a) $y = x^3 \ln x + cx^2$

(b) $y = x^3 \ln x + cx^3$

(c) $y = x^2 \ln x + cx^3$

(d) $y = x^2 \ln x + cx^2$

(e) $y = x^3 \ln x + cx$

14. Let

$$A = \begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

If $e^{At} = \begin{bmatrix} 1 & f(t) & h(t) \\ 0 & 1 & 3t \\ 0 & 0 & 1 \end{bmatrix}$, then $f(2) + h(2) =$

(a) 26

(b) 32

(c) 24

(d) 30

(e) 28

15. If $y(x)$ is the solution of the initial value problem

$$\frac{dy}{dx} = \frac{2x+1}{2y},\ y(-2) = -1, \text{ then } y(2) =$$

(a) $\sqrt{3}$
(b) $-\sqrt{3}$
(c) 0
(d) $-\sqrt{5}$
(e) $\sqrt{5}$

16. If $X = c_1 \begin{bmatrix} 5 \\ -6 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{4t}$ is the solution of the initial value problem

$$X' = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} X,\ X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

then $c_2^2 - c_1^2 =$

(a) 36
(b) 32
(c) 37
(d) 34
(e) 35

17. The general solution of the first order homogeneous system $X' = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix} X$ is given by

$$X = c_1 \begin{bmatrix} a \\ b \\ -2 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} \alpha \\ \beta \\ 1 \end{bmatrix} e^{\lambda t} + c_3 \begin{bmatrix} e \\ f \\ 13 \end{bmatrix}$$

then $a \cdot b \cdot \lambda =$

(a) -24
(b) -12
(c) 0
(d) 24
(e) 12

18. The general solution of the differential equation

$$y''' + 3y'' - 4y = 0$$

is

(a) $y(x) = c_1 e^{x} + c_2 e^{-2x} + c_3 x e^{-2x}$
(b) $y(x) = c_1 e^{x} + c_2 e^{-2x} + c_3 e^{3x}$
(c) $y(x) = c_1 e^{x} + c_2 e^{-2x} + c_3 e^{4x}$
(d) $y(x) = c_1 e^{-x} + c_2 e^{-2x} + c_3 x e^{-2x}$
(e) $y(x) = c_1 e^{-x} + c_2 e^{3x} + c_3 x e^{3x}$

19. The matrix $A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$ has only one eigenvalue $\lambda = -1$ which is defective of defect 2. If we choose $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A + I)^3\, v_3 = 0$, and $(A + I)^2 v_3 \neq 0$, then the general solution of $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(b) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 - \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 + \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(d) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} + 1 \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(e) $X = \left(c_1 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ -t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} + 1 \\ t^2 \\ t \end{bmatrix} \right) e^{-t}$

20. The characteristic equation of a matrix A is $(\lambda+1)(\lambda-5)^3=0$, where we have only two linearly independent eigenvectors corresponding to $\lambda=5$. The Jordan normal form of A is

(a) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

(e) $\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE04 **CODE04**

Math 208
Final Exam
221
December 24, 2022
Net Time Allowed: 180 Minutes

Name	
ID	
Sec	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $(x, y, z) = (a, b, c)$ is the solution of the system

$$\begin{cases} 2x + 8y + 3z = 2 \\ x + 3y + 2z = 5 \\ 2x + 7y + 4z = 8 \end{cases}$$

then $a + b + c =$

(a) 5

(b) 7

(c) 3

(d) 0

(e) 6

2. A possible fundamental matrix for the system $X' = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} X$ is

(a) $\Phi(t) = \begin{bmatrix} e^{-2t} & 2e^{5t} \\ 0 & e^{5t} \end{bmatrix}$

(b) $\Phi(t) = \begin{bmatrix} e^{-2t} & e^{5t} \\ e^{-2t} & 2e^{5t} \end{bmatrix}$

(c) $\Phi(t) = \begin{bmatrix} e^{-2t} & 2e^{5t} \\ -3e^{-2t} & e^{5t} \end{bmatrix}$

(d) $\Phi(t) = \begin{bmatrix} e^{-2t} & e^{5t} \\ -3e^{-2t} & 0 \end{bmatrix}$

(e) $\Phi(t) = \begin{bmatrix} 3e^{-2t} & 0 \\ 5e^{-2t} & e^{5t} \end{bmatrix}$

3. Let $A = \begin{bmatrix} 9 & -8 \\ 6 & -5 \end{bmatrix}$. A diagonalization matrix P, such that $P^{-1}AP$ is diagonal is

