King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208
Exam II
231
November 07, 2023

EXAM COVER

Number of versions: 4
Number of questions: 20

King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208

Exam II

231

November 07, 2023

Net Time Allowed: 120 Minutes

MASTER VERSION

1. A value of k for which the following vectors

$$v_1 = (k, 0, k),\ v_2 = (2k, -3, 4) \text{ and } v_3 = (3, 5, 2)$$

are linearly dependent is

(a) $\frac{17}{10}$ ____ (correct)
(b) $\frac{7}{5}$
(c) $\frac{-8}{5}$
(d) $\frac{-16}{5}$
(e) $\frac{10}{5}$

2. Consider the vectors

$$v_1 = (-1, 2, 3),\ v_2 = (3, 1, -2),\ v_3 = (2, 3, 0) \text{ and } w = (1, 5, 8).$$

If $w = av_1 + bv_2 + cv_3$, then $a + b - c =$

(a) 15 ____ (correct)
(b) 17
(c) 19
(d) 21
(e) 13

3. Consider the subspace S of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d)|a = b + c + d\}$. A basis of S is consisting of the vectors

(a) $v_1 = (1, 1, 0, 0)$, $v_2 = (1, 0, 1, 0)$, $v_3 = (1, 0, 0, 1)$ ________________ (correct)

(b) $v_1 = (1, 1, 1, 0)$, $v_2 = (0, 1, 1, 1)$, $v_3 = (0, 1, 1, 0)$

(c) $v_1 = (0, 1, 0, 0)$, $v_2 = (0, 0, 1, 0)$, $v_3 = (0, 0, 0, 1)$

(d) $v_1 = (1, 1, 1, 1)$, $v_2 = (0, 0, 1, 0)$, $v_3 = (1, 0, 0, 1)$

(e) $v_1 = (1, 1, 0, 0)$, $v_2 = (0, 0, 1, 1)$, $v_3 = (0, 1, 0, 1)$

4. Let the solution space of the system

$$\begin{aligned} x_1 - 2x_2 - 9x_3 + 7x_4 &= 0 \\ x_1 + x_2 + 3x_3 + 4x_4 &= 0 \\ x_1 + 4x_2 + 15x_3 + x_4 &= 0 \end{aligned}$$

has all linear combinations of the two vectors

$$u = (1, -4, 1, 0) \text{ and } v = (\alpha, 1, \beta, 1).$$

Then $\alpha + \beta =$

(a) -5 ________________ (correct)

(b) 5

(c) -3

(d) 3

(e) 0

5. The rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & -9 & 9 \\ 3 & 6 & 2 & 1 & 6 \\ 2 & 4 & 1 & 2 & 3 \\ 4 & 8 & 3 & 0 & 9 \end{bmatrix}$$

is

(a) 2 ____(correct)

(b) 3

(c) 1

(d) 5

(e) 4

6. The solution of the initial-value problem

$$y'' - y = 0, \;\; y(0) = 0 \;\text{ and }\; y'(0) = 5$$

is

(a) $y = \frac{5}{2}e^{x} - \frac{5}{2}e^{-x}$ ____(correct)

(b) $y = \frac{-5}{2}e^{x} + \frac{5}{2}e^{-x}$

(c) $y = e^{x} - e^{-x}$

(d) $y = e^{-x} - e^{x}$

(e) $y = \frac{3}{2}e^{x} - \frac{3}{2}e^{-x}$

7. The Wronskian of the functions

$$f(x) = 1,\ g(x) = x,\ h(x) = x^2$$

is

(a) 2 ____ (correct)

(b) 3

(c) 0

(d) 4

(e) -4

8. The general solution of the differential equation

$$4y'' - 12y' + 9y = 0$$

is

(a) $y = c_1 e^{\frac{3}{2}x} + c_2 x e^{\frac{3}{2}x}$ ____ (correct)

(b) $y = c_1 e^{x} + c_2 x e^{x}$

(c) $y = c_1 e^{\frac{3}{2}x} + c_2 x e^{-\frac{3}{2}x}$

(d) $y = c_1 e^{3x} + c_2 x e^{3x}$

(e) $y = c_1 e^{3x} + c_2 e^{-3x}$

9. The general solution of the differential equation

$$y^{(3)} + 3y'' - 4y = 0$$

is

(a) $y = c_1e^{x} + c_2e^{-2x} + c_3xe^{-2x}$ ______________________________ (correct)
(b) $y = c_1e^{-x} + c_2e^{-2x} + c_3xe^{-2x}$
(c) $y = c_1e^{x} + c_2e^{2x} + c_3xe^{2x}$
(d) $y = c_1e^{-x} + c_2e^{2x} + c_3xe^{2x}$
(e) $y = c_1e^{x} + c_2e^{3x} + c_3xe^{3x}$

10. The general solution of the differential equation

$$y^{(4)} + 18y'' + 81y = 0$$

is

(a) $y = (c_1 + c_2x)\cos(3x) + (c_3 + c_4x)\sin(3x)$ ______________________ (correct)
(b) $y = (c_1 + c_2x)\cos(2x) + (c_3 + c_4x)\sin(2x)$
(c) $y = (c_1 + c_2x)e^{x}\cos(3x) + (c_3 + c_4x)e^{x}\sin(3x)$
(d) $y = (c_1 + c_2x)e^{x}\cos(2x) + (c_3 + c_4x)e^{x}\sin(2x)$
(e) $y = c_1\cos(3x) + c_2\sin(3x) + c_3\cos(2x) + c_4\sin(2x)$

11. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = e^{-4x}(c_1 \cos(3x) + c_2 \sin(3x))$$

is

(a) $y'' + 8y' + 25y = 0$ ________________ (correct)
(b) $y'' - 8y' + 25y = 0$
(c) $y'' + 6y' + 25y = 0$
(d) $y'' - 8y' - 25y = 0$
(e) $y'' - 6y' + 25y = 0$

