King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208
Major Exam I
231
October 02, 2023

EXAM COVER

Number of versions: 4
Number of questions: 20

King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 208
Major Exam I
231
October 02, 2023
Net Time Allowed: 120 Minutes

MASTER VERSION

1. Find the sum of all values of r such that $y = e^{rx}$ is a solution of the differential equation $5y'' - 6y' - y = 0$.

(a) $\frac{6}{5}$ (correct)

(b) $-\frac{4}{5}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

(e) $-\frac{6}{5}$

2. Find a value of c such that $y = \tan(x^3 + c)$ is a solution of the initial value problem $y' = 3x^2(y^2 + 1)$, $y(0) = 1$.

(a) $\frac{5\pi}{4}$ (correct)

(b) $\frac{\pi}{3}$

(c) $\frac{5\pi}{3}$

(d) $-\frac{\pi}{4}$

(e) $\frac{3\pi}{4}$

3. A particle is moving in a straight line with acceleration $a(t) = 18\cos(3t)$, and initial position $x(0) = -7$, and an initial velocity $v(0) = 4$.

   Find $x(\pi)$ (the position of the particle at $t = \pi$).

   (a) $4\pi - 3$ (correct)
   (b) $4\pi + 3$
   (c) $2\pi - 3$
   (d) $2\pi + 3$
   (e) $4\pi - 7$

4. Find the general solution of the differential equation $\dfrac{dy}{dx} = xe^{-x}$.

   (a) $y(x) = -(x+1)e^{-x} + c$ (correct)
   (b) $y(x) = 2(x+1)e^{-x} + c$
   (c) $y(x) = xe^{-x} + c$
   (d) $y(x) = -xe^{-x} + x + c$
   (e) $y(x) = (x+2)e^{-x} + c$

5. Find a general solution of the separable differential equation $x^2\dfrac{dy}{dx} = 1-x^2+y^2-x^2y^2$.

(a) $y(x) = \tan\left(c - \dfrac{1}{x} - x\right)$ (correct)

(b) $y(x) = \tan\left(c - \dfrac{2}{x} + x\right)$

(c) $y(x) = \tan\left(c + \dfrac{2}{x} + x\right)$

(d) $y(x) = \tan\left(c + \dfrac{1}{x} + x\right)$

(e) $y(x) = \tan\left(c - \dfrac{3}{x} + x\right)$

6. In a certain culture of bacteria, the number of bacteria doubled after 3 hours. How long did it take for the population to triple?

(a) $\dfrac{\ln 27}{\ln 2}$ (correct)

(b) $\dfrac{\ln 9}{\ln 2}$

(c) $\dfrac{\ln 3}{\ln 2}$

(d) $\dfrac{\ln 12}{\ln 2}$

(e) $\dfrac{\ln 16}{2}$

7. Solve the initial-value problem $y' + y = 2,\ y(0) = 0$.

(a) $y(x) = 2 - 2e^{-x}$ _____ (correct)
(b) $y(x) = 1 - e^{-x}$
(c) $y(x) = 1 + e^{-x}$
(d) $y(x) = 2 - e^{-x}$
(e) $y(x) = 3 - 2e^{-x}$

8. Find a general solution of the linear differential equation $x\dfrac{dy}{dx} = 2y + x^3 \cos x$.

(a) $y(x) = x^2(\sin x + c)$ _____ (correct)
(b) $y(x) = x^2(\cos x + c)$
(c) $y(x) = x^3(\sin x + c)$
(d) $y(x) = x^3(\cos x + c)$
(e) $y(x) = x(\cos x + c)$

9. Find a general solution of the differential equation $xy\dfrac{dy}{dx} = x^2 + 3y^2$.

(a) $2y^2 = cx^6 - x^2$ _____(correct)

(b) $2y^2 = cx^6 + x^2$

(c) $2y^2 = cx^5 - x^2$

(d) $2y^2 = cx^5 + x^2$

(e) $2y^2 = cx^4 + x^3$

10. Find a constant A that makes the differential equation $(2x+Ay)\,dx+(3x+2y)\,dy = 0$ exact.

(a) 3 _____(correct)

(b) -3

(c) 2

(d) -2

(e) 0

11. Find a general solution of the exact differential equation

$$(1 + ye^{xy})\,dx + (2y + xe^{xy})\,dy = 0.$$

(a) $x + e^{xy} + y^2 = c$ (correct)
(b) $x - e^{xy} + y^2 = c$
(c) $x + e^{xy} - y^2 = c$
(d) $x^2 + e^{xy} + y^2 = c$
(e) $x^2 - e^{xy} + y^2 = c$

12. Find a general solution of the differential equation $y'' = 2y(y')^3$.

(a) $y^3 + 3x + Ay + B = 0$ (correct)
(b) $y^3 - 3x + Ay + B = 0$
(c) $y^3 - 3x + Ay + B = 0$
(d) $y^2 + 3x + Ay + B = 0$
(e) $y^3 + 2x + Ay + B = 0$

13. By making a suitable substitution, the differential equation $x^2y' + 2xy = 5y^3$ can be transformed into a linear differential equation

(a) $v' - \frac{4}{x}v = -\frac{10}{x^2}$ _____(correct)