   (a) $P = \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$

   (b) $P = \begin{bmatrix} 1 & 2 \\ 1 & 5 \end{bmatrix}$

   (c) $P = \begin{bmatrix} -1 & 4 \\ 1 & -3 \end{bmatrix}$

   (d) $P = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

   (e) $P = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

4. Let $A = \begin{bmatrix} 3 & 5 & -2 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$. An eigenvector corresponding to the eigenvalue $\lambda = 2$ of A is

   (a) $\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$

   (b) $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

   (c) $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

   (d) $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

   (e) $\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

5. The general solution of the first order homogeneous system $X' = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix} X$ is given by

$$X = c_1 \begin{bmatrix} a \\ b \\ -2 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} \alpha \\ \beta \\ 1 \end{bmatrix} e^{\lambda t} + c_3 \begin{bmatrix} e \\ f \\ 13 \end{bmatrix}$$

then $a \cdot b \cdot \lambda =$

(a) -12
(b) 12
(c) 24
(d) -24
(e) 0

6. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & -2 & 2 \end{bmatrix}$. Using Cayley-Hamilton Theorem,
$A^4 = aA^3 + bA^2 + cA.$
$a + b + c =$

(a) 4
(b) 1
(c) 3
(d) 0
(e) 2

7. If $y(x)$ is the solution of the initial value problem

$$\frac{dy}{dx} = \frac{2x+1}{2y},\ y(-2) = -1, \text{ then } y(2) =$$

(a) $\sqrt{5}$
(b) $-\sqrt{3}$
(c) 0
(d) $\sqrt{3}$
(e) $-\sqrt{5}$

8. The general solution of the exact differential equation

$$(3x^2 + 2y^2)\,dx + (4xy + 6y^2)\,dy = 0$$

is

(a) $x^3 - 2x^2y + 2y^3 = c$
(b) $x^3 + 2xy^2 + 2y^3 = c$
(c) $x^3 - 2xy^2 + 2y^3 = c$
(d) $x^3 - 2xy^2 - 2y^3 = c$
(e) $x^3 - 2x^2y - 2y^3 = c$

9. The general solution of the differential equation

$$y''' + 3y'' - 4y = 0$$

is

(a) $y(x) = c_1 e^{x} + c_2 e^{-2x} + c_3 e^{3x}$
(b) $y(x) = c_1 e^{x} + c_2 e^{-2x} + c_3 x e^{-2x}$
(c) $y(x) = c_1 e^{-x} + c_2 e^{3x} + c_3 x e^{3x}$
(d) $y(x) = c_1 e^{-x} + c_2 e^{-2x} + c_3 x e^{-2x}$
(e) $y(x) = c_1 e^{x} + c_2 e^{-2x} + c_3 e^{4x}$

10. If $X = c_1 \begin{bmatrix} 5 \\ -6 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{4t}$ is the solution of the initial value problem

$$X' = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} X,\ X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

then $c_2^2 - c_1^2 =$

(a) 37
(b) 34
(c) 36
(d) 35
(e) 32

11. By using the method of variation of parameters, a particular solution y_p of the differential equation

$$y'' - 9y = \frac{9x}{e^{3x}}$$

is

(a) $y_p = -\frac{3}{4}x^2e^{-3x} - \frac{1}{4}xe^{-3x} - \frac{1}{24}\,e^{-3x}$

(b) $y_p = \frac{1}{4}x^3e^{-3x} + \frac{1}{4}xe^{-3x} - \frac{1}{24}\,e^{-3x}$

(c) $y_p = -\frac{3}{4}xe^{-3x} + \frac{1}{4}xe^{-3x} - \frac{1}{24}\,e^{-3x}$

(d) $y_p = \frac{3}{4}x^2e^{-3x} + \frac{1}{2}xe^{-3x} - \frac{1}{24}\,e^{-3x}$

(e) $y_p = \frac{1}{2}x^2e^{-3x} + \frac{1}{4}x^3e^{-3x} - \frac{1}{24}\,e^{-3x}$

12. If $y(x)$ is the solution of the initial-value problem

$$y'' + 4y = 2x;\; y(0) = 1,\; y'(0) = 2$$

then $y(\pi) =$

(a) $1 + \frac{\pi}{4}$

(b) $1 - \frac{\pi}{4}$

(c) $1 + \frac{\pi}{3}$

(d) $1 + \frac{\pi}{2}$

(e) $1 - \frac{\pi}{2}$

13. The solution of $X' = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} X$, $X(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ at $t = \frac{\pi}{4}$ equals

(a) $\begin{pmatrix} -2 \\ 0 \end{pmatrix} e^{\frac{\pi}{4}}$

(b) $\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{\frac{\pi}{4}}$

(c) $\begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{\frac{\pi}{4}}$

(d) $\begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{\frac{\pi}{4}}$

(e) $\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-\frac{\pi}{4}}$

14. Let S be a subspace of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d) | a = b + c + d\}$. A basis for the subspace is

(a) $\{(1, 1, 0, 0),\ (1, 0, 1, 0),\ (2, 0, 1, 2)\}$

(b) $\{(1, 1, 0, 0),\ (1, 0, 1, 0)\}$

(c) $\{(1, 1, 0, 0),\ (1, 0, 1, 0),\ (1, 0, 0, 1)\}$

(d) $\{(1, 1, 1, 0),\ (1, 0, 1, 0),\ (1, 0, 1, 1)\}$

(e) $\{(1, 0, 1, 0),\ (1, 0, 0, 1)\}$

15. Using variation of parameters to find a particular solution X_p of the nonhomogeneous system $X' = AX + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t$ where $X_c = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$ form a general solution of the associated homogeneous system, then $X_p(1) =$

(a) $\begin{bmatrix} 3e \\ 2e \end{bmatrix}$

(b) $\begin{bmatrix} e \\ 5e \end{bmatrix}$

(c) $\begin{bmatrix} 7e \\ e \end{bmatrix}$

(d) $\begin{bmatrix} 7e \\ 5e \end{bmatrix}$

(e) $\begin{bmatrix} 0 \\ 3e \end{bmatrix}$

16. The differential equation

$$t^3 x''' - 2t^2 x'' + 3tx' + 5x = \ln t$$

is equivalent to the system of first-order equations.

(a) $x_1' = x_2,\ x_2' = x_3,\ t^3\, x_3' = -5x_1 - 3tx_2 + 2t^2 x_3 + \ln t$

(b) $x_1' = x_2,\ x_2' = x_3,\ x_3' = -5x_1 - 3tx_2 - 2x_3 + \ln t$

(c) $x_1' = x_2',\ x_2' = x_1,\ t^3\, x_3' = 5x_1 + 3tx_2 + 2t^2 x_3 + \ln t$

(d) $x_1' = x_2,\ x_2' = x_3,\ x_3' = -5x_1 - 3tx_2 + 2t^2 x_3 - \ln t$

(e) $x_1' = x_1,\ x_2' = x_2,\ x_3' = -5x_1 - 3tx_2 + 2t^2 x_3 + \ln t$

17. The general solution of the differential equation

$$x\frac{dy}{dx} - 3y = x^3$$

is

(a) $y = x^3 \ln x + cx^2$
(b) $y = x^3 \ln x + cx$
(c) $y = x^2 \ln x + cx^3$
(d) $y = x^3 \ln x + cx^3$
(e) $y = x^2 \ln x + cx^2$

18. Let

$$A = \begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

If $e^{At} = \begin{bmatrix} 1 & f(t) & h(t) \\ 0 & 1 & 3t \\ 0 & 0 & 1 \end{bmatrix}$, then $f(2) + h(2) =$

(a) 32
(b) 24
(c) 28
(d) 26
(e) 30

19. The matrix $A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$ has only one eigenvalue $\lambda = -1$ which is defective of defect 2. If we choose $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A+I)^3 v_3 = 0$, and $(A+I)^2 v_3 \neq 0$, then the general solution of $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 - \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(b) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 + \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(d) $X = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} + 1 \\ \frac{t^2}{2} \\ t \end{bmatrix} \right) e^{-t}$

(e) $X = \left(c_1 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ -t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} + 1 \\ t^2 \\ t \end{bmatrix} \right) e^{-t}$

20. The characteristic equation of a matrix A is $(\lambda+1)(\lambda-5)^3=0$, where we have only two linearly independent eigenvectors corresponding to $\lambda=5$. The Jordan normal form of A is

(a) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 1 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

(e) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	E 12	C 8	B 4	A 4
2	A	D 14	D 14	D 6	C 16
3	A	C 10	C 7	D 12	A 11
4	A	C 7	C 12	D 10	D 10
5	A	E 11	A 16	B 17	D 9
6	A	E 18	A 15	E 16	B 8
7	A	D 5	D 10	A 8	E 1
8	A	E 6	E 9	D 3	B 3
9	A	B 4	A 1	C 11	B 18
10	A	A 3	B 6	E 14	D 13
11	A	E 13	B 13	B 7	A 7
12	A	B 15	B 5	D 5	D 6
13	A	B 1	D 2	B 2	C 14
14	A	A 8	B 11	B 15	C 5
15	A	B 16	D 18	D 1	D 17
16	A	C 2	B 17	E 13	A 12
17	A	B 17	B 3	A 9	D 2
18	A	A 9	B 4	A 18	A 15
19	A	E 20	E 19	D 20	D 20
20	A	B 19	E 20	A 19	E 19

Answer Counts

V	A	B	C	D	E
1	3	6	3	2	6
2	3	7	3	4	3
3	4	5	1	7	3
4	5	3	3	7	2