King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208
Final Exam
231
December 19, 2023

EXAM COVER

Number of versions: 4
Number of questions: 28

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Department of Mathematics

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Final Exam

231

December 19, 2023

Net Time Allowed: 180 Minutes

MASTER VERSION

1. If $y(x)$ is the solution of the initial value problem $e^y\,dx - e^{-x}\,dy = 0,\ y(0) = 0$, then $y(x) =$

   (a) $y = -\ln(2 - e^x)$ ________________________________ (correct)

   (b) $y = \ln(2 - e^x)$

   (c) $y = 2\ln(2 - e^x)$

   (d) $y = 3\ln(2 - e^x)$

   (e) $y = -3\ln(2 - e^x)$

2. The solution of the linear differential equation $x\dfrac{dy}{dx} - y = x^2 \sin x$ is given by

   (a) $y = cx - x\cos x$ ________________________________ (correct)

   (b) $y = cx^2 + x\cos x$

   (c) $y = cx - x^2\cos x$

   (d) $y = cx + x\sin x$

   (e) $y = cx^2 + x\sin x$

3. The solution of the exact differential equation $(2xy^2 - 3)\,dx + (2x^2y + 4)\,dy = 0$ is given by

   (a) $x^2y^2 - 3x + 4y = c$ ______________________________ (correct)

   (b) $x^2y^3 + 3x + 4y = c$

   (c) $x^2y^2 + 4x + 3y = c$

   (d) $x^2y^3 - 3x - 4y = c$

   (e) $x^2y^2 + 3x - 3y = c$

4. The solution of the homogeneous differential equation $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$ is given by

   (a) $\ln(x^2+y^2) + 2\tan^{-1}\left(\dfrac{y}{x}\right) = c$ ______________________________ (correct)

   (b) $\ln(x^2+y^2) + 3\tan^{-1}\left(\dfrac{y}{x}\right) = c$

   (c) $\ln(x^2+y^2) - 4\tan^{-1}\left(\dfrac{y}{x}\right) = c$

   (d) $\ln(x^2+y^2) + 5\tan^{-1}\left(\dfrac{y}{x}\right) = c$

   (e) $\ln(x^2+y^2) - \tan^{-1}\left(\dfrac{y}{x}\right) = c$

5. Let $t = (2, -7, 9)$, $u = (1, -2, 2)$, $v = (3, 0, 1)$, $w = (1, -1, 2)$ be four vectors in $\mathbb{R}^3$. If $t = au + bv + cw$, then $a^2 + b^2 + c^2 =$

(a) 14 ________________ (correct)

(b) 15

(c) 16

(d) 17

(e) 18

6. The rank of the matrix $A = \begin{bmatrix} 1 & -3 & 0 & -5 \\ -1 & 4 & 1 & 7 \\ 2 & 1 & 7 & 4 \\ 2 & -2 & 4 & -2 \end{bmatrix}$ is equal to

(a) 2 ________________ (correct)

(b) 3

(c) 1

(d) 4

(e) 0

7. A basis for the subspace

$$W = \{(x, y, z) : x - 2y + 5z = 0\}$$

of $\mathbb{R}^3$ consist of

(a) $v_1 = (2, 1, 0)$ and $v_2 = (-5, 0, 1)$ ________________ (correct)
(b) $v_1 = (2, 1, 1)$ and $v_2 = (-5, 0, 1)$
(c) $v_1 = (2, 1, 0)$ and $v_2 = (-5, 1, 1)$
(d) $v_1 = (0, 1, 0)$ and $v_2 = (-5, 0, 1)$
(e) $v_1 = (2, 1, 0)$ and $v_2 = (1, 0, 1)$

8. The general solution of the differential equation

$$(D + 1)(D - 2)^2(D^2 - 4D + 13)y = 0$$

is given by

(a) $y = c_1e^{-x} + c_2e^{2x} + c_3xe^{2x} + c_4e^{2x}\cos(3x) + c_5e^{2x}\sin(3x)$ ________________ (correct)
(b) $y = c_1e^{-x} + c_2e^{2x} + c_3e^{2x}\cos(3x) + c_4e^{2x}\sin(3x)$
(c) $y = c_1e^{-x} + c_2e^{2x} + c_3xe^{2x} + c_4e^{3x}\cos(2x) + c_5e^{3x}\sin(3x)$
(d) $y = c_1e^{x} + c_2e^{2x} + c_3xe^{2x} + c_4e^{3x}\cos(2x) + c_5e^{3x}\sin(2x)$
(e) $y = c_1e^{-x} + c_2e^{-2x} + c_3xe^{-2x} + c_4e^{2x}\cos(3x) + c_5e^{2x}\sin(3x)$

9. An appropriate form of a particular solution of the differential equation

$$y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$$

is

(a) $y_p = Ax + Bx^2 + (Cx + Dx^2)\, e^{-3x}$ ______ (correct)
(b) $y_p = A + Bx + (Cx + Dx^2)\, e^{-3x}$
(c) $y_p = Ax + Bx^2 + (C + Dx^2)\, e^{-3x}$
(d) $y_p = Ax + Bx^3 + (Cx + Dx^2)\, e^{-3x}$
(e) $y_p = Ax + Bx^2 + (Cx^2 + Dx^3)\, e^{-3x}$

10. If $y(x)$ is the solution of the initial-value problem $y'' + 4y = 2x$, $y(0) = 1$, $y'(0) = 2$, then $y(2\pi) =$

(a) $1 + \pi$ ______ (correct)
(b) $1 - \pi$
(c) $2 + \pi$
(d) $2 - \pi$
(e) π

11. Using variation of parameters, the differential equation $y'' - 2y' + y = \frac{e^x}{1+x^2}$ has a particular solution $y_p = u_1(x)e^x + u_2(x)xe^x$, then $u_1(x) =$

(a) $-\dfrac{1}{2}\ln(1+x^2)$ ________________ (correct)

(b) $\dfrac{3}{2}\ln(1+x^2)$

(c) $\ln(1+x^2)$

(d) $3\ \ln(1+x^2)$

(e) $-3\ln(1+x^2)$

12. The largest eigenvalue of the matrix

$$A = \begin{bmatrix} 4 & -3 & 1 \\ 2 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

is equal to

(a) 2 ________________ (correct)

(b) 3

(c) 5

(d) 1

(e) -4

13. If $V = \begin{bmatrix} \alpha \\ 1 \\ \beta \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 4 & -3 & 1 \\ 2 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ associated with the eigenvalue $\lambda = 1$, then $10\alpha + 6\beta =$

(a) 10 ________________ (correct)