(b) $v' - \frac{3}{x}v = -\frac{5}{x^2}$

(c) $v' - \frac{4}{x}v = \frac{5}{x^2}$

(d) $v' + \frac{3}{x}v = \frac{5}{x^2}$

(e) $v' - \frac{5}{x}v = \frac{10}{x^2}$

14. Find a general solution of the differential equation $y' = \sqrt{x + y + 1}$.

(a) $2\sqrt{x+y+1} - 2\ln(1 + \sqrt{x+y+1}) = x + c$ _____(correct)

(b) $2\sqrt{x+y+1} + 2\ln(1 + \sqrt{x+y+1}) = -x + c$

(c) $3\sqrt{x+y+1} - 2\ln(1 + \sqrt{x+y+1}) = x + c$

(d) $3\sqrt{x+y+1} + 2\ln(1 + \sqrt{x+y+1}) = -x + c$

(e) $3\sqrt{x+y+1} + 2\ln(1 + \sqrt{x+y+1}) = x + c$

15. Find the order of the differential equation

$$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^8 + \sin^4 x = 0.$$

(a) 3 (correct)

(b) 8

(c) 4

(d) 1

(e) 5

16. Let $\mathbf{u} = (5, 1)$, $\mathbf{v} = (-2, -1)$, $\mathbf{w} = (5, -2)$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a^2 + b^2 =$

(a) 34 (correct)

(b) 8

(c) 25

(d) 16

(e) 36

17. Find the value of k for which the vectors of $\mathbb{R}^3$

$$\mathbf{u} = (1, 1, 0),\ \mathbf{v} = (k, 3, 1),\ \mathbf{w} = (3, -2, -4)$$

are linearly dependent.

(a) $k = \dfrac{7}{4}$ (correct)
(b) $k = 0$
(c) $k = \dfrac{3}{4}$
(d) $k = -\dfrac{3}{4}$
(e) $k = 2$

18. Which one of the following statements is true about the subset V of $\mathbb{R}^3$ defined by

$$V = \{(x_1, x_2, x_3) : x_1 = 0\}$$

(a) V is a subspace of $\mathbb{R}^3$ (correct)
(b) V is not closed under addition
(c) V is not closed under multiplication by scalars
(d) V is closed under addition but not closed under multiplication by scalars
(e) V is not closed under addition but closed under multiplication by scalars

19. The solution of the system

$$\begin{aligned} x_1 - 3x_2 - 7x_3 + 5x_4 &= 0 \\ 3x_1 + x_2 + 9x_3 - 5x_4 &= 0 \\ x_1 - 2x_2 - 4x_3 + 4x_4 &= 0 \end{aligned}$$

is the set of all scalars multiples of a vector $\mathbf{u}$ where $\mathbf{u} =$

(a) $(-2, -3, 1, 0)$ _____ (correct)

(b) $(-1, -6, 1, 0)$

(c) $(-2, 3, 0, 1)$

(d) $(-1, 3, 1, 0)$

(e) $(-2, 3, 1, 2)$

20. If $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^3$ and r and s are real numbers, then which one of the following statements is not true?

(a) $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|$ _____ (correct)

(b) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(c) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

(d) $r(\mathbf{u} + \mathbf{v}) = r\mathbf{u} + r\mathbf{v}$

(e) $(r + s)\mathbf{u} = r\mathbf{u} + s\mathbf{u}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE01 **CODE01**

Math 208
Major Exam I
231
October 02, 2023
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Find the general solution of the differential equation $\dfrac{dy}{dx} = xe^{-x}$.

   (a) $y(x) = 2(x+1)e^{-x} + c$
   (b) $y(x) = -(x+1)e^{-x} + c$
   (c) $y(x) = -xe^{-x} + x + c$
   (d) $y(x) = xe^{-x} + c$
   (e) $y(x) = (x+2)e^{-x} + c$

2. Find the value of k for which the vectors of $\mathbb{R}^3$

   $$\mathbf{u} = (1,1,0),\ \mathbf{v} = (k,3,1),\ \mathbf{w} = (3,-2,-4)$$

   are linearly dependent.

   (a) $k = 0$
   (b) $k = \dfrac{3}{4}$
   (c) $k = 2$
   (d) $k = \dfrac{7}{4}$
   (e) $k = -\dfrac{3}{4}$

3. If $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^3$ and r and s are real numbers, then which one of the following statements is not true?

   (a) $r(\mathbf{u}+\mathbf{v}) = r\mathbf{u} + r\mathbf{v}$

   (b) $\mathbf{u}+\mathbf{v} = \mathbf{v}+\mathbf{u}$

   (c) $(\mathbf{u}+\mathbf{v})+\mathbf{w} = \mathbf{u}+(\mathbf{v}+\mathbf{w})$

   (d) $\mathbf{u}\cdot\mathbf{u} = |\mathbf{u}|$

   (e) $(r+s)\mathbf{u} = r\mathbf{u} + s\mathbf{u}$

4. Find a general solution of the separable differential equation $x^2\dfrac{dy}{dx} = 1-x^2+y^2-x^2y^2$.

   (a) $y(x) = \tan\left(c - \dfrac{1}{x} - x\right)$

   (b) $y(x) = \tan\left(c + \dfrac{1}{x} + x\right)$

   (c) $y(x) = \tan\left(c - \dfrac{3}{x} + x\right)$

   (d) $y(x) = \tan\left(c - \dfrac{2}{x} + x\right)$

   (e) $y(x) = \tan\left(c + \dfrac{2}{x} + x\right)$

5. Find a general solution of the exact differential equation

$$(1 + ye^{xy})\,dx + (2y + xe^{xy})\,dy = 0.$$

(a) $x - e^{xy} + y^2 = c$
(b) $x + e^{xy} + y^2 = c$
(c) $x + e^{xy} - y^2 = c$
(d) $x^2 + e^{xy} + y^2 = c$
(e) $x^2 - e^{xy} + y^2 = c$

6. Find a general solution of the differential equation $xy\dfrac{dy}{dx} = x^2 + 3y^2$.

(a) $2y^2 = cx^6 - x^2$
(b) $2y^2 = cx^4 + x^3$
(c) $2y^2 = cx^6 + x^2$
(d) $2y^2 = cx^5 - x^2$
(e) $2y^2 = cx^5 + x^2$

7. Find the order of the differential equation

$$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^8 + \sin^4 x = 0.$$

(a) 4
(b) 3
(c) 8
(d) 1
(e) 5

8. Let $\mathbf{u} = (5, 1)$, $\mathbf{v} = (-2, -1)$, $\mathbf{w} = (5, -2)$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a^2 + b^2 =$

(a) 8
(b) 25
(c) 36
(d) 16
(e) 34

9. Find a general solution of the differential equation $y' = \sqrt{x+y+1}$.

(a) $3\sqrt{x+y+1} - 2\ln(1+\sqrt{x+y+1}) = x + c$
(b) $3\sqrt{x+y+1} + 2\ln(1+\sqrt{x+y+1}) = -x + c$
(c) $3\sqrt{x+y+1} + 2\ln(1+\sqrt{x+y+1}) = x + c$
(d) $2\sqrt{x+y+1} - 2\ln(1+\sqrt{x+y+1}) = x + c$
(e) $2\sqrt{x+y+1} + 2\ln(1+\sqrt{x+y+1}) = -x + c$

10. The solution of the system

$$\begin{aligned} x_1 - 3x_2 - 7x_3 + 5x_4 &= 0 \\ 3x_1 + x_2 + 9x_3 - 5x_4 &= 0 \\ x_1 - 2x_2 - 4x_3 + 4x_4 &= 0 \end{aligned}$$

is the set of all scalars multiples of a vector $\mathbf{u}$ where $\mathbf{u} =$

(a) $(-2, 3, 0, 1)$
(b) $(-1, 3, 1, 0)$
(c) $(-2, -3, 1, 0)$
(d) $(-2, 3, 1, 2)$
(e) $(-1, -6, 1, 0)$

11. Find a general solution of the differential equation $y'' = 2y(y')^3$.

(a) $y^3 + 3x + Ay + B = 0$
(b) $y^2 + 3x + Ay + B = 0$
(c) $y^3 + 2x + Ay + B = 0$
(d) $y^3 - 3x + Ay + B = 0$
(e) $y^3 - 3x + Ay + B = 0$

12. Find a value of c such that $y = \tan(x^3 + c)$ is a solution of the initial value problem $y' = 3x^2(y^2 + 1)$, $y(0) = 1$.

(a) $\frac{5\pi}{4}$
(b) $\frac{\pi}{3}$
(c) $-\frac{\pi}{4}$
(d) $\frac{5\pi}{3}$
(e) $\frac{3\pi}{4}$

13. Find a constant A that makes the differential equation $(2x+Ay)\,dx+(3x+2y)\,dy=0$ exact.

(a) -2
(b) 2
(c) 3
(d) 0
(e) -3

14. By making a suitable substitution, the differential equation $x^2y'+2xy=5y^3$ can be transformed into a linear differential equation

(a) $v'-\dfrac{3}{x}v=-\dfrac{5}{x^2}$
(b) $v'-\dfrac{5}{x}v=\dfrac{10}{x^2}$
(c) $v'+\dfrac{3}{x}v=\dfrac{5}{x^2}$
(d) $v'-\dfrac{4}{x}v=\dfrac{5}{x^2}$
(e) $v'-\dfrac{4}{x}v=-\dfrac{10}{x^2}$

15. Solve the initial-value problem $y' + y = 2,\ y(0) = 0$.

(a) $y(x) = 1 - e^{-x}$
(b) $y(x) = 1 + e^{-x}$
(c) $y(x) = 3 - 2e^{-x}$
(d) $y(x) = 2 - e^{-x}$
(e) $y(x) = 2 - 2e^{-x}$

16. In a certain culture of bacteria, the number of bacteria doubled after 3 hours. How long did it take for the population to triple?

(a) $\dfrac{\ln 9}{\ln 2}$
(b) $\dfrac{\ln 3}{\ln 2}$
(c) $\dfrac{\ln 27}{\ln 2}$
(d) $\dfrac{\ln 16}{2}$
(e) $\dfrac{\ln 12}{\ln 2}$

17. A particle is moving in a straight line with acceleration $a(t) = 18\cos(3t)$, and initial position $x(0) = -7$, and an initial velocity $v(0) = 4$.

    Find $x(\pi)$ (the position of the particle at $t = \pi$).

    (a) $4\pi + 3$
    (b) $2\pi + 3$
    (c) $2\pi - 3$
    (d) $4\pi - 3$
    (e) $4\pi - 7$

18. Find the sum of all values of r such that $y = e^{rx}$ is a solution of the differential equation $5y'' - 6y' - y = 0$.

    (a) $\frac{3}{5}$
    (b) $-\frac{4}{5}$
    (c) $\frac{4}{5}$
    (d) $-\frac{6}{5}$
    (e) $\frac{6}{5}$

19. Find a general solution of the linear differential equation $x\dfrac{dy}{dx} = 2y + x^3 \cos x$.

(a) $y(x) = x^2(\sin x + c)$
(b) $y(x) = x(\cos x + c)$
(c) $y(x) = x^3(\cos x + c)$
(d) $y(x) = x^2(\cos x + c)$
(e) $y(x) = x^3(\sin x + c)$

20. Which one of the following statements is true about the subset V of $\mathbb{R}^3$ defined by

$$V = \{(x_1, x_2, x_3) : x_1 = 0\}$$

(a) V is a subspace of $\mathbb{R}^3$
(b) V is not closed under addition
(c) V is not closed under addition but closed under multiplication by scalars
(d) V is closed under addition but not closed under multiplication by scalars
(e) V is not closed under multiplication by scalars

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE02 **CODE02**

Math 208
Major Exam I
231
October 02, 2023
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Find the value of k for which the vectors of $\mathbb{R}^3$

$$\mathbf{u} = (1, 1, 0),\ \mathbf{v} = (k, 3, 1),\ \mathbf{w} = (3, -2, -4)$$

are linearly dependent.

(a) $k = \dfrac{7}{4}$
(b) $k = 2$
(c) $k = \dfrac{3}{4}$
(d) $k = 0$
(e) $k = -\dfrac{3}{4}$

2. Find a general solution of the exact differential equation

$$(1 + ye^{xy})\,dx + (2y + xe^{xy})\,dy = 0.$$

(a) $x^2 - e^{xy} + y^2 = c$
(b) $x - e^{xy} + y^2 = c$
(c) $x + e^{xy} - y^2 = c$
(d) $x + e^{xy} + y^2 = c$
(e) $x^2 + e^{xy} + y^2 = c$

3. By making a suitable substitution, the differential equation $x^2y' + 2xy = 5y^3$ can be transformed into a linear differential equation

(a) $v' - \frac{4}{x}v = \frac{5}{x^2}$
(b) $v' - \frac{4}{x}v = -\frac{10}{x^2}$
(c) $v' - \frac{3}{x}v = -\frac{5}{x^2}$
(d) $v' + \frac{3}{x}v = \frac{5}{x^2}$
(e) $v' - \frac{5}{x}v = \frac{10}{x^2}$

4. Find a general solution of the differential equation $y' = \sqrt{x + y + 1}$.

(a) $2\sqrt{x+y+1} + 2\ln(1 + \sqrt{x+y+1}) = -x + c$
(b) $3\sqrt{x+y+1} - 2\ln(1 + \sqrt{x+y+1}) = x + c$
(c) $3\sqrt{x+y+1} + 2\ln(1 + \sqrt{x+y+1}) = -x + c$
(d) $3\sqrt{x+y+1} + 2\ln(1 + \sqrt{x+y+1}) = x + c$
(e) $2\sqrt{x+y+1} - 2\ln(1 + \sqrt{x+y+1}) = x + c$

5. A particle is moving in a straight line with acceleration $a(t) = 18\cos(3t)$, and initial position $x(0) = -7$, and an initial velocity $v(0) = 4$.

   Find $x(\pi)$ (the position of the particle at $t = \pi$).

   (a) $2\pi - 3$
   (b) $2\pi + 3$
   (c) $4\pi - 3$
   (d) $4\pi + 3$
   (e) $4\pi - 7$

6. Find a general solution of the linear differential equation $x\dfrac{dy}{dx} = 2y + x^3\cos x$.

   (a) $y(x) = x^2(\sin x + c)$
   (b) $y(x) = x^3(\sin x + c)$
   (c) $y(x) = x^3(\cos x + c)$
   (d) $y(x) = x^2(\cos x + c)$
   (e) $y(x) = x(\cos x + c)$

7. Find a general solution of the differential equation $xy\dfrac{dy}{dx} = x^2 + 3y^2$.

(a) $2y^2 = cx^5 - x^2$
(b) $2y^2 = cx^6 + x^2$
(c) $2y^2 = cx^4 + x^3$
(d) $2y^2 = cx^5 + x^2$
(e) $2y^2 = cx^6 - x^2$

8. Find the general solution of the differential equation $\dfrac{dy}{dx} = xe^{-x}$.

(a) $y(x) = -(x+1)e^{-x} + c$
(b) $y(x) = -xe^{-x} + x + c$
(c) $y(x) = (x+2)e^{-x} + c$
(d) $y(x) = xe^{-x} + c$
(e) $y(x) = 2(x+1)e^{-x} + c$

9. In a certain culture of bacteria, the number of bacteria doubled after 3 hours. How long did it take for the population to triple?

(a) $\dfrac{\ln 9}{\ln 2}$
(b) $\dfrac{\ln 27}{\ln 2}$
(c) $\dfrac{\ln 12}{\ln 2}$
(d) $\dfrac{\ln 3}{\ln 2}$
(e) $\dfrac{\ln 16}{2}$

10. Find a general solution of the differential equation $y'' = 2y(y')^3$.

(a) $y^3 + 3x + Ay + B = 0$
(b) $y^3 + 2x + Ay + B = 0$
(c) $y^3 - 3x + Ay + B = 0$
(d) $y^3 - 3x + Ay + B = 0$
(e) $y^2 + 3x + Ay + B = 0$

11. The solution of the system

$$\begin{aligned} x_1 - 3x_2 - 7x_3 + 5x_4 &= 0 \\ 3x_1 + x_2 + 9x_3 - 5x_4 &= 0 \\ x_1 - 2x_2 - 4x_3 + 4x_4 &= 0 \end{aligned}$$

is the set of all scalars multiples of a vector $\mathbf{u}$ where $\mathbf{u} =$

(a) $(-1, -6, 1, 0)$
(b) $(-2, -3, 1, 0)$
(c) $(-2, 3, 0, 1)$
(d) $(-1, 3, 1, 0)$
(e) $(-2, 3, 1, 2)$

12. Solve the initial-value problem $y' + y = 2,\ y(0) = 0$.

(a) $y(x) = 3 - 2e^{-x}$
(b) $y(x) = 1 - e^{-x}$
(c) $y(x) = 1 + e^{-x}$
(d) $y(x) = 2 - e^{-x}$
(e) $y(x) = 2 - 2e^{-x}$

13. Find the order of the differential equation

$$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^8 + \sin^4 x = 0.$$

(a) 8
(b) 4
(c) 5
(d) 3
(e) 1

14. Find a constant A that makes the differential equation $(2x+Ay)\,dx+(3x+2y)\,dy = 0$ exact.

(a) 0
(b) 3
(c) -3
(d) -2
(e) 2

15. If $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^3$ and r and s are real numbers, then which one of the following statements is not true?

(a) $r(\mathbf{u}+\mathbf{v}) = r\mathbf{u} + r\mathbf{v}$

(b) $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|$

(c) $(r+s)\mathbf{u} = r\mathbf{u} + s\mathbf{u}$

(d) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(e) $(\mathbf{u}+\mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v}+\mathbf{w})$

16. Find the sum of all values of r such that $y = e^{rx}$ is a solution of the differential equation $5y'' - 6y' - y = 0$.

(a) $-\frac{6}{5}$

(b) $\frac{3}{5}$

(c) $\frac{4}{5}$

(d) $-\frac{4}{5}$

(e) $\frac{6}{5}$

17. Find a general solution of the separable differential equation $x^2\dfrac{dy}{dx} = 1-x^2+y^2-x^2y^2$.

(a) $y(x) = \tan\left(c - \dfrac{3}{x} + x\right)$

(b) $y(x) = \tan\left(c - \dfrac{1}{x} - x\right)$

(c) $y(x) = \tan\left(c + \dfrac{2}{x} + x\right)$

(d) $y(x) = \tan\left(c + \dfrac{1}{x} + x\right)$

(e) $y(x) = \tan\left(c - \dfrac{2}{x} + x\right)$

18. Let $\mathbf{u} = (5, 1)$, $\mathbf{v} = (-2, -1)$, $\mathbf{w} = (5, -2)$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a^2 + b^2 =$

(a) 34

(b) 8

(c) 25

(d) 36

(e) 16

19. Find a value of c such that $y = \tan(x^3 + c)$ is a solution of the initial value problem $y' = 3x^2(y^2 + 1)$, $y(0) = 1$.

(a) $\frac{\pi}{3}$
(b) $\frac{3\pi}{4}$
(c) $\frac{5\pi}{3}$
(d) $\frac{5\pi}{4}$
(e) $-\frac{\pi}{4}$

20. Which one of the following statements is true about the subset V of $\mathbb{R}^3$ defined by

$$V = \{(x_1, x_2, x_3) : x_1 = 0\}$$

(a) V is not closed under addition
(b) V is a subspace of $\mathbb{R}^3$
(c) V is closed under addition but not closed under multiplication by scalars
(d) V is not closed under addition but closed under multiplication by scalars
(e) V is not closed under multiplication by scalars

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE03 **CODE03**

Math 208
Major Exam I
231
October 02, 2023
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Find the order of the differential equation

$$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^8 + \sin^4 x = 0.$$

(a) 8
(b) 4
(c) 5
(d) 1
(e) 3

2. The solution of the system

$$\begin{aligned} x_1 - 3x_2 - 7x_3 + 5x_4 &= 0 \\ 3x_1 + x_2 + 9x_3 - 5x_4 &= 0 \\ x_1 - 2x_2 - 4x_3 + 4x_4 &= 0 \end{aligned}$$

is the set of all scalars multiples of a vector $\mathbf{u}$ where $\mathbf{u} =$

(a) $(-1, 3, 1, 0)$
(b) $(-2, -3, 1, 0)$
(c) $(-2, 3, 1, 2)$
(d) $(-2, 3, 0, 1)$
(e) $(-1, -6, 1, 0)$

3. Find a general solution of the differential equation $y' = \sqrt{x+y+1}$.

(a) $3\sqrt{x+y+1} + 2\ln(1+\sqrt{x+y+1}) = x + c$
(b) $3\sqrt{x+y+1} + 2\ln(1+\sqrt{x+y+1}) = -x + c$
(c) $2\sqrt{x+y+1} + 2\ln(1+\sqrt{x+y+1}) = -x + c$
(d) $3\sqrt{x+y+1} - 2\ln(1+\sqrt{x+y+1}) = x + c$
(e) $2\sqrt{x+y+1} - 2\ln(1+\sqrt{x+y+1}) = x + c$

4. Let $\mathbf{u} = (5,1)$, $\mathbf{v} = (-2,-1)$, $\mathbf{w} = (5,-2)$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a^2 + b^2 =$

(a) 8
(b) 25
(c) 36
(d) 34
(e) 16

5. Find a constant A that makes the differential equation $(2x+Ay)\,dx+(3x+2y)\,dy=0$ exact.

(a) -3
(b) -2
(c) 0
(d) 3
(e) 2

6. Find the value of k for which the vectors of $\mathbb{R}^3$

$$\mathbf{u}=(1,1,0),\ \mathbf{v}=(k,3,1),\ \mathbf{w}=(3,-2,-4)$$

are linearly dependent.

(a) $k=-\dfrac{3}{4}$
(b) $k=\dfrac{7}{4}$
(c) $k=0$
(d) $k=\dfrac{3}{4}$
(e) $k=2$

7. Find the general solution of the differential equation $\dfrac{dy}{dx} = xe^{-x}$.

(a) $y(x) = xe^{-x} + c$
(b) $y(x) = -xe^{-x} + x + c$
(c) $y(x) = -(x+1)e^{-x} + c$
(d) $y(x) = 2(x+1)e^{-x} + c$
(e) $y(x) = (x+2)e^{-x} + c$

8. Find a general solution of the exact differential equation

$$(1 + ye^{xy})\,dx + (2y + xe^{xy})\,dy = 0.$$

(a) $x - e^{xy} + y^2 = c$
(b) $x + e^{xy} - y^2 = c$
(c) $x^2 - e^{xy} + y^2 = c$
(d) $x^2 + e^{xy} + y^2 = c$
(e) $x + e^{xy} + y^2 = c$

9. Find a general solution of the differential equation $y'' = 2y(y')^3$.

(a) $y^3 + 2x + Ay + B = 0$
(b) $y^3 - 3x + Ay + B = 0$
(c) $y^2 + 3x + Ay + B = 0$
(d) $y^3 - 3x + Ay + B = 0$
(e) $y^3 + 3x + Ay + B = 0$

10. If $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^3$ and r and s are real numbers, then which one of the following statements is not true?

(a) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
(b) $(r + s)\mathbf{u} = r\mathbf{u} + s\mathbf{u}$
(c) $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|$
(d) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
(e) $r(\mathbf{u} + \mathbf{v}) = r\mathbf{u} + r\mathbf{v}$

11. Which one of the following statements is true about the subset V of $\mathbb{R}^3$ defined by

$$V = \{(x_1, x_2, x_3) : x_1 = 0\}$$

(a) V is closed under addition but not closed under multiplication by scalars

(b) V is not closed under addition

(c) V is not closed under addition but closed under multiplication by scalars

(d) V is not closed under multiplication by scalars

(e) V is a subspace of $\mathbb{R}^3$

12. A particle is moving in a straight line with acceleration $a(t) = 18\cos(3t)$, and initial position $x(0) = -7$, and an initial velocity $v(0) = 4$.

Find $x(\pi)$ (the position of the particle at $t = \pi$).

(a) $4\pi + 3$

(b) $4\pi - 7$

(c) $2\pi + 3$

(d) $4\pi - 3$

(e) $2\pi - 3$

13. Find a value of c such that $y = \tan(x^3 + c)$ is a solution of the initial value problem $y' = 3x^2(y^2 + 1)$, $y(0) = 1$.

(a) $\dfrac{3\pi}{4}$
(b) $-\dfrac{\pi}{4}$
(c) $\dfrac{5\pi}{4}$
(d) $\dfrac{\pi}{3}$
(e) $\dfrac{5\pi}{3}$

14. In a certain culture of bacteria, the number of bacteria doubled after 3 hours. How long did it take for the population to triple?

(a) $\dfrac{\ln 12}{\ln 2}$
(b) $\dfrac{\ln 27}{\ln 2}$
(c) $\dfrac{\ln 9}{\ln 2}$
(d) $\dfrac{\ln 3}{\ln 2}$
(e) $\dfrac{\ln 16}{2}$

15. Solve the initial-value problem $y' + y = 2,\ y(0) = 0$.

(a) $y(x) = 2 - e^{-x}$
(b) $y(x) = 1 - e^{-x}$
(c) $y(x) = 1 + e^{-x}$
(d) $y(x) = 2 - 2e^{-x}$
(e) $y(x) = 3 - 2e^{-x}$

16. Find the sum of all values of r such that $y = e^{rx}$ is a solution of the differential equation $5y'' - 6y' - y = 0$.

(a) $-\frac{4}{5}$
(b) $-\frac{6}{5}$
(c) $\frac{4}{5}$
(d) $\frac{3}{5}$
(e) $\frac{6}{5}$

17. Find a general solution of the differential equation $xy\dfrac{dy}{dx} = x^2 + 3y^2$.

(a) $2y^2 = cx^6 - x^2$
(b) $2y^2 = cx^5 - x^2$
(c) $2y^2 = cx^5 + x^2$
(d) $2y^2 = cx^4 + x^3$
(e) $2y^2 = cx^6 + x^2$

18. By making a suitable substitution, the differential equation $x^2y' + 2xy = 5y^3$ can be transformed into a linear differential equation

(a) $v' - \dfrac{4}{x}v = \dfrac{5}{x^2}$
(b) $v' - \dfrac{5}{x}v = \dfrac{10}{x^2}$
(c) $v' - \dfrac{3}{x}v = -\dfrac{5}{x^2}$
(d) $v' + \dfrac{3}{x}v = \dfrac{5}{x^2}$
(e) $v' - \dfrac{4}{x}v = -\dfrac{10}{x^2}$

19. Find a general solution of the separable differential equation $x^2\dfrac{dy}{dx} = 1-x^2+y^2-x^2y^2$.

(a) $y(x) = \tan\left(c + \dfrac{1}{x} + x\right)$

(b) $y(x) = \tan\left(c - \dfrac{1}{x} - x\right)$

(c) $y(x) = \tan\left(c - \dfrac{3}{x} + x\right)$

(d) $y(x) = \tan\left(c + \dfrac{2}{x} + x\right)$

(e) $y(x) = \tan\left(c - \dfrac{2}{x} + x\right)$

20. Find a general solution of the linear differential equation $x\dfrac{dy}{dx} = 2y + x^3\cos x$.

(a) $y(x) = x(\cos x + c)$

(b) $y(x) = x^3(\sin x + c)$

(c) $y(x) = x^2(\cos x + c)$

(d) $y(x) = x^2(\sin x + c)$

(e) $y(x) = x^3(\cos x + c)$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE04 **CODE04**

Math 208
Major Exam I
231
October 02, 2023
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The solution of the system

$$\begin{aligned} x_1 - 3x_2 - 7x_3 + 5x_4 &= 0 \\ 3x_1 + x_2 + 9x_3 - 5x_4 &= 0 \\ x_1 - 2x_2 - 4x_3 + 4x_4 &= 0 \end{aligned}$$

is the set of all scalars multiples of a vector $\mathbf{u}$ where $\mathbf{u} =$

(a) $(-2, -3, 1, 0)$
(b) $(-1, 3, 1, 0)$
(c) $(-2, 3, 1, 2)$
(d) $(-2, 3, 0, 1)$
(e) $(-1, -6, 1, 0)$

2. By making a suitable substitution, the differential equation $x^2y' + 2xy = 5y^3$ can be transformed into a linear differential equation

(a) $v' - \frac{3}{x}v = -\frac{5}{x^2}$
(b) $v' - \frac{5}{x}v = \frac{10}{x^2}$
(c) $v' - \frac{4}{x}v = \frac{5}{x^2}$
(d) $v' + \frac{3}{x}v = \frac{5}{x^2}$
(e) $v' - \frac{4}{x}v = -\frac{10}{x^2}$

3. Find a general solution of the linear differential equation $x\dfrac{dy}{dx} = 2y + x^3 \cos x$.

(a) $y(x) = x^3(\cos x + c)$
(b) $y(x) = x^2(\cos x + c)$
(c) $y(x) = x(\cos x + c)$
(d) $y(x) = x^3(\sin x + c)$
(e) $y(x) = x^2(\sin x + c)$

4. Find the sum of all values of r such that $y = e^{rx}$ is a solution of the differential equation $5y'' - 6y' - y = 0$.

(a) $\dfrac{3}{5}$
(b) $-\dfrac{4}{5}$
(c) $\dfrac{4}{5}$
(d) $-\dfrac{6}{5}$
(e) $\dfrac{6}{5}$

5. Find a general solution of the differential equation $y'' = 2y(y')^3$.

(a) $y^3 + 2x + Ay + B = 0$
(b) $y^3 + 3x + Ay + B = 0$
(c) $y^2 + 3x + Ay + B = 0$
(d) $y^3 - 3x + Ay + B = 0$
(e) $y^3 - 3x + Ay + B = 0$

6. Find the order of the differential equation

$$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^8 + \sin^4 x = 0.$$

(a) 5
(b) 8
(c) 3
(d) 1
(e) 4

7. Find the general solution of the differential equation $\dfrac{dy}{dx} = xe^{-x}$.

(a) $y(x) = xe^{-x} + c$
(b) $y(x) = 2(x+1)e^{-x} + c$
(c) $y(x) = -xe^{-x} + x + c$
(d) $y(x) = -(x+1)e^{-x} + c$
(e) $y(x) = (x+2)e^{-x} + c$

8. Find a general solution of the exact differential equation

$$(1 + ye^{xy})\,dx + (2y + xe^{xy})\,dy = 0.$$

(a) $x^2 - e^{xy} + y^2 = c$
(b) $x + e^{xy} - y^2 = c$
(c) $x - e^{xy} + y^2 = c$
(d) $x^2 + e^{xy} + y^2 = c$
(e) $x + e^{xy} + y^2 = c$

9. Find a constant A that makes the differential equation $(2x+Ay)\,dx+(3x+2y)\,dy=0$ exact.

(a) -3
(b) -2
(c) 3
(d) 0
(e) 2

10. In a certain culture of bacteria, the number of bacteria doubled after 3 hours. How long did it take for the population to triple?

(a) $\dfrac{\ln 9}{\ln 2}$
(b) $\dfrac{\ln 12}{\ln 2}$
(c) $\dfrac{\ln 27}{\ln 2}$
(d) $\dfrac{\ln 16}{2}$
(e) $\dfrac{\ln 3}{\ln 2}$

11. Which one of the following statements is true about the subset V of $\mathbb{R}^3$ defined by

$$V = \{(x_1, x_2, x_3) : x_1 = 0\}$$

(a) V is not closed under addition

(b) V is a subspace of $\mathbb{R}^3$

(c) V is not closed under addition but closed under multiplication by scalars

(d) V is closed under addition but not closed under multiplication by scalars

(e) V is not closed under multiplication by scalars

12. A particle is moving in a straight line with acceleration $a(t) = 18\cos(3t)$, and initial position $x(0) = -7$, and an initial velocity $v(0) = 4$.

Find $x(\pi)$ (the position of the particle at $t = \pi$).

(a) $4\pi - 7$

(b) $4\pi + 3$

(c) $2\pi - 3$

(d) $4\pi - 3$

(e) $2\pi + 3$

13. Find a general solution of the differential equation $xy\dfrac{dy}{dx} = x^2 + 3y^2$.

(a) $2y^2 = cx^4 + x^3$
(b) $2y^2 = cx^6 + x^2$
(c) $2y^2 = cx^5 + x^2$
(d) $2y^2 = cx^5 - x^2$
(e) $2y^2 = cx^6 - x^2$

14. Find a general solution of the separable differential equation $x^2\dfrac{dy}{dx} = 1 - x^2 + y^2 - x^2y^2$.

(a) $y(x) = \tan\left(c - \dfrac{3}{x} + x\right)$
(b) $y(x) = \tan\left(c - \dfrac{1}{x} - x\right)$
(c) $y(x) = \tan\left(c + \dfrac{2}{x} + x\right)$
(d) $y(x) = \tan\left(c + \dfrac{1}{x} + x\right)$
(e) $y(x) = \tan\left(c - \dfrac{2}{x} + x\right)$

15. Solve the initial-value problem $y' + y = 2,\ y(0) = 0$.

(a) $y(x) = 1 + e^{-x}$
(b) $y(x) = 2 - 2e^{-x}$
(c) $y(x) = 2 - e^{-x}$
(d) $y(x) = 1 - e^{-x}$
(e) $y(x) = 3 - 2e^{-x}$

16. Find a value of c such that $y = \tan(x^3 + c)$ is a solution of the initial value problem $y' = 3x^2(y^2 + 1)$, $y(0) = 1$.

(a) $\dfrac{3\pi}{4}$
(b) $\dfrac{\pi}{3}$
(c) $\dfrac{5\pi}{3}$
(d) $\dfrac{5\pi}{4}$
(e) $-\dfrac{\pi}{4}$

17. Find the value of k for which the vectors of $\mathbb{R}^3$

$$\mathbf{u} = (1, 1, 0),\ \mathbf{v} = (k, 3, 1),\ \mathbf{w} = (3, -2, -4)$$

are linearly dependent.

(a) $k = -\dfrac{3}{4}$
(b) $k = 0$
(c) $k = \dfrac{7}{4}$
(d) $k = \dfrac{3}{4}$
(e) $k = 2$

18. Find a general solution of the differential equation $y' = \sqrt{x + y + 1}$.

(a) $2\sqrt{x+y+1} + 2\ln(1 + \sqrt{x+y+1}) = -x + c$
(b) $3\sqrt{x+y+1} + 2\ln(1 + \sqrt{x+y+1}) = x + c$
(c) $2\sqrt{x+y+1} - 2\ln(1 + \sqrt{x+y+1}) = x + c$
(d) $3\sqrt{x+y+1} + 2\ln(1 + \sqrt{x+y+1}) = -x + c$
(e) $3\sqrt{x+y+1} - 2\ln(1 + \sqrt{x+y+1}) = x + c$

19. If $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^3$ and r and s are real numbers, then which one of the following statements is not true?

(a) $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|$

(b) $(r+s)\mathbf{u} = r\mathbf{u} + s\mathbf{u}$

(c) $(\mathbf{u}+\mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v}+\mathbf{w})$

(d) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(e) $r(\mathbf{u}+\mathbf{v}) = r\mathbf{u} + r\mathbf{v}$

20. Let $\mathbf{u} = (5,1)$, $\mathbf{v} = (-2,-1)$, $\mathbf{w} = (5,-2)$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a^2 + b^2 =$

(a) 36

(b) 34

(c) 16

(d) 25

(e) 8

Math 208, 231, Major Exam I **Answer KEY**

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	B 4	A 17	E 15	A 19
2	A	D 17	D 11	B 19	E 13
3	A	D 20	B 13	E 14	E 8
4	A	A 5	E 14	D 16	E 1
5	A	B 11	C 3	D 10	B 12
6	A	A 9	A 8	B 17	C 15
7	A	B 15	E 9	C 4	D 4
8	A	E 16	A 4	E 11	E 11
9	A	D 14	B 6	E 12	C 10
10	A	C 19	A 12	C 20	C 6
11	A	A 12	B 19	E 18	B 18
12	A	A 2	E 7	D 3	D 3
13	A	C 10	D 15	C 2	E 9
14	A	E 13	B 10	B 6	B 5
15	A	E 7	B 20	D 7	B 7
16	A	C 6	E 1	E 1	D 2
17	A	D 3	B 5	A 9	C 17
18	A	E 1	A 16	E 13	C 14
19	A	A 8	D 2	B 5	A 20
20	A	A 18	B 18	D 8	B 16

Answer Counts

V	A	B	C	D	E
1	6	3	3	4	4
2	5	7	1	3	4
3	1	4	3	5	7
4	2	5	5	3	5