12. If $y_p = A + Bx$ is a particular solution of the differential equation

$$y'' - y' - 2y = 3x + 4, \text{ then } 4A + 4B =$$

(a) -11 ________________ (correct)
(b) -10
(c) 12
(d) 9
(e) 0

13. An appropriate form of a particular solution y_p for the non-homogeneous differential equation $y'' + 4y = 3x\ \cos(2x)$ is given by $y_p(x) =$

(a) $(Ax + Bx^2)\ \cos(2x) + (Cx + Dx^2)\ \sin(2x)$ ________________ (correct)

(b) $(Ax + Bx)\ \cos(2x) + (C + Dx)\ \sin(2x)$

(c) $(Ax + Bx^2)\ \cos(2x) + (C + Dx)\ \sin(2x)$

(d) $(A + Bx)\ \cos(2x) + (Cx + Dx^2)\ \sin(2x)$

(e) $(Ax^2 + Bx^3)\ \cos(2x) + (C + Dx)\ \sin(2x)$

14. Given that $y_p = u_1(x)\,(\cos(3x)) + u_2(x)\,(\sin(3x))$ is a particular solution of the differential equation

$$y'' + 9y = 2\sec(3x), \text{ then } u_1(x) =$$

(a) $\frac{2}{9}\ln|\cos(3x)|$ ________________ (correct)

(b) $\frac{1}{9}\ln|\cos(3x)|$

(c) $\frac{2}{7}\ln|\cos(3x)|$

(d) $\frac{1}{3}\ln|\cos(3x)|$

(e) $\frac{1}{7}\ln|\cos(3x)|$

15. The characteristic polynomial of the matrix $\begin{bmatrix} 3 & 5 & -2 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ is $p(\lambda) =$

(a) $-\lambda^3 + 6\lambda^2 - 11\lambda + 6$ ________________ (correct)

(b) $\lambda^3 - 5\lambda^2 + 11\lambda + 6$

(c) $\lambda^3 - 6\lambda^2 + 9\lambda + 6$

(d) $\lambda^3 + 4\lambda^2 - 2\lambda + 6$

(e) $\lambda^3 + 6\lambda^2 - 11\lambda + 4$

16. The eigenvector associated with the eigenvalue $\lambda = -1$ of the matrix $A = \begin{bmatrix} 7 & -8 \\ 6 & -7 \end{bmatrix}$ is $\begin{bmatrix} a \\ 1 \end{bmatrix}$, where $a =$

(a) 1 ________________ (correct)

(b) -1

(c) 2

(d) -2

(e) 0

17. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 5 & -6 & 3 \\ 6 & -7 & 3 \\ 6 & -6 & 2 \end{bmatrix}$ is $p(\lambda) = -(\lambda+1)^2(\lambda-2)$, then a basis for the eigenspace of $\lambda = -1$ is $v_1 = \begin{bmatrix} m \\ 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 0 \\ n \end{bmatrix}$, where $m + n =$

(a) 3 ____ (correct)
(b) 4
(c) 5
(d) 0
(e) 1

18. If the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & -9 \\ 2 & 5 & m \end{bmatrix}$ is equal to 2, then $m =$

(a) 2 ____ (correct)
(b) 5
(c) -5
(d) 4
(e) 3

19. Which one of the following set of functions are linearly dependent

(a) $y_1(x) = 1,\ y_2(x) = x,\ y_3(x) = 2x + 3$ ________________(correct)

(b) $y_1(x) = 1,\ y_2(x) = x,\ y_3(x) = x^2$

(c) $y_1(x) = 2,\ y_2(x) = 3x,\ y_3(x) = x^3$

(d) $y_1(x) = x,\ y_2(x) = x^2,\ y_3(x) = x^3$

(e) $y_1(x) = x,\ y_2(x) = x^2 + x,\ y_3(x) = x^3 - 1$

20. If the matrix $A = \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix},\ D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ ________________(correct)

(b) $P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix},\ D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(c) $P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix},\ D = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix},\ D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix},\ D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

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Department of Mathematics

CODE01 **CODE01**

Math 208
Exam II
231
November 07, 2023
Net Time Allowed: 120 Minutes

Name	

ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

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2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & -9 \\ 2 & 5 & m \end{bmatrix}$ is equal to 2, then $m =$

   (a) -5
   (b) 5
   (c) 2
   (d) 3
   (e) 4

2. Consider the subspace S of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d) | a = b + c + d\}$. A basis of S is consisting of the vectors

   (a) $v_1 = (1, 1, 0, 0)$, $v_2 = (0, 0, 1, 1)$, $v_3 = (0, 1, 0, 1)$
   (b) $v_1 = (0, 1, 0, 0)$, $v_2 = (0, 0, 1, 0)$, $v_3 = (0, 0, 0, 1)$
   (c) $v_1 = (1, 1, 1, 1)$, $v_2 = (0, 0, 1, 0)$, $v_3 = (1, 0, 0, 1)$
   (d) $v_1 = (1, 1, 0, 0)$, $v_2 = (1, 0, 1, 0)$, $v_3 = (1, 0, 0, 1)$
   (e) $v_1 = (1, 1, 1, 0)$, $v_2 = (0, 1, 1, 1)$, $v_3 = (0, 1, 1, 0)$

3. The rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & -9 & 9 \\ 3 & 6 & 2 & 1 & 6 \\ 2 & 4 & 1 & 2 & 3 \\ 4 & 8 & 3 & 0 & 9 \end{bmatrix}$$

is

(a) 4
(b) 3
(c) 2
(d) 1
(e) 5

4. The solution of the initial-value problem

$$y'' - y = 0, \ \ y(0) = 0 \ \text{ and } \ y'(0) = 5$$

is

(a) $y = e^{-x} - e^{x}$
(b) $y = e^{x} - e^{-x}$
(c) $y = \dfrac{-5}{2}e^{x} + \dfrac{5}{2}e^{-x}$
(d) $y = \dfrac{3}{2}e^{x} - \dfrac{3}{2}e^{-x}$
(e) $y = \dfrac{5}{2}e^{x} - \dfrac{5}{2}e^{-x}$

5. The Wronskian of the functions

$$f(x) = 1,\ g(x) = x,\ h(x) = x^2$$

is

(a) 3
(b) 2
(c) -4
(d) 0
(e) 4

6. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = e^{-4x}(c_1 \cos(3x) + c_2 \sin(3x))$$

is

(a) $y'' - 8y' - 25y = 0$
(b) $y'' - 8y' + 25y = 0$
(c) $y'' - 6y' + 25y = 0$
(d) $y'' + 6y' + 25y = 0$
(e) $y'' + 8y' + 25y = 0$

7. Which one of the following set of functions are linearly dependent

(a) $y_1(x) = x,\ y_2(x) = x^2,\ y_3(x) = x^3$

(b) $y_1(x) = 1,\ y_2(x) = x,\ y_3(x) = 2x + 3$

(c) $y_1(x) = x,\ y_2(x) = x^2 + x,\ y_3(x) = x^3 - 1$

(d) $y_1(x) = 1,\ y_2(x) = x,\ y_3(x) = x^2$

(e) $y_1(x) = 2,\ y_2(x) = 3x,\ y_3(x) = x^3$

8. The general solution of the differential equation

$$y^{(4)} + 18y'' + 81y = 0$$

is

(a) $y = c_1 \cos(3x) + c_2 \sin(3x) + c_3 \cos(2x) + c_4 \sin(2x)$

(b) $y = (c_1 + c_2x)e^x \cos(2x) + (c_3 + c_4x)e^x \sin(2x)$

(c) $y = (c_1 + c_2x)e^x \cos(3x) + (c_3 + c_4x)e^x \sin(3x)$

(d) $y = (c_1 + c_2x) \cos(2x) + (c_3 + c_4x) \sin(2x)$

(e) $y = (c_1 + c_2x) \cos(3x) + (c_3 + c_4x) \sin(3x)$

9. Let the solution space of the system

$$\begin{aligned} x_1 - 2x_2 - 9x_3 + 7x_4 &= 0 \\ x_1 + x_2 + 3x_3 + 4x_4 &= 0 \\ x_1 + 4x_2 + 15x_3 + x_4 &= 0 \end{aligned}$$

has all linear combinations of the two vectors

$$u = (1, -4, 1, 0) \text{ and } v = (\alpha, 1, \beta, 1).$$

Then $\alpha + \beta =$

(a) 5
(b) -5
(c) 0
(d) -3
(e) 3

10. The general solution of the differential equation

$$4y'' - 12y' + 9y = 0$$

is

(a) $y = c_1 e^{3x} + c_2 e^{-3x}$
(b) $y = c_1 e^{\frac{3}{2}x} + c_2 x e^{-\frac{3}{2}x}$
(c) $y = c_1 e^{\frac{3}{2}x} + c_2 x e^{\frac{3}{2}x}$
(d) $y = c_1 e^{3x} + c_2 x e^{3x}$
(e) $y = c_1 e^{x} + c_2 x e^{x}$

11. A value of k for which the following vectors

$$v_1 = (k, 0, k),\ v_2 = (2k, -3, 4) \text{ and } v_3 = (3, 5, 2)$$

are linearly dependent is

(a) $\frac{7}{5}$

(b) $\frac{-8}{5}$

(c) $\frac{-16}{5}$

(d) $\frac{17}{10}$

(e) $\frac{10}{5}$

12. The eigenvector associated with the eigenvalue $\lambda = -1$ of the matrix $A = \begin{bmatrix} 7 & -8 \\ 6 & -7 \end{bmatrix}$ is $\begin{bmatrix} a \\ 1 \end{bmatrix}$, where $a =$

(a) -2

(b) 1

(c) -1

(d) 2

(e) 0

13. If $y_p = A + Bx$ is a particular solution of the differential equation

$$y'' - y' - 2y = 3x + 4, \text{ then } 4A + 4B =$$

(a) -10
(b) 12
(c) 0
(d) 9
(e) -11

14. Consider the vectors

$$v_1 = (-1, 2, 3),\ v_2 = (3, 1, -2),\ v_3 = (2, 3, 0) \text{ and } w = (1, 5, 8).$$

If $w = av_1 + bv_2 + cv_3$, then $a + b - c =$

(a) 21
(b) 19
(c) 13
(d) 17
(e) 15

15. If the matrix $A = \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(b) $P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(e) $P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$

16. Given that $y_p = u_1(x)\,(\cos(3x)) + u_2(x)\,(\sin(3x))$ is a particular solution of the differential equation

$y'' + 9y = 2\sec(3x)$, then $u_1(x) =$

(a) $\frac{1}{7}\ln|\cos(3x)|$

(b) $\frac{1}{3}\ln|\cos(3x)|$

(c) $\frac{2}{7}\ln|\cos(3x)|$

(d) $\frac{1}{9}\ln|\cos(3x)|$

(e) $\frac{2}{9}\ln|\cos(3x)|$

17. The characteristic polynomial of the matrix $\begin{bmatrix} 3 & 5 & -2 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ is $p(\lambda) =$

(a) $\lambda^3 + 4\lambda^2 - 2\lambda + 6$
(b) $\lambda^3 - 6\lambda^2 + 9\lambda + 6$
(c) $\lambda^3 - 5\lambda^2 + 11\lambda + 6$
(d) $-\lambda^3 + 6\lambda^2 - 11\lambda + 6$
(e) $\lambda^3 + 6\lambda^2 - 11\lambda + 4$

18. An appropriate form of a particular solution y_p for the non-homogeneous differential equation $y'' + 4y = 3x\,\cos(2x)$ is given by $y_p(x) =$

(a) $(Ax + Bx)\,\cos(2x) + (C + Dx)\,\sin(2x)$
(b) $(Ax + Bx^2)\,\cos(2x) + (C + Dx)\,\sin(2x)$
(c) $(A + Bx)\,\cos(2x) + (Cx + Dx^2)\,\sin(2x)$
(d) $(Ax^2 + Bx^3)\,\cos(2x) + (C + Dx)\,\sin(2x)$
(e) $(Ax + Bx^2)\,\cos(2x) + (Cx + Dx^2)\,\sin(2x)$

19. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 5 & -6 & 3 \\ 6 & -7 & 3 \\ 6 & -6 & 2 \end{bmatrix}$ is $p(\lambda) = -(\lambda+1)^2(\lambda-2)$, then a basis for the eigenspace of $\lambda = -1$ is $v_1 = \begin{bmatrix} m \\ 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 0 \\ n \end{bmatrix}$, where $m + n =$

(a) 5
(b) 0
(c) 1
(d) 4
(e) 3

20. The general solution of the differential equation

$$y^{(3)} + 3y'' - 4y = 0$$

is

(a) $y = c_1e^{-x} + c_2e^{2x} + c_3xe^{2x}$
(b) $y = c_1e^{x} + c_2e^{-2x} + c_3xe^{-2x}$
(c) $y = c_1e^{x} + c_2e^{3x} + c_3xe^{3x}$
(d) $y = c_1e^{x} + c_2e^{2x} + c_3xe^{2x}$
(e) $y = c_1e^{-x} + c_2e^{-2x} + c_3xe^{-2x}$

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CODE02 **CODE02**

Math 208
Exam II
231
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Name	
ID		**Sec**	

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1. Consider the subspace S of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d) | a = b + c + d\}$. A basis of S is consisting of the vectors

   (a) $v_1 = (0, 1, 0, 0)$, $v_2 = (0, 0, 1, 0)$, $v_3 = (0, 0, 0, 1)$
   (b) $v_1 = (1, 1, 1, 1)$, $v_2 = (0, 0, 1, 0)$, $v_3 = (1, 0, 0, 1)$
   (c) $v_1 = (1, 1, 0, 0)$, $v_2 = (1, 0, 1, 0)$, $v_3 = (1, 0, 0, 1)$
   (d) $v_1 = (1, 1, 1, 0)$, $v_2 = (0, 1, 1, 1)$, $v_3 = (0, 1, 1, 0)$
   (e) $v_1 = (1, 1, 0, 0)$, $v_2 = (0, 0, 1, 1)$, $v_3 = (0, 1, 0, 1)$

2. Let the solution space of the system

$$\begin{aligned} x_1 - 2x_2 - 9x_3 + 7x_4 &= 0 \\ x_1 + x_2 + 3x_3 + 4x_4 &= 0 \\ x_1 + 4x_2 + 15x_3 + x_4 &= 0 \end{aligned}$$

   has all linear combinations of the two vectors

$$u = (1, -4, 1, 0) \text{ and } v = (\alpha, 1, \beta, 1).$$

   Then $\alpha + \beta =$

   (a) 5
   (b) 3
   (c) -5
   (d) -3
   (e) 0

3. Given that $y_p = u_1(x)\,(\cos(3x)) + u_2(x)\,(\sin(3x))$ is a particular solution of the differential equation

$$y'' + 9y = 2\sec(3x), \text{ then } u_1(x) =$$

(a) $\frac{1}{7}\ln|\cos(3x)|$
(b) $\frac{1}{3}\ln|\cos(3x)|$
(c) $\frac{2}{7}\ln|\cos(3x)|$
(d) $\frac{2}{9}\ln|\cos(3x)|$
(e) $\frac{1}{9}\ln|\cos(3x)|$

4. Which one of the following set of functions are linearly dependent

(a) $y_1(x) = 2,\ y_2(x) = 3x,\ y_3(x) = x^3$
(b) $y_1(x) = 1,\ y_2(x) = x,\ y_3(x) = 2x + 3$
(c) $y_1(x) = x,\ y_2(x) = x^2 + x,\ y_3(x) = x^3 - 1$
(d) $y_1(x) = x,\ y_2(x) = x^2,\ y_3(x) = x^3$
(e) $y_1(x) = 1,\ y_2(x) = x,\ y_3(x) = x^2$

5. If the matrix $A = \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(d) $P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(e) $P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$

6. The general solution of the differential equation

$$4y'' - 12y' + 9y = 0$$

is

(a) $y = c_1e^{x} + c_2xe^{x}$

(b) $y = c_1e^{\frac{3}{2}x} + c_2xe^{-\frac{3}{2}x}$

(c) $y = c_1e^{3x} + c_2e^{-3x}$

(d) $y = c_1e^{3x} + c_2xe^{3x}$

(e) $y = c_1e^{\frac{3}{2}x} + c_2xe^{\frac{3}{2}x}$

7. If $y_p = A + Bx$ is a particular solution of the differential equation

$$y'' - y' - 2y = 3x + 4, \text{ then } 4A + 4B =$$

(a) 12
(b) -10
(c) 0
(d) 9
(e) -11

8. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 5 & -6 & 3 \\ 6 & -7 & 3 \\ 6 & -6 & 2 \end{bmatrix}$ is

$p(\lambda) = -(\lambda+1)^2(\lambda-2)$, then a basis for the eigenspace of $\lambda = -1$ is $v_1 = \begin{bmatrix} m \\ 1 \\ 0 \end{bmatrix}$

and $v_2 = \begin{bmatrix} -1 \\ 0 \\ n \end{bmatrix}$, where $m + n =$

(a) 5
(b) 3
(c) 4
(d) 0
(e) 1

9. The eigenvector associated with the eigenvalue $\lambda = -1$ of the matrix $A = \begin{bmatrix} 7 & -8 \\ 6 & -7 \end{bmatrix}$ is $\begin{bmatrix} a \\ 1 \end{bmatrix}$, where $a =$

   (a) -2
   (b) 2
   (c) 0
   (d) 1
   (e) -1

10. The general solution of the differential equation

$$y^{(3)} + 3y'' - 4y = 0$$

is

   (a) $y = c_1 e^{-x} + c_2 e^{2x} + c_3 x e^{2x}$
   (b) $y = c_1 e^{x} + c_2 e^{3x} + c_3 x e^{3x}$
   (c) $y = c_1 e^{x} + c_2 e^{-2x} + c_3 x e^{-2x}$
   (d) $y = c_1 e^{-x} + c_2 e^{-2x} + c_3 x e^{-2x}$
   (e) $y = c_1 e^{x} + c_2 e^{2x} + c_3 x e^{2x}$

11. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = e^{-4x}(c_1 \cos(3x) + c_2 \sin(3x))$$

is

(a) $y'' - 8y' + 25y = 0$
(b) $y'' + 8y' + 25y = 0$
(c) $y'' - 6y' + 25y = 0$
(d) $y'' - 8y' - 25y = 0$
(e) $y'' + 6y' + 25y = 0$

12. The rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & -9 & 9 \\ 3 & 6 & 2 & 1 & 6 \\ 2 & 4 & 1 & 2 & 3 \\ 4 & 8 & 3 & 0 & 9 \end{bmatrix}$$

is

(a) 3
(b) 2
(c) 5
(d) 1
(e) 4

13. An appropriate form of a particular solution y_p for the non-homogeneous differential equation $y'' + 4y = 3x\,\cos(2x)$ is given by $y_p(x) =$

(a) $(Ax^2 + Bx^3)\,\cos(2x) + (C + Dx)\,\sin(2x)$
(b) $(Ax + Bx^2)\,\cos(2x) + (C + Dx)\,\sin(2x)$
(c) $(Ax + Bx^2)\,\cos(2x) + (Cx + Dx^2)\,\sin(2x)$
(d) $(Ax + Bx)\,\cos(2x) + (C + Dx)\,\sin(2x)$
(e) $(A + Bx)\,\cos(2x) + (Cx + Dx^2)\,\sin(2x)$

14. If the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & -9 \\ 2 & 5 & m \end{bmatrix}$ is equal to 2, then $m =$

(a) -5
(b) 4
(c) 3
(d) 2
(e) 5

15. The solution of the initial-value problem

$$y'' - y = 0, \;\; y(0) = 0 \text{ and } y'(0) = 5$$

is

(a) $y = \frac{5}{2}e^x - \frac{5}{2}e^{-x}$

(b) $y = \frac{3}{2}e^x - \frac{3}{2}e^{-x}$

(c) $y = e^{-x} - e^x$

(d) $y = \frac{-5}{2}e^x + \frac{5}{2}e^{-x}$

(e) $y = e^x - e^{-x}$

16. Consider the vectors

$$v_1 = (-1, 2, 3),\; v_2 = (3, 1, -2),\; v_3 = (2, 3, 0) \text{ and } w = (1, 5, 8).$$

If $w = av_1 + bv_2 + cv_3$, then $a + b - c =$

(a) 15

(b) 17

(c) 13

(d) 19

(e) 21

17. A value of k for which the following vectors

$$v_1 = (k, 0, k),\ v_2 = (2k, -3, 4) \text{ and } v_3 = (3, 5, 2)$$

are linearly dependent is

(a) $\frac{-16}{5}$
(b) $\frac{7}{5}$
(c) $\frac{17}{10}$
(d) $\frac{-8}{5}$
(e) $\frac{10}{5}$

18. The Wronskian of the functions

$$f(x) = 1,\ g(x) = x,\ h(x) = x^2$$

is

(a) 0
(b) 4
(c) 3
(d) -4
(e) 2

19. The general solution of the differential equation

$$y^{(4)} + 18y'' + 81y = 0$$

is

(a) $y = c_1 \cos(3x) + c_2 \sin(3x) + c_3 \cos(2x) + c_4 \sin(2x)$
(b) $y = (c_1 + c_2 x)e^x \cos(2x) + (c_3 + c_4 x)e^x \sin(2x)$
(c) $y = (c_1 + c_2 x) \cos(3x) + (c_3 + c_4 x) \sin(3x)$
(d) $y = (c_1 + c_2 x) \cos(2x) + (c_3 + c_4 x) \sin(2x)$
(e) $y = (c_1 + c_2 x)e^x \cos(3x) + (c_3 + c_4 x)e^x \sin(3x)$

20. The characteristic polynomial of the matrix $\begin{bmatrix} 3 & 5 & -2 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ is $p(\lambda) =$

(a) $-\lambda^3 + 6\lambda^2 - 11\lambda + 6$
(b) $\lambda^3 + 4\lambda^2 - 2\lambda + 6$
(c) $\lambda^3 + 6\lambda^2 - 11\lambda + 4$
(d) $\lambda^3 - 5\lambda^2 + 11\lambda + 6$
(e) $\lambda^3 - 6\lambda^2 + 9\lambda + 6$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE03 **CODE03**

Math 208
Exam II
231
November 07, 2023
Net Time Allowed: 120 Minutes

Name	

ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The general solution of the differential equation

$$y^{(4)} + 18y'' + 81y = 0$$

is

(a) $y = (c_1 + c_2x)\cos(3x) + (c_3 + c_4x)\sin(3x)$
(b) $y = (c_1 + c_2x)e^x\cos(2x) + (c_3 + c_4x)e^x\sin(2x)$
(c) $y = (c_1 + c_2x)\cos(2x) + (c_3 + c_4x)\sin(2x)$
(d) $y = c_1\cos(3x) + c_2\sin(3x) + c_3\cos(2x) + c_4\sin(2x)$
(e) $y = (c_1 + c_2x)e^x\cos(3x) + (c_3 + c_4x)e^x\sin(3x)$

2. The eigenvector associated with the eigenvalue $\lambda = -1$ of the matrix $A = \begin{bmatrix} 7 & -8 \\ 6 & -7 \end{bmatrix}$ is $\begin{bmatrix} a \\ 1 \end{bmatrix}$, where $a =$

(a) -1
(b) -2
(c) 1
(d) 0
(e) 2

3. A value of k for which the following vectors

$$v_1 = (k, 0, k),\ v_2 = (2k, -3, 4) \text{ and } v_3 = (3, 5, 2)$$

are linearly dependent is

(a) $\frac{7}{5}$
(b) $\frac{-16}{5}$
(c) $\frac{10}{5}$
(d) $\frac{-8}{5}$
(e) $\frac{17}{10}$

4. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 5 & -6 & 3 \\ 6 & -7 & 3 \\ 6 & -6 & 2 \end{bmatrix}$ is $p(\lambda) = -(\lambda+1)^2(\lambda-2)$, then a basis for the eigenspace of $\lambda = -1$ is $v_1 = \begin{bmatrix} m \\ 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 0 \\ n \end{bmatrix}$, where $m + n =$

(a) 3
(b) 5
(c) 4
(d) 0
(e) 1

5. The general solution of the differential equation

$$4y'' - 12y' + 9y = 0$$

is

(a) $y = c_1 e^{x} + c_2 x e^{x}$
(b) $y = c_1 e^{\frac{3}{2}x} + c_2 x e^{\frac{3}{2}x}$
(c) $y = c_1 e^{3x} + c_2 x e^{3x}$
(d) $y = c_1 e^{3x} + c_2 e^{-3x}$
(e) $y = c_1 e^{\frac{3}{2}x} + c_2 x e^{-\frac{3}{2}x}$

6. If the matrix $A = \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}, \ D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
(b) $P = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}, \ D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
(c) $P = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}, \ D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
(d) $P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, \ D = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$
(e) $P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, \ D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

7. The characteristic polynomial of the matrix $\begin{bmatrix} 3 & 5 & -2 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ is $p(\lambda) =$

(a) $-\lambda^3 + 6\lambda^2 - 11\lambda + 6$
(b) $\lambda^3 + 6\lambda^2 - 11\lambda + 4$
(c) $\lambda^3 - 6\lambda^2 + 9\lambda + 6$
(d) $\lambda^3 + 4\lambda^2 - 2\lambda + 6$
(e) $\lambda^3 - 5\lambda^2 + 11\lambda + 6$

8. Consider the subspace S of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d) | a = b + c + d\}$. A basis of S is consisting of the vectors

(a) $v_1 = (1, 1, 1, 0)$, $v_2 = (0, 1, 1, 1)$, $v_3 = (0, 1, 1, 0)$
(b) $v_1 = (1, 1, 0, 0)$, $v_2 = (0, 0, 1, 1)$, $v_3 = (0, 1, 0, 1)$
(c) $v_1 = (1, 1, 0, 0)$, $v_2 = (1, 0, 1, 0)$, $v_3 = (1, 0, 0, 1)$
(d) $v_1 = (1, 1, 1, 1)$, $v_2 = (0, 0, 1, 0)$, $v_3 = (1, 0, 0, 1)$
(e) $v_1 = (0, 1, 0, 0)$, $v_2 = (0, 0, 1, 0)$, $v_3 = (0, 0, 0, 1)$

9. Let the solution space of the system

$$\begin{aligned} x_1 - 2x_2 - 9x_3 + 7x_4 &= 0 \\ x_1 + x_2 + 3x_3 + 4x_4 &= 0 \\ x_1 + 4x_2 + 15x_3 + x_4 &= 0 \end{aligned}$$

has all linear combinations of the two vectors

$$u = (1, -4, 1, 0) \text{ and } v = (\alpha, 1, \beta, 1).$$

Then $\alpha + \beta =$

(a) -5
(b) 5
(c) 0
(d) -3
(e) 3

10. The Wronskian of the functions

$$f(x) = 1,\ g(x) = x,\ h(x) = x^2$$

is

(a) -4
(b) 4
(c) 3
(d) 0
(e) 2

11. If $y_p = A + Bx$ is a particular solution of the differential equation

$$y'' - y' - 2y = 3x + 4, \text{ then } 4A + 4B =$$

(a) 9

(b) 0

(c) -10

(d) 12

(e) -11

12. An appropriate form of a particular solution y_p for the non-homogeneous differential equation $y'' + 4y = 3x\cos(2x)$ is given by $y_p(x) =$

(a) $(Ax^2 + Bx^3)\cos(2x) + (C + Dx)\sin(2x)$

(b) $(Ax + Bx^2)\cos(2x) + (Cx + Dx^2)\sin(2x)$

(c) $(Ax + Bx^2)\cos(2x) + (C + Dx)\sin(2x)$

(d) $(A + Bx)\cos(2x) + (Cx + Dx^2)\sin(2x)$

(e) $(Ax + Bx)\cos(2x) + (C + Dx)\sin(2x)$

13. Given that $y_p = u_1(x)\,(\cos(3x)) + u_2(x)\,(\sin(3x))$ is a particular solution of the differential equation

$$y'' + 9y = 2\sec(3x), \text{ then } u_1(x) =$$

(a) $\frac{1}{9}\ln|\cos(3x)|$

(b) $\frac{1}{7}\ln|\cos(3x)|$

(c) $\frac{1}{3}\ln|\cos(3x)|$

(d) $\frac{2}{9}\ln|\cos(3x)|$

(e) $\frac{2}{7}\ln|\cos(3x)|$

14. The rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & -9 & 9 \\ 3 & 6 & 2 & 1 & 6 \\ 2 & 4 & 1 & 2 & 3 \\ 4 & 8 & 3 & 0 & 9 \end{bmatrix}$$

is

(a) 2

(b) 5

(c) 3

(d) 4

(e) 1

15. If the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & -9 \\ 2 & 5 & m \end{bmatrix}$ is equal to 2, then $m =$

(a) 3

(b) 2

(c) 4

(d) -5

(e) 5

16. Which one of the following set of functions are linearly dependent

(a) $y_1(x) = x,\ y_2(x) = x^2,\ y_3(x) = x^3$

(b) $y_1(x) = 1,\ y_2(x) = x,\ y_3(x) = x^2$

(c) $y_1(x) = x,\ y_2(x) = x^2 + x,\ y_3(x) = x^3 - 1$

(d) $y_1(x) = 1,\ y_2(x) = x,\ y_3(x) = 2x + 3$

(e) $y_1(x) = 2,\ y_2(x) = 3x,\ y_3(x) = x^3$

17. The general solution of the differential equation

$$y^{(3)} + 3y'' - 4y = 0$$

is

(a) $y = c_1e^{-x} + c_2e^{-2x} + c_3xe^{-2x}$
(b) $y = c_1e^{x} + c_2e^{3x} + c_3xe^{3x}$
(c) $y = c_1e^{x} + c_2e^{2x} + c_3xe^{2x}$
(d) $y = c_1e^{-x} + c_2e^{2x} + c_3xe^{2x}$
(e) $y = c_1e^{x} + c_2e^{-2x} + c_3xe^{-2x}$

18. The solution of the initial-value problem

$$y'' - y = 0, \;\; y(0) = 0 \text{ and } y'(0) = 5$$

is

(a) $y = \frac{-5}{2}e^{x} + \frac{5}{2}e^{-x}$
(b) $y = \frac{5}{2}e^{x} - \frac{5}{2}e^{-x}$
(c) $y = e^{x} - e^{-x}$
(d) $y = \frac{3}{2}e^{x} - \frac{3}{2}e^{-x}$
(e) $y = e^{-x} - e^{x}$

19. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = e^{-4x}(c_1 \cos(3x) + c_2 \sin(3x))$$

is

(a) $y'' + 8y' + 25y = 0$
(b) $y'' - 8y' - 25y = 0$
(c) $y'' - 6y' + 25y = 0$
(d) $y'' + 6y' + 25y = 0$
(e) $y'' - 8y' + 25y = 0$

20. Consider the vectors

$$v_1 = (-1, 2, 3),\ v_2 = (3, 1, -2),\ v_3 = (2, 3, 0) \text{ and } w = (1, 5, 8).$$

If $w = av_1 + bv_2 + cv_3$, then $a + b - c =$

(a) 17
(b) 21
(c) 13
(d) 19
(e) 15

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE04		CODE04

Math 208
Exam II
231
November 07, 2023
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
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3. Use a good eraser. DO NOT use the erasers attached to the pencil.
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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The characteristic polynomial of the matrix $\begin{bmatrix} 3 & 5 & -2 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ is $p(\lambda) =$

(a) $\lambda^3 + 4\lambda^2 - 2\lambda + 6$
(b) $\lambda^3 - 6\lambda^2 + 9\lambda + 6$
(c) $-\lambda^3 + 6\lambda^2 - 11\lambda + 6$
(d) $\lambda^3 - 5\lambda^2 + 11\lambda + 6$
(e) $\lambda^3 + 6\lambda^2 - 11\lambda + 4$

2. If the matrix $A = \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$
(b) $P = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
(c) $P = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
(d) $P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
(e) $P = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

3. The rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & -9 & 9 \\ 3 & 6 & 2 & 1 & 6 \\ 2 & 4 & 1 & 2 & 3 \\ 4 & 8 & 3 & 0 & 9 \end{bmatrix}$$

is

(a) 1
(b) 4
(c) 3
(d) 5
(e) 2

4. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 5 & -6 & 3 \\ 6 & -7 & 3 \\ 6 & -6 & 2 \end{bmatrix}$ is $p(\lambda) = -(\lambda+1)^2(\lambda-2)$, then a basis for the eigenspace of $\lambda = -1$ is $v_1 = \begin{bmatrix} m \\ 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 0 \\ n \end{bmatrix}$, where $m + n =$

(a) 4
(b) 1
(c) 0
(d) 5
(e) 3

5. Which one of the following set of functions are linearly dependent

(a) $y_1(x) = x,\ y_2(x) = x^2,\ y_3(x) = x^3$
(b) $y_1(x) = 1,\ y_2(x) = x,\ y_3(x) = x^2$
(c) $y_1(x) = 1,\ y_2(x) = x,\ y_3(x) = 2x + 3$
(d) $y_1(x) = x,\ y_2(x) = x^2 + x,\ y_3(x) = x^3 - 1$
(e) $y_1(x) = 2,\ y_2(x) = 3x,\ y_3(x) = x^3$

6. The general solution of the differential equation

$$y^{(3)} + 3y'' - 4y = 0$$

is

(a) $y = c_1 e^{x} + c_2 e^{2x} + c_3 x e^{2x}$
(b) $y = c_1 e^{-x} + c_2 e^{-2x} + c_3 x e^{-2x}$
(c) $y = c_1 e^{-x} + c_2 e^{2x} + c_3 x e^{2x}$
(d) $y = c_1 e^{x} + c_2 e^{-2x} + c_3 x e^{-2x}$
(e) $y = c_1 e^{x} + c_2 e^{3x} + c_3 x e^{3x}$

7. The Wronskian of the functions

$$f(x) = 1,\ g(x) = x,\ h(x) = x^2$$

is

(a) 2
(b) 4
(c) 3
(d) -4
(e) 0

8. The solution of the initial-value problem

$$y'' - y = 0, \quad y(0) = 0 \ \text{ and } \ y'(0) = 5$$

is

(a) $y = e^x - e^{-x}$
(b) $y = e^{-x} - e^x$
(c) $y = \dfrac{-5}{2}e^x + \dfrac{5}{2}e^{-x}$
(d) $y = \dfrac{5}{2}e^x - \dfrac{5}{2}e^{-x}$
(e) $y = \dfrac{3}{2}e^x - \dfrac{3}{2}e^{-x}$

9. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = e^{-4x}(c_1 \cos(3x) + c_2 \sin(3x))$$

is

(a) $y'' - 8y' - 25y = 0$
(b) $y'' - 6y' + 25y = 0$
(c) $y'' + 8y' + 25y = 0$
(d) $y'' + 6y' + 25y = 0$
(e) $y'' - 8y' + 25y = 0$

10. An appropriate form of a particular solution y_p for the non-homogeneous differential equation $y'' + 4y = 3x\ \cos(2x)$ is given by $y_p(x) =$

(a) $(A + Bx)\ \cos(2x) + (Cx + Dx^2)\ \sin(2x)$
(b) $(Ax + Bx^2)\ \cos(2x) + (C + Dx)\ \sin(2x)$
(c) $(Ax + Bx^2)\ \cos(2x) + (Cx + Dx^2)\ \sin(2x)$
(d) $(Ax^2 + Bx^3)\ \cos(2x) + (C + Dx)\ \sin(2x)$
(e) $(Ax + Bx)\ \cos(2x) + (C + Dx)\ \sin(2x)$

11. Let the solution space of the system

$$\begin{aligned} x_1 - 2x_2 - 9x_3 + 7x_4 &= 0 \\ x_1 + x_2 + 3x_3 + 4x_4 &= 0 \\ x_1 + 4x_2 + 15x_3 + x_4 &= 0 \end{aligned}$$

has all linear combinations of the two vectors

$$u = (1, -4, 1, 0) \text{ and } v = (\alpha, 1, \beta, 1).$$

Then $\alpha + \beta =$

(a) 0
(b) 3
(c) -5
(d) -3
(e) 5

12. The general solution of the differential equation

$$y^{(4)} + 18y'' + 81y = 0$$

is

(a) $y = (c_1 + c_2x)e^x \cos(2x) + (c_3 + c_4x)e^x \sin(2x)$
(b) $y = (c_1 + c_2x) \cos(3x) + (c_3 + c_4x) \sin(3x)$
(c) $y = (c_1 + c_2x) \cos(2x) + (c_3 + c_4x) \sin(2x)$
(d) $y = (c_1 + c_2x)e^x \cos(3x) + (c_3 + c_4x)e^x \sin(3x)$
(e) $y = c_1 \cos(3x) + c_2 \sin(3x) + c_3 \cos(2x) + c_4 \sin(2x)$

13. If the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & -9 \\ 2 & 5 & m \end{bmatrix}$ is equal to 2, then $m =$

(a) 4
(b) 2
(c) 3
(d) -5
(e) 5

14. Consider the vectors

$$v_1 = (-1, 2, 3),\ v_2 = (3, 1, -2),\ v_3 = (2, 3, 0) \text{ and } w = (1, 5, 8).$$

If $w = av_1 + bv_2 + cv_3$, then $a + b - c =$

(a) 19
(b) 21
(c) 17
(d) 13
(e) 15

15. Consider the subspace S of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d)|a = b + c + d\}$. A basis of S is consisting of the vectors

(a) $v_1 = (1, 1, 1, 1)$, $v_2 = (0, 0, 1, 0)$, $v_3 = (1, 0, 0, 1)$
(b) $v_1 = (0, 1, 0, 0)$, $v_2 = (0, 0, 1, 0)$, $v_3 = (0, 0, 0, 1)$
(c) $v_1 = (1, 1, 0, 0)$, $v_2 = (0, 0, 1, 1)$, $v_3 = (0, 1, 0, 1)$
(d) $v_1 = (1, 1, 1, 0)$, $v_2 = (0, 1, 1, 1)$, $v_3 = (0, 1, 1, 0)$
(e) $v_1 = (1, 1, 0, 0)$, $v_2 = (1, 0, 1, 0)$, $v_3 = (1, 0, 0, 1)$

16. A value of k for which the following vectors

$$v_1 = (k, 0, k),\ v_2 = (2k, -3, 4) \text{ and } v_3 = (3, 5, 2)$$

are linearly dependent is

(a) $\dfrac{7}{5}$
(b) $\dfrac{-16}{5}$
(c) $\dfrac{17}{10}$
(d) $\dfrac{-8}{5}$
(e) $\dfrac{10}{5}$

17. The eigenvector associated with the eigenvalue $\lambda = -1$ of the matrix $A = \begin{bmatrix} 7 & -8 \\ 6 & -7 \end{bmatrix}$ is $\begin{bmatrix} a \\ 1 \end{bmatrix}$, where $a =$

(a) -1
(b) 1
(c) 2
(d) 0
(e) -2

18. Given that $y_p = u_1(x)\,(\cos(3x)) + u_2(x)\,(\sin(3x))$ is a particular solution of the differential equation

$$y'' + 9y = 2\sec(3x), \text{ then } u_1(x) =$$

(a) $\frac{1}{9}\ln|\cos(3x)|$
(b) $\frac{1}{7}\ln|\cos(3x)|$
(c) $\frac{2}{7}\ln|\cos(3x)|$
(d) $\frac{2}{9}\ln|\cos(3x)|$
(e) $\frac{1}{3}\ln|\cos(3x)|$

19. The general solution of the differential equation

$$4y'' - 12y' + 9y = 0$$

is

(a) $y = c_1e^{3x} + c_2xe^{3x}$
(b) $y = c_1e^{x} + c_2xe^{x}$
(c) $y = c_1e^{\frac{3}{2}x} + c_2xe^{-\frac{3}{2}x}$
(d) $y = c_1e^{3x} + c_2e^{-3x}$
(e) $y = c_1e^{\frac{3}{2}x} + c_2xe^{\frac{3}{2}x}$

20. If $y_p = A + Bx$ is a particular solution of the differential equation

$$y'' - y' - 2y = 3x + 4, \text{ then } 4A + 4B =$$

(a) 0
(b) 12
(c) -10
(d) -11
(e) 9

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	C 18	C 3	A 10	C 15
2	A	D 3	C 4	C 16	B 20
3	A	C 5	D 14	E 1	E 5
4	A	E 6	B 19	A 17	E 17
5	A	B 7	A 20	B 8	C 19
6	A	E 11	E 8	C 20	D 9
7	A	B 19	E 12	A 15	A 7
8	A	E 10	B 17	C 3	D 6
9	A	B 4	D 16	A 4	C 11
10	A	C 8	C 9	E 7	C 13
11	A	D 1	B 11	E 12	C 4
12	A	B 16	B 5	B 13	B 10
13	A	E 12	C 13	D 14	B 18
14	A	E 2	D 18	A 5	E 2
15	A	A 20	A 6	B 18	E 3
16	A	E 14	A 2	D 19	C 1
17	A	D 15	C 1	E 9	B 16
18	A	E 13	E 7	B 6	D 14
19	A	E 17	C 10	A 11	E 8
20	A	B 9	A 15	E 2	D 12

Answer Counts

V	A	B	C	D	E
1	1	5	3	3	8
2	4	4	6	3	3
3	6	4	3	2	5
4	1	4	6	4	5