(b) 8

(c) 6

(d) 4

(e) 0

14. An eigenvector associated with eigenvalue $\lambda = 2i$ of the matrix $A = \begin{bmatrix} 2 & 8 \\ -1 & -2 \end{bmatrix}$ is

(a) $\begin{bmatrix} 2+2i \\ -1 \end{bmatrix}$ ________________ (correct)

(b) $\begin{bmatrix} 2+2i \\ -2 \end{bmatrix}$

(c) $\begin{bmatrix} 2+2i \\ -3 \end{bmatrix}$

(d) $\begin{bmatrix} 2+2i \\ -4 \end{bmatrix}$

(e) $\begin{bmatrix} 2+2i \\ -5 \end{bmatrix}$

15. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 7 & 2 \\ 10 & -15 & -4 \end{bmatrix}$. A basis for the eigenspace of A associated with the eigenvalue $\lambda = 1$ of A is

(a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \right\}$ ________________________________(correct)

(b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

16. The roots of the indicial equation at $x = 0$ for the differential equation $2xy'' - y' + 2y = 0$ are

(a) $r_1 = 0$, $r_2 = \frac{3}{2}$ ________________________________(correct)

(b) $r_1 = 1$, $r_2 = \frac{5}{2}$

(c) $r_1 = 0$, $r_2 = \frac{5}{2}$

(d) $r_1 = 1$, $r_2 = \frac{3}{2}$

(e) $r_1 = 0$, $r_2 = -\frac{5}{2}$

17. If the general solution of the system $X' = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix} X$ is given by

$$X = c_1 \begin{bmatrix} a \\ b \\ -2 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} \alpha \\ \beta \\ 1 \end{bmatrix} e^{\lambda t} + c_3 \begin{bmatrix} e \\ f \\ 13 \end{bmatrix}$$

then $a \cdot b \cdot \lambda =$

(a) -24 ____(correct)

(b) 12

(c) 24

(d) -12

(e) 0

18. Let $F(t) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, $e^{At} = \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}$. A particular solution for the system $X' = AX + F(t)$ is

(a) $X_p = \begin{bmatrix} -3 \\ \frac{1}{2} \end{bmatrix}$ ____(correct)

(b) $X_p = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

(c) $X_p = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

(d) $X_p = \begin{bmatrix} 3 \\ \frac{1}{2} \end{bmatrix}$

(e) $X_p = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

19. Let $A = \begin{bmatrix} 6 & 4 \\ -9 & -6 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} 1+6t & 4t \\ -9t & 1-6t \end{bmatrix}$ ———————————— (correct)

(b) $\begin{bmatrix} 1-6t & 4t \\ 9t & 1-6t \end{bmatrix}$

(c) $\begin{bmatrix} 1+6t & 4t \\ 9t & 1+6t \end{bmatrix}$

(d) $\begin{bmatrix} 1+6t & -4t \\ -9t & 1-6t \end{bmatrix}$

(e) $\begin{bmatrix} 1+6t & -4t \\ 9t & 1+6t \end{bmatrix}$

20. A possible fundamental matrix for the system $X' = \begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix} X$ is

(a) $\Phi(t) = \begin{bmatrix} 1 & e^{4t} \\ 2 & -2e^{4t} \end{bmatrix}$ ———————————— (correct)

(b) $\Phi(t) = \begin{bmatrix} -1 & e^{4t} \\ 2 & -2e^{4t} \end{bmatrix}$

(c) $\Phi(t) = \begin{bmatrix} 1 & e^{4t} \\ -2 & -2e^{4t} \end{bmatrix}$

(d) $\Phi(t) = \begin{bmatrix} 1 & -e^{4t} \\ 2 & 3e^{4t} \end{bmatrix}$

(e) $\Phi(t) = \begin{bmatrix} 1 & e^{4t} \\ 2 & 2e^{4t} \end{bmatrix}$

21. If $y = \sum_{n=0}^{\infty} C_n x^n$ is a power series solution about the ordinary point $x = 0$ of the differential equation $y'' - 2xy' + y = 0$, then the coefficients C_n satisfy

(a) $C_{n+2} = \dfrac{2n-1}{(n+2)(n+1)} C_n,\ n \geq 1$ ______________________________ (correct)

(b) $C_{n+2} = \dfrac{2n+1}{(n+2)(n+1)} C_n,\ n \geq 1$

(c) $C_{n+2} = \dfrac{2n}{(n+1)(n+2)} C_n,\ n \geq 1$

(d) $C_{n+2} = \dfrac{2}{(n+2)(n+1)} C_{n-1},\ n \geq 1$

(e) $C_{n+2} = \dfrac{3}{(n+1)(n+2)} C_{n-2},\ n \geq 1$

22. The minimum radius of convergence of the power series solutions for the differential equation $(x^2 - 2x + 5)\, y'' + xy' - y = 0$ about the ordinary point $x = 0$ is

(a) $\sqrt{5}$ ______________________________ (correct)

(b) ∞

(c) 0

(d) $\sqrt{8}$

(e) 2

23. The general solution of the system $X' = \begin{bmatrix} 4 & 1 \\ 6 & -1 \end{bmatrix} X$ is given by

(a) $X = c_1 \begin{bmatrix} 1 \\ -6 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{5t}$ ________________________________ (correct)

(b) $X = c_1 \begin{bmatrix} 1 \\ 6 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t}$

(c) $X = c_1 \begin{bmatrix} 1 \\ -6 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t}$

(d) $X = c_1 \begin{bmatrix} 1 \\ 6 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t}$

(e) $X = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} e^{-2t}$

24. If $X = c_1 \begin{bmatrix} 5 \\ -6 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{4t}$ is the solution of the initial value problem

$$X' = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} X, \; X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

then $c_2 - c_1 =$

(a) 7 ________________________________ (correct)

(b) 6

(c) 4

(d) 8

(e) 3

25. Let $A = \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix}$. If P is diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$ ______ (correct)

(b) $P = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ 1 & -2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$

26. A 2×2 real matrix A has an eigenvector $\begin{bmatrix} 2+2i \\ -1 \end{bmatrix}$ associated with the eigenvalue $\lambda = 2i$ of A. Then the general solution of the system $X' = AX$ is

(a) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$ ______ (correct)

(b) $X = c_1 \begin{bmatrix} \cos(2t) - 2\sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} \cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(c) $X = c_1 \begin{bmatrix} 2\cos(2t) - \sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + \sin(2t) \\ -\sin(2t) \end{bmatrix}$

(d) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -2\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(e) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -2\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -2\sin(2t) \end{bmatrix}$

27. Let

$$A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}, \ \Phi(t) = \begin{bmatrix} e^t & 4e^{2t} \\ e^t & 3e^{2t} \end{bmatrix}.$$

If $\Phi(t)$ is a fundamental matrix for the system $X' = AX$, then $e^{At} =$

(a) $\begin{bmatrix} -3e^t + 4e^{2t} & 4e^t - 4e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$ ________________ (correct)

(b) $\begin{bmatrix} 3e^t - 4e^{2t} & 4e^t - 4e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

(c) $\begin{bmatrix} -3e^t + 4e^{2t} & e^t - e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

(d) $\begin{bmatrix} -3e^t + 4e^{2t} & 4e^t - 4e^{2t} \\ e^t + 3e^{2t} & 4e^t - e^{2t} \end{bmatrix}$

(e) $\begin{bmatrix} -3e^t + 4e^{2t} & 5e^t - 5e^{2t} \\ -2e^t + 2e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

28. The matrix $A = \begin{bmatrix} 5 & 1 & 6 \\ -1 & 1 & -3 \\ 0 & 1 & 3 \end{bmatrix}$ has a defective eigenvalue $\lambda = 3$ of defect 2. Choosing $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - 3I)^3 V_3 = 0$ and $(A - 3I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 2+3t \\ -1 \\ -t \end{bmatrix} + c_3 \begin{bmatrix} 1 + 2t + \frac{3t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$ ____________(correct)

(b) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ -1 \\ -t \end{bmatrix} + c_3 \begin{bmatrix} 1 + 2t + \frac{t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

(c) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 + 2t + \frac{3t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

(d) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} \\ t \\ t^2 \end{bmatrix} \right) e^{3t}$

(e) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2+3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 + t + \frac{t^2}{2} \\ t \\ 1 \end{bmatrix} \right) e^{3t}$

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CODE01 **CODE01**

Math 208
Final Exam
231
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Net Time Allowed: 180 Minutes

Name	
ID		**Sec**	

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Let $t = (2, -7, 9)$, $u = (1, -2, 2)$, $v = (3, 0, 1)$, $w = (1, -1, 2)$ be four vectors in $\mathbb{R}^3$. If $t = au + bv + cw$, then $a^2 + b^2 + c^2 =$

   (a) 15
   (b) 18
   (c) 14
   (d) 17
   (e) 16

2. Let

$$A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}, \ \Phi(t) = \begin{bmatrix} e^t & 4e^{2t} \\ e^t & 3e^{2t} \end{bmatrix}.$$

   If $\Phi(t)$ is a fundamental matrix for the system $X' = AX$, then $e^{At} =$

   (a) $\begin{bmatrix} -3e^t + 4e^{2t} & e^t - e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

   (b) $\begin{bmatrix} -3e^t + 4e^{2t} & 4e^t - 4e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

   (c) $\begin{bmatrix} -3e^t + 4e^{2t} & 5e^t - 5e^{2t} \\ -2e^t + 2e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

   (d) $\begin{bmatrix} -3e^t + 4e^{2t} & 4e^t - 4e^{2t} \\ e^t + 3e^{2t} & 4e^t - e^{2t} \end{bmatrix}$

   (e) $\begin{bmatrix} 3e^t - 4e^{2t} & 4e^t - 4e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

3. If $V = \begin{bmatrix} \alpha \\ 1 \\ \beta \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 4 & -3 & 1 \\ 2 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ associated with the eigenvalue $\lambda = 1$, then $10\alpha + 6\beta =$

(a) 0
(b) 8
(c) 6
(d) 10
(e) 4

4. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 7 & 2 \\ 10 & -15 & -4 \end{bmatrix}$. A basis for the eigenspace of A associated with the eigenvalue $\lambda = 1$ of A is

(a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \right\}$

5. Using variation of parameters, the differential equation $y'' - 2y' + y = \frac{e^x}{1+x^2}$ has a particular solution $y_p = u_1(x)e^x + u_2(x)xe^x$, then $u_1(x) =$

   (a) $-3\ln(1+x^2)$
   (b) $-\dfrac{1}{2}\ln(1+x^2)$
   (c) $\ln(1+x^2)$
   (d) $3\ln(1+x^2)$
   (e) $\dfrac{3}{2}\ln(1+x^2)$

6. The solution of the homogeneous differential equation $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$ is given by

   (a) $\ln(x^2+y^2) + 2\tan^{-1}\left(\dfrac{y}{x}\right) = c$
   (b) $\ln(x^2+y^2) + 5\tan^{-1}\left(\dfrac{y}{x}\right) = c$
   (c) $\ln(x^2+y^2) - \tan^{-1}\left(\dfrac{y}{x}\right) = c$
   (d) $\ln(x^2+y^2) - 4\tan^{-1}\left(\dfrac{y}{x}\right) = c$
   (e) $\ln(x^2+y^2) + 3\tan^{-1}\left(\dfrac{y}{x}\right) = c$

7. If $X = c_1 \begin{bmatrix} 5 \\ -6 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{4t}$ is the solution of the initial value problem

$$X' = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} X, \; X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

then $c_2 - c_1 =$

(a) 7

(b) 3

(c) 4

(d) 8

(e) 6

8. If $y = \sum_{n=0}^{\infty} C_n x^n$ is a power series solution about the ordinary point $x = 0$ of the differential equation $y'' - 2xy' + y = 0$, then the coefficients C_n satisfy

(a) $C_{n+2} = \dfrac{2}{(n+2)(n+1)} C_{n-1}, \; n \geq 1$

(b) $C_{n+2} = \dfrac{2n+1}{(n+2)(n+1)} C_n, \; n \geq 1$

(c) $C_{n+2} = \dfrac{2n-1}{(n+2)(n+1)} C_n, \; n \geq 1$

(d) $C_{n+2} = \dfrac{3}{(n+1)(n+2)} C_{n-2}, \; n \geq 1$

(e) $C_{n+2} = \dfrac{2n}{(n+1)(n+2)} C_n, \; n \geq 1$

9. If $y(x)$ is the solution of the initial-value problem $y'' + 4y = 2x$, $y(0) = 1$, $y'(0) = 2$, then $y(2\pi) =$

(a) π

(b) $1 - \pi$

(c) $1 + \pi$

(d) $2 + \pi$

(e) $2 - \pi$

10. A possible fundamental matrix for the system $X' = \begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix} X$ is

(a) $\Phi(t) = \begin{bmatrix} 1 & e^{4t} \\ 2 & 2e^{4t} \end{bmatrix}$

(b) $\Phi(t) = \begin{bmatrix} 1 & e^{4t} \\ 2 & -2e^{4t} \end{bmatrix}$

(c) $\Phi(t) = \begin{bmatrix} 1 & e^{4t} \\ -2 & -2e^{4t} \end{bmatrix}$

(d) $\Phi(t) = \begin{bmatrix} 1 & -e^{4t} \\ 2 & 3e^{4t} \end{bmatrix}$

(e) $\Phi(t) = \begin{bmatrix} -1 & e^{4t} \\ 2 & -2e^{4t} \end{bmatrix}$

11. The solution of the linear differential equation $x\dfrac{dy}{dx} - y = x^2 \sin x$ is given by

(a) $y = cx - x^2 \cos x$

(b) $y = cx^2 + x \sin x$

(c) $y = cx^2 + x \cos x$

(d) $y = cx - x \cos x$

(e) $y = cx + x \sin x$

12. A 2×2 real matrix A has an eigenvector $\begin{bmatrix} 2+2i \\ -1 \end{bmatrix}$ associated with the eigenvalue $\lambda = 2i$ of A. Then the general solution of the system $X' = AX$ is

(a) $X = c_1 \begin{bmatrix} 2\cos(2t) - \sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + \sin(2t) \\ -\sin(2t) \end{bmatrix}$

(b) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(c) $X = c_1 \begin{bmatrix} \cos(2t) - 2\sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} \cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(d) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -2\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(e) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -2\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -2\sin(2t) \end{bmatrix}$

13. The general solution of the differential equation

$$(D+1)(D-2)^2(D^2-4D+13)y=0$$

is given by

(a) $y=c_1e^{-x}+c_2e^{2x}+c_3xe^{2x}+c_4e^{3x}\cos(2x)+c_5e^{3x}\sin(3x)$
(b) $y=c_1e^{x}+c_2e^{2x}+c_3xe^{2x}+c_4e^{3x}\cos(2x)+c_5e^{3x}\sin(2x)$
(c) $y=c_1e^{-x}+c_2e^{2x}+c_3e^{2x}\cos(3x)+c_4e^{2x}\sin(3x)$
(d) $y=c_1e^{-x}+c_2e^{-2x}+c_3xe^{-2x}+c_4e^{2x}\cos(3x)+c_5e^{2x}\sin(3x)$
(e) $y=c_1e^{-x}+c_2e^{2x}+c_3xe^{2x}+c_4e^{2x}\cos(3x)+c_5e^{2x}\sin(3x)$

14. The rank of the matrix $A=\begin{bmatrix}1&-3&0&-5\\-1&4&1&7\\2&1&7&4\\2&-2&4&-2\end{bmatrix}$ is equal to

(a) 2
(b) 4
(c) 3
(d) 1
(e) 0

15. An eigenvector associated with eigenvalue $\lambda = 2i$ of the matrix $A = \begin{bmatrix} 2 & 8 \\ -1 & -2 \end{bmatrix}$ is

(a) $\begin{bmatrix} 2+2i \\ -4 \end{bmatrix}$

(b) $\begin{bmatrix} 2+2i \\ -5 \end{bmatrix}$

(c) $\begin{bmatrix} 2+2i \\ -3 \end{bmatrix}$

(d) $\begin{bmatrix} 2+2i \\ -1 \end{bmatrix}$

(e) $\begin{bmatrix} 2+2i \\ -2 \end{bmatrix}$

16. The largest eigenvalue of the matrix

$$A = \begin{bmatrix} 4 & -3 & 1 \\ 2 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

is equal to

(a) 5

(b) 1

(c) -4

(d) 2

(e) 3

17. A basis for the subspace

$$W = \{(x, y, z) : x - 2y + 5z = 0\}$$

of $\mathbb{R}^3$ consist of

(a) $v_1 = (0, 1, 0)$ and $v_2 = (-5, 0, 1)$
(b) $v_1 = (2, 1, 0)$ and $v_2 = (-5, 0, 1)$
(c) $v_1 = (2, 1, 0)$ and $v_2 = (1, 0, 1)$
(d) $v_1 = (2, 1, 1)$ and $v_2 = (-5, 0, 1)$
(e) $v_1 = (2, 1, 0)$ and $v_2 = (-5, 1, 1)$

18. If $y(x)$ is the solution of the initial value problem $e^y\,dx - e^{-x}\,dy = 0,\ y(0) = 0$, then $y(x) =$

(a) $y = -3\ln(2 - e^x)$
(b) $y = 3\ln(2 - e^x)$
(c) $y = -\ln(2 - e^x)$
(d) $y = \ln(2 - e^x)$
(e) $y = 2\ln(2 - e^x)$

19. An appropriate form of a particular solution of the differential equation

$$y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$$

is

(a) $y_p = A + Bx + (Cx + Dx^2)\,e^{-3x}$
(b) $y_p = Ax + Bx^2 + (Cx + Dx^2)\,e^{-3x}$
(c) $y_p = Ax + Bx^3 + (Cx + Dx^2)\,e^{-3x}$
(d) $y_p = Ax + Bx^2 + (C + Dx^2)\,e^{-3x}$
(e) $y_p = Ax + Bx^2 + (Cx^2 + Dx^3)\,e^{-3x}$

20. If the general solution of the system $X' = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix} X$ is given by

$$X = c_1 \begin{bmatrix} a \\ b \\ -2 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} \alpha \\ \beta \\ 1 \end{bmatrix} e^{\lambda t} + c_3 \begin{bmatrix} e \\ f \\ 13 \end{bmatrix}$$

then $a \cdot b \cdot \lambda =$

(a) -24
(b) -12
(c) 0
(d) 24
(e) 12

21. Let $A = \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix}$. If P is diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 3 \\ 1 & -2 \end{bmatrix}$

22. Let $A = \begin{bmatrix} 6 & 4 \\ -9 & -6 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} 1+6t & -4t \\ 9t & 1+6t \end{bmatrix}$

(b) $\begin{bmatrix} 1+6t & 4t \\ 9t & 1+6t \end{bmatrix}$

(c) $\begin{bmatrix} 1+6t & 4t \\ -9t & 1-6t \end{bmatrix}$

(d) $\begin{bmatrix} 1-6t & 4t \\ 9t & 1-6t \end{bmatrix}$

(e) $\begin{bmatrix} 1+6t & -4t \\ -9t & 1-6t \end{bmatrix}$

23. Let $F(t) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, $e^{At} = \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}$. A particular solution for the system $X' = AX + F(t)$ is

(a) $X_p = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

(b) $X_p = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

(c) $X_p = \begin{bmatrix} 3 \\ \frac{1}{2} \end{bmatrix}$

(d) $X_p = \begin{bmatrix} -3 \\ \frac{1}{2} \end{bmatrix}$

(e) $X_p = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

24. The minimum radius of convergence of the power series solutions for the differential equation $(x^2 - 2x + 5)\, y'' + xy' - y = 0$ about the ordinary point $x = 0$ is

(a) $\sqrt{5}$

(b) ∞

(c) 2

(d) $\sqrt{8}$

(e) 0

25. The solution of the exact differential equation $(2xy^2 - 3)\,dx + (2x^2y + 4)\,dy = 0$ is given by

(a) $x^2y^3 + 3x + 4y = c$
(b) $x^2y^2 + 3x - 3y = c$
(c) $x^2y^2 - 3x + 4y = c$
(d) $x^2y^3 - 3x - 4y = c$
(e) $x^2y^2 + 4x + 3y = c$

26. The roots of the indicial equation at $x = 0$ for the differential equation $2xy'' - y' + 2y = 0$ are

(a) $r_1 = 0,\ r_2 = \dfrac{5}{2}$
(b) $r_1 = 0,\ r_2 = \dfrac{3}{2}$
(c) $r_1 = 0,\ r_2 = -\dfrac{5}{2}$
(d) $r_1 = 1,\ r_2 = \dfrac{5}{2}$
(e) $r_1 = 1,\ r_2 = \dfrac{3}{2}$

27. The general solution of the system $X' = \begin{bmatrix} 4 & 1 \\ 6 & -1 \end{bmatrix} X$ is given by

(a) $X = c_1 \begin{bmatrix} 1 \\ -6 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{5t}$

(b) $X = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} e^{-2t}$

(c) $X = c_1 \begin{bmatrix} 1 \\ 6 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t}$

(d) $X = c_1 \begin{bmatrix} 1 \\ -6 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t}$

(e) $X = c_1 \begin{bmatrix} 1 \\ 6 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t}$

28. The matrix $A = \begin{bmatrix} 5 & 1 & 6 \\ -1 & 1 & -3 \\ 0 & 1 & 3 \end{bmatrix}$ has a defective eigenvalue $\lambda = 3$ of defect 2. Choosing $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - 3I)^3\, V_3 = 0$ and $(A - 3I)^2\, V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ -1 \\ -t \end{bmatrix} + c_3 \begin{bmatrix} 1 + 2t + \frac{t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

(b) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 2 + 3t \\ -1 \\ -t \end{bmatrix} + c_3 \begin{bmatrix} 1 + 2t + \frac{3t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

(c) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 + 2t + \frac{3t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

(d) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} \\ t \\ t^2 \end{bmatrix} \right) e^{3t}$

(e) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 + 3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 + t + \frac{t^2}{2} \\ t \\ 1 \end{bmatrix} \right) e^{3t}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE02 **CODE02**

Math 208
Final Exam
231
December 19, 2023
Net Time Allowed: 180 Minutes

Name	

ID		**Sec**	

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. A basis for the subspace

$$W = \{(x, y, z) : x - 2y + 5z = 0\}$$

of $\mathbb{R}^3$ consist of

(a) $v_1 = (0, 1, 0)$ and $v_2 = (-5, 0, 1)$
(b) $v_1 = (2, 1, 0)$ and $v_2 = (-5, 1, 1)$
(c) $v_1 = (2, 1, 1)$ and $v_2 = (-5, 0, 1)$
(d) $v_1 = (2, 1, 0)$ and $v_2 = (1, 0, 1)$
(e) $v_1 = (2, 1, 0)$ and $v_2 = (-5, 0, 1)$

2. A 2×2 real matrix A has an eigenvector $\begin{bmatrix} 2+2i \\ -1 \end{bmatrix}$ associated with the eigenvalue $\lambda = 2i$ of A. Then the general solution of the system $X' = AX$ is

(a) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(b) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -2\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(c) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -2\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -2\sin(2t) \end{bmatrix}$

(d) $X = c_1 \begin{bmatrix} 2\cos(2t) - \sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + \sin(2t) \\ -\sin(2t) \end{bmatrix}$

(e) $X = c_1 \begin{bmatrix} \cos(2t) - 2\sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} \cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

3. The roots of the indicial equation at $x = 0$ for the differential equation $2xy'' - y' + 2y = 0$ are

(a) $r_1 = 1,\ r_2 = \frac{5}{2}$
(b) $r_1 = 0,\ r_2 = \frac{5}{2}$
(c) $r_1 = 1,\ r_2 = \frac{3}{2}$
(d) $r_1 = 0,\ r_2 = -\frac{5}{2}$
(e) $r_1 = 0,\ r_2 = \frac{3}{2}$

4. Let $t = (2, -7, 9)$, $u = (1, -2, 2)$, $v = (3, 0, 1)$, $w = (1, -1, 2)$ be four vectors in $\mathbb{R}^3$. If $t = au + bv + cw$, then $a^2 + b^2 + c^2 =$

(a) 17
(b) 15
(c) 16
(d) 18
(e) 14

5. The solution of the linear differential equation $x\dfrac{dy}{dx} - y = x^2 \sin x$ is given by

   (a) $y = cx - x\cos x$
   (b) $y = cx + x\sin x$
   (c) $y = cx - x^2\cos x$
   (d) $y = cx^2 + x\cos x$
   (e) $y = cx^2 + x\sin x$

6. An appropriate form of a particular solution of the differential equation

   $$y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$$

   is

   (a) $y_p = Ax + Bx^2 + (C + Dx^2)\, e^{-3x}$
   (b) $y_p = Ax + Bx^3 + (Cx + Dx^2)\, e^{-3x}$
   (c) $y_p = A + Bx + (Cx + Dx^2)\, e^{-3x}$
   (d) $y_p = Ax + Bx^2 + (Cx + Dx^2)\, e^{-3x}$
   (e) $y_p = Ax + Bx^2 + (Cx^2 + Dx^3)\, e^{-3x}$

7. The general solution of the system $X' = \begin{bmatrix} 4 & 1 \\ 6 & -1 \end{bmatrix} X$ is given by

(a) $X = c_1 \begin{bmatrix} 1 \\ -6 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{5t}$

(b) $X = c_1 \begin{bmatrix} 1 \\ -6 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t}$

(c) $X = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} e^{-2t}$

(d) $X = c_1 \begin{bmatrix} 1 \\ 6 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t}$

(e) $X = c_1 \begin{bmatrix} 1 \\ 6 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t}$

8. The solution of the exact differential equation $(2xy^2 - 3)\,dx + (2x^2y + 4)\,dy = 0$ is given by

(a) $x^2y^2 - 3x + 4y = c$

(b) $x^2y^2 + 4x + 3y = c$

(c) $x^2y^2 + 3x - 3y = c$

(d) $x^2y^3 - 3x - 4y = c$

(e) $x^2y^3 + 3x + 4y = c$

9. Let

$$A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}, \ \Phi(t) = \begin{bmatrix} e^t & 4e^{2t} \\ e^t & 3e^{2t} \end{bmatrix}.$$

If $\Phi(t)$ is a fundamental matrix for the system $X' = AX$, then $e^{At} =$

(a) $\begin{bmatrix} -3e^t + 4e^{2t} & 5e^t - 5e^{2t} \\ -2e^t + 2e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

(b) $\begin{bmatrix} 3e^t - 4e^{2t} & 4e^t - 4e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

(c) $\begin{bmatrix} -3e^t + 4e^{2t} & 4e^t - 4e^{2t} \\ e^t + 3e^{2t} & 4e^t - e^{2t} \end{bmatrix}$

(d) $\begin{bmatrix} -3e^t + 4e^{2t} & 4e^t - 4e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

(e) $\begin{bmatrix} -3e^t + 4e^{2t} & e^t - e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

10. The rank of the matrix $A = \begin{bmatrix} 1 & -3 & 0 & -5 \\ -1 & 4 & 1 & 7 \\ 2 & 1 & 7 & 4 \\ 2 & -2 & 4 & -2 \end{bmatrix}$ is equal to

(a) 4
(b) 2
(c) 1
(d) 0
(e) 3

11. The solution of the homogeneous differential equation $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$ is given by

(a) $\ln(x^2+y^2) + 2\tan^{-1}\left(\dfrac{y}{x}\right) = c$

(b) $\ln(x^2+y^2) - 4\tan^{-1}\left(\dfrac{y}{x}\right) = c$

(c) $\ln(x^2+y^2) + 3\tan^{-1}\left(\dfrac{y}{x}\right) = c$

(d) $\ln(x^2+y^2) + 5\tan^{-1}\left(\dfrac{y}{x}\right) = c$

(e) $\ln(x^2+y^2) - \tan^{-1}\left(\dfrac{y}{x}\right) = c$

12. Let $F(t) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, $e^{At} = \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}$. A particular solution for the system $X' = AX + F(t)$ is

(a) $X_p = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

(b) $X_p = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

(c) $X_p = \begin{bmatrix} 3 \\ \frac{1}{2} \end{bmatrix}$

(d) $X_p = \begin{bmatrix} -3 \\ \frac{1}{2} \end{bmatrix}$

(e) $X_p = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

13. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 7 & 2 \\ 10 & -15 & -4 \end{bmatrix}$. A basis for the eigenspace of A associated with the eigenvalue $\lambda = 1$ of A is

(a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

14. The minimum radius of convergence of the power series solutions for the differential equation $(x^2 - 2x + 5)\, y'' + xy' - y = 0$ about the ordinary point $x = 0$ is

(a) 2
(b) ∞
(c) $\sqrt{5}$
(d) $\sqrt{8}$
(e) 0

15. If $y = \sum_{n=0}^{\infty} C_n x^n$ is a power series solution about the ordinary point $x = 0$ of the differential equation $y'' - 2xy' + y = 0$, then the coefficients C_n satisfy

(a) $C_{n+2} = \dfrac{3}{(n+1)(n+2)} C_{n-2},\ n \geq 1$

(b) $C_{n+2} = \dfrac{2n-1}{(n+2)(n+1)} C_n,\ n \geq 1$

(c) $C_{n+2} = \dfrac{2n+1}{(n+2)(n+1)} C_n,\ n \geq 1$

(d) $C_{n+2} = \dfrac{2n}{(n+1)(n+2)} C_n,\ n \geq 1$

(e) $C_{n+2} = \dfrac{2}{(n+2)(n+1)} C_{n-1},\ n \geq 1$

16. If $y(x)$ is the solution of the initial-value problem $y'' + 4y = 2x$, $y(0) = 1$, $y'(0) = 2$, then $y(2\pi) =$

(a) $1 + \pi$

(b) $1 - \pi$

(c) $2 - \pi$

(d) $2 + \pi$

(e) π

17. If $V = \begin{bmatrix} \alpha \\ 1 \\ \beta \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 4 & -3 & 1 \\ 2 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ associated with the eigenvalue $\lambda = 1$, then $10\alpha + 6\beta =$

(a) 4
(b) 8
(c) 10
(d) 6
(e) 0

18. If $X = c_1 \begin{bmatrix} 5 \\ -6 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{4t}$ is the solution of the initial value problem

$$X' = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} X, \; X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

then $c_2 - c_1 =$

(a) 8
(b) 6
(c) 4
(d) 3
(e) 7

19. A possible fundamental matrix for the system $X' = \begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix} X$ is

(a) $\Phi(t) = \begin{bmatrix} 1 & e^{4t} \\ -2 & -2e^{4t} \end{bmatrix}$

(b) $\Phi(t) = \begin{bmatrix} 1 & e^{4t} \\ 2 & 2e^{4t} \end{bmatrix}$

(c) $\Phi(t) = \begin{bmatrix} 1 & -e^{4t} \\ 2 & 3e^{4t} \end{bmatrix}$

(d) $\Phi(t) = \begin{bmatrix} -1 & e^{4t} \\ 2 & -2e^{4t} \end{bmatrix}$

(e) $\Phi(t) = \begin{bmatrix} 1 & e^{4t} \\ 2 & -2e^{4t} \end{bmatrix}$

20. Using variation of parameters, the differential equation $y'' - 2y' + y = \frac{e^x}{1+x^2}$ has a particular solution $y_p = u_1(x)e^x + u_2(x)xe^x$, then $u_1(x) =$

(a) $\frac{3}{2}\ln(1+x^2)$

(b) $-\frac{1}{2}\ln(1+x^2)$

(c) $-3\ln(1+x^2)$

(d) $3\ln(1+x^2)$

(e) $\ln(1+x^2)$

21. Let $A = \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix}$. If P is diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 3 \\ 1 & -2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

22. Let $A = \begin{bmatrix} 6 & 4 \\ -9 & -6 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} 1+6t & -4t \\ 9t & 1+6t \end{bmatrix}$

(b) $\begin{bmatrix} 1+6t & -4t \\ -9t & 1-6t \end{bmatrix}$

(c) $\begin{bmatrix} 1-6t & 4t \\ 9t & 1-6t \end{bmatrix}$

(d) $\begin{bmatrix} 1+6t & 4t \\ -9t & 1-6t \end{bmatrix}$

(e) $\begin{bmatrix} 1+6t & 4t \\ 9t & 1+6t \end{bmatrix}$

23. If $y(x)$ is the solution of the initial value problem $e^y\,dx - e^{-x}\,dy = 0,\ y(0) = 0$, then $y(x) =$

(a) $y = -3\ln(2 - e^x)$

(b) $y = 3\ln(2 - e^x)$

(c) $y = \ln(2 - e^x)$

(d) $y = -\ln(2 - e^x)$

(e) $y = 2\ln(2 - e^x)$

24. The largest eigenvalue of the matrix

$$A = \begin{bmatrix} 4 & -3 & 1 \\ 2 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

is equal to

(a) -4

(b) 5

(c) 3

(d) 2

(e) 1

25. If the general solution of the system $X' = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix} X$ is given by

$$X = c_1 \begin{bmatrix} a \\ b \\ -2 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} \alpha \\ \beta \\ 1 \end{bmatrix} e^{\lambda t} + c_3 \begin{bmatrix} e \\ f \\ 13 \end{bmatrix}$$

then $a \cdot b \cdot \lambda =$

(a) 12
(b) 0
(c) -24
(d) -12
(e) 24

26. The general solution of the differential equation

$$(D+1)(D-2)^2(D^2-4D+13)y = 0$$

is given by

(a) $y = c_1e^{-x} + c_2e^{2x} + c_3xe^{2x} + c_4e^{3x}\cos(2x) + c_5e^{3x}\sin(3x)$
(b) $y = c_1e^{-x} + c_2e^{2x} + c_3xe^{2x} + c_4e^{2x}\cos(3x) + c_5e^{2x}\sin(3x)$
(c) $y = c_1e^{x} + c_2e^{2x} + c_3xe^{2x} + c_4e^{3x}\cos(2x) + c_5e^{3x}\sin(2x)$
(d) $y = c_1e^{-x} + c_2e^{2x} + c_3e^{2x}\cos(3x) + c_4e^{2x}\sin(3x)$
(e) $y = c_1e^{-x} + c_2e^{-2x} + c_3xe^{-2x} + c_4e^{2x}\cos(3x) + c_5e^{2x}\sin(3x)$

27. An eigenvector associated with eigenvalue $\lambda = 2i$ of the matrix $A = \begin{bmatrix} 2 & 8 \\ -1 & -2 \end{bmatrix}$ is

(a) $\begin{bmatrix} 2+2i \\ -3 \end{bmatrix}$

(b) $\begin{bmatrix} 2+2i \\ -2 \end{bmatrix}$

(c) $\begin{bmatrix} 2+2i \\ -5 \end{bmatrix}$

(d) $\begin{bmatrix} 2+2i \\ -1 \end{bmatrix}$

(e) $\begin{bmatrix} 2+2i \\ -4 \end{bmatrix}$

28. The matrix $A = \begin{bmatrix} 5 & 1 & 6 \\ -1 & 1 & -3 \\ 0 & 1 & 3 \end{bmatrix}$ has a defective eigenvalue $\lambda = 3$ of defect 2. Choosing $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - 3I)^3 V_3 = 0$ and $(A - 3I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ -1 \\ -t \end{bmatrix} + c_3 \begin{bmatrix} 1 + 2t + \frac{t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

(b) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 + 3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 + t + \frac{t^2}{2} \\ t \\ 1 \end{bmatrix} \right) e^{3t}$

(c) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 2 + 3t \\ -1 \\ -t \end{bmatrix} + c_3 \begin{bmatrix} 1 + 2t + \frac{3t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

(d) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 + 2t + \frac{3t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

(e) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} \\ t \\ t^2 \end{bmatrix} \right) e^{3t}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE03 **CODE03**

Math 208
Final Exam
231
December 19, 2023
Net Time Allowed: 180 Minutes

Name	

ID		**Sec**	

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. An eigenvector associated with eigenvalue $\lambda = 2i$ of the matrix $A = \begin{bmatrix} 2 & 8 \\ -1 & -2 \end{bmatrix}$ is

(a) $\begin{bmatrix} 2+2i \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 2+2i \\ -4 \end{bmatrix}$

(c) $\begin{bmatrix} 2+2i \\ -5 \end{bmatrix}$

(d) $\begin{bmatrix} 2+2i \\ -3 \end{bmatrix}$

(e) $\begin{bmatrix} 2+2i \\ -2 \end{bmatrix}$

2. A basis for the subspace

$$W = \{(x, y, z) : x - 2y + 5z = 0\}$$

of $\mathbb{R}^3$ consist of

(a) $v_1 = (2, 1, 0)$ and $v_2 = (1, 0, 1)$

(b) $v_1 = (2, 1, 0)$ and $v_2 = (-5, 1, 1)$

(c) $v_1 = (2, 1, 0)$ and $v_2 = (-5, 0, 1)$

(d) $v_1 = (2, 1, 1)$ and $v_2 = (-5, 0, 1)$

(e) $v_1 = (0, 1, 0)$ and $v_2 = (-5, 0, 1)$

3. The solution of the homogeneous differential equation $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$ is given by

   (a) $\ln(x^2+y^2) - 4\tan^{-1}\left(\frac{y}{x}\right) = c$

   (b) $\ln(x^2+y^2) - \tan^{-1}\left(\frac{y}{x}\right) = c$

   (c) $\ln(x^2+y^2) + 5\tan^{-1}\left(\frac{y}{x}\right) = c$

   (d) $\ln(x^2+y^2) + 3\tan^{-1}\left(\frac{y}{x}\right) = c$

   (e) $\ln(x^2+y^2) + 2\tan^{-1}\left(\frac{y}{x}\right) = c$

4. The roots of the indicial equation at $x = 0$ for the differential equation $2xy'' - y' + 2y = 0$ are

   (a) $r_1 = 0,\ r_2 = \frac{5}{2}$

   (b) $r_1 = 0,\ r_2 = -\frac{5}{2}$

   (c) $r_1 = 0,\ r_2 = \frac{3}{2}$

   (d) $r_1 = 1,\ r_2 = \frac{5}{2}$

   (e) $r_1 = 1,\ r_2 = \frac{3}{2}$

5. Let $F(t) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, $e^{At} = \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}$. A particular solution for the system $X' = AX + F(t)$ is

(a) $X_p = \begin{bmatrix} 3 \\ \frac{1}{2} \end{bmatrix}$

(b) $X_p = \begin{bmatrix} -3 \\ \frac{1}{2} \end{bmatrix}$

(c) $X_p = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

(d) $X_p = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

(e) $X_p = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

6. Let $A = \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix}$. If P is diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 3 \\ 1 & -2 \end{bmatrix}$

7. Using variation of parameters, the differential equation $y'' - 2y' + y = \frac{e^x}{1+x^2}$ has a particular solution $y_p = u_1(x)e^x + u_2(x)xe^x$, then $u_1(x) =$

(a) $-\dfrac{1}{2}\ln(1+x^2)$
(b) $3\ln(1+x^2)$
(c) $\dfrac{3}{2}\ln(1+x^2)$
(d) $-3\ln(1+x^2)$
(e) $\ln(1+x^2)$

8. An appropriate form of a particular solution of the differential equation

$$y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$$

is

(a) $y_p = Ax + Bx^2 + (Cx + Dx^2)\,e^{-3x}$
(b) $y_p = Ax + Bx^2 + (C + Dx^2)\,e^{-3x}$
(c) $y_p = A + Bx + (Cx + Dx^2)\,e^{-3x}$
(d) $y_p = Ax + Bx^3 + (Cx + Dx^2)\,e^{-3x}$
(e) $y_p = Ax + Bx^2 + (Cx^2 + Dx^3)\,e^{-3x}$

9. If $y = \sum_{n=0}^{\infty} C_n x^n$ is a power series solution about the ordinary point $x = 0$ of the differential equation $y'' - 2xy' + y = 0$, then the coefficients C_n satisfy

(a) $C_{n+2} = \dfrac{2n}{(n+1)(n+2)} C_n,\ n \geq 1$

(b) $C_{n+2} = \dfrac{3}{(n+1)(n+2)} C_{n-2},\ n \geq 1$

(c) $C_{n+2} = \dfrac{2n+1}{(n+2)(n+1)} C_n,\ n \geq 1$

(d) $C_{n+2} = \dfrac{2n-1}{(n+2)(n+1)} C_n,\ n \geq 1$

(e) $C_{n+2} = \dfrac{2}{(n+2)(n+1)} C_{n-1},\ n \geq 1$

10. The general solution of the differential equation

$$(D+1)(D-2)^2(D^2-4D+13)y = 0$$

is given by

(a) $y = c_1 e^{x} + c_2 e^{2x} + c_3 x e^{2x} + c_4 e^{3x}\cos(2x) + c_5 e^{3x}\sin(2x)$

(b) $y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{2x}\cos(3x) + c_4 e^{2x}\sin(3x)$

(c) $y = c_1 e^{-x} + c_2 e^{2x} + c_3 x e^{2x} + c_4 e^{3x}\cos(2x) + c_5 e^{3x}\sin(3x)$

(d) $y = c_1 e^{-x} + c_2 e^{-2x} + c_3 x e^{-2x} + c_4 e^{2x}\cos(3x) + c_5 e^{2x}\sin(3x)$

(e) $y = c_1 e^{-x} + c_2 e^{2x} + c_3 x e^{2x} + c_4 e^{2x}\cos(3x) + c_5 e^{2x}\sin(3x)$

11. The rank of the matrix $A = \begin{bmatrix} 1 & -3 & 0 & -5 \\ -1 & 4 & 1 & 7 \\ 2 & 1 & 7 & 4 \\ 2 & -2 & 4 & -2 \end{bmatrix}$ is equal to

(a) 0
(b) 4
(c) 2
(d) 1
(e) 3

12. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 7 & 2 \\ 10 & -15 & -4 \end{bmatrix}$. A basis for the eigenspace of A associated with the eigenvalue $\lambda = 1$ of A is

(a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

13. The solution of the exact differential equation $(2xy^2 - 3)\,dx + (2x^2y + 4)\,dy = 0$ is given by

    (a) $x^2y^2 + 4x + 3y = c$
    (b) $x^2y^3 - 3x - 4y = c$
    (c) $x^2y^2 - 3x + 4y = c$
    (d) $x^2y^2 + 3x - 3y = c$
    (e) $x^2y^3 + 3x + 4y = c$

14. The minimum radius of convergence of the power series solutions for the differential equation $(x^2 - 2x + 5)\,y'' + xy' - y = 0$ about the ordinary point $x = 0$ is

    (a) ∞
    (b) 2
    (c) $\sqrt{5}$
    (d) 0
    (e) $\sqrt{8}$

15. A possible fundamental matrix for the system $X' = \begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix} X$ is

(a) $\Phi(t) = \begin{bmatrix} 1 & e^{4t} \\ -2 & -2e^{4t} \end{bmatrix}$

(b) $\Phi(t) = \begin{bmatrix} 1 & e^{4t} \\ 2 & 2e^{4t} \end{bmatrix}$

(c) $\Phi(t) = \begin{bmatrix} -1 & e^{4t} \\ 2 & -2e^{4t} \end{bmatrix}$

(d) $\Phi(t) = \begin{bmatrix} 1 & e^{4t} \\ 2 & -2e^{4t} \end{bmatrix}$

(e) $\Phi(t) = \begin{bmatrix} 1 & -e^{4t} \\ 2 & 3e^{4t} \end{bmatrix}$

16. Let $A = \begin{bmatrix} 6 & 4 \\ -9 & -6 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} 1+6t & 4t \\ -9t & 1-6t \end{bmatrix}$

(b) $\begin{bmatrix} 1+6t & 4t \\ 9t & 1+6t \end{bmatrix}$

(c) $\begin{bmatrix} 1-6t & 4t \\ 9t & 1-6t \end{bmatrix}$

(d) $\begin{bmatrix} 1+6t & -4t \\ 9t & 1+6t \end{bmatrix}$

(e) $\begin{bmatrix} 1+6t & -4t \\ -9t & 1-6t \end{bmatrix}$

17. If $X = c_1 \begin{bmatrix} 5 \\ -6 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{4t}$ is the solution of the initial value problem

$$X' = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} X, \; X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

then $c_2 - c_1 =$

(a) 7
(b) 3
(c) 6
(d) 4
(e) 8

18. If $y(x)$ is the solution of the initial value problem $e^y\,dx - e^{-x}\,dy = 0,\; y(0) = 0$, then $y(x) =$

(a) $y = -\ln(2 - e^x)$
(b) $y = \ln(2 - e^x)$
(c) $y = 3\ln(2 - e^x)$
(d) $y = -3\ln(2 - e^x)$
(e) $y = 2\ln(2 - e^x)$

19. Let

$$A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}, \ \Phi(t) = \begin{bmatrix} