King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208

Exam II

222

March 27, 2023

Net Time Allowed: 120 Minutes

USE THIS AS A TEMPLATE

Write your questions, once you are satisfied upload this file.

1. Which one of the following sets V is a subspace of $\mathbb{R}^3$?

   (a) V is the set of all (x, y, z) such that $z = 2x + 3y$
   (b) V is the set of all (x, y, z) such that $y = 1$
   (c) V is the set of all (x, y, z) such that $x + y + z = 3$
   (d) V is the set of all (x, y, z) such that $z \geq 0$
   (e) V is the set of all (x, y, z) such that $xyz = 1$

2. Consider the vectors $\vec{t} = (4, 20, 23)$, $\vec{u} = (1, 3, 2)$, $\vec{v} = (2, 8, 7)$ and $\vec{w} = (1, 7, 9)$. If $\vec{t} = a\vec{u} + b\vec{v} + c\vec{w}$, then $a + b - c =$

   (a) 0
   (b) 1
   (c) 2
   (d) -1
   (e) -2

3. The vectors $v_1 = (1, 0, 1)$, $v_2 = (2, -3, 4)$ and $v_3 = (3, 5, a)$ are linearly dependent if $3a =$

   (a) -1
   (b) 0
   (c) 2
   (d) 1
   (e) -2

4. Given that the vectors $v_1 = (2, 0, 0, 0)$, $v_2 = (0, 3, 0, 0)$, $v_3 = (0, 0, 7, 6)$ and $v_4 = (0, 0, 4, a)$ form a basis for $\mathbb{R}^4$, then $7a \neq$

   (a) 24
   (b) 18
   (c) 12
   (d) 6
   (e) 0

5. The dimension of the subspace consisting of the set of all vectors of the form (a, b, c, d) for which $a + 2b = c + 3d = 0$ is

   (a) 2
   (b) 3
   (c) 4
   (d) 1
   (e) 0

6. The rank of the matrix $A = \begin{bmatrix} 1 & -2 & -3 & -5 \\ 1 & 4 & 9 & 2 \\ 1 & 3 & 7 & 1 \\ 2 & 2 & 6 & -3 \end{bmatrix}$ is

(a) 3
(b) 4
(c) 2
(d) 1
(e) 0

7. If $y = c_1 \cos(\ln x) + c_2 \sin(\ln x)$ is a solution of the initial-value problem

$$\begin{aligned} &x^2y'' + xy' + y = 0 \\ &y(1) = 7,\ y'(1) = 2, \end{aligned}$$

then $c_1 + c_2 =$

(a) 9
(b) 5
(c) -5
(d) -9
(e) 0

8. Which of the following sets of solutions of homogeneous linear differential equations, is linearly independent?

   (a) $\{1, e^x, \sinh x\}$
   (b) $\{0, \sin x, e^x\}$
   (c) $\{17, 2\sin^2 x, 3\cos^2 x\}$
   (d) $\{e^x, \cosh x, \sinh x\}$
   (e) $\{17, \cos^2 x, \cos 2x\}$

9. The general solution of the differential equation $y^{(4)} + 3y'' - 4y = 0$ is $y =$

   (a) $c_1 e^x + c_2 e^{-x} + c_3 \sin 2x + c_4 \cos 2x$
   (b) $c_1 e^x + c_2 x e^x + c_3 \sin 2x + c_4 \cos 2x$
   (c) $c_1 e^{-x} + c_2 x e^{-x} + c_3 \sin 2x + c_4 \cos 2x$
   (d) $c_1 e^x + c_2 e^{-x} + c_3 \sin 2x$
   (e) $c_1 e^x + c_2 e^{-x} + c_3 \sin 4x + c_4 \cos 4x$

10. If $y = c_1 e^{ax} + c_2 e^{-x} \sin x + c_3 g(x)$ is the general solution of the differential equation

$$9y''' + 11y'' + 4y' - 14y = 0,$$

then $a =$

(a) $\frac{7}{9}$
(b) $\frac{5}{9}$
(c) $\frac{1}{3}$
(d) -1
(e) 1

11. A linear homogeneous constant-coefficient equation which has the general solution $y(x) = (A + Bx)e^{2x} + Ce^{-2x}$ is

(a) $y''' - 2y'' - 4y' + 8y = 0$
(b) $y''' + 6y'' - 4y' + 8y = 0$
(c) $y''' - 2y'' + 4y' + 8y = 0$
(d) $y''' - 2y'' - 4y' - 8y = 0$
(e) $y'' - 4y = 0$

12. An appropriate form of a particular solution y_p for the non-homogeneous differential equation

$$(D-1)^3(D^2-4)y = xe^x + e^{2x} + e^{-2x}$$

is given by $y_p(x) =$

(a) $Axe^{2x} + Bxe^{-2x} + Cx^3e^x + Dx^4e^x$

(b) $Ae^{2x} + Be^{-2x} + Ce^x + Dxe^x$

(c) $Axe^{2x} + Bxe^{-2x} + Ce^x + Dx^2e^x + Ex^3e^x$

(d) $Axe^{2x} + Be^{-2x} + Cx^3e^x + Dx^4e^x$

(e) $Axe^{2x} + Bxe^{-2x} + Ce^x + Dxe^x + Ex^2e^x$

13. Given that $y_p = u_1 \cos x - \cos x \sin x$ is a particular solution of the differential equation $y'' + y = \tan x$, then $u_1(x) =$

(a) $\sin x - \ln|\sec x + \tan x|$

(b) $\cos x - \ln|\csc x + \cot x|$

(c) $\sin x + \ln|\sec x + \tan x|$

(d) $\sin x - \ln|\sec x|$

(e) $\cos x + \ln|\cot x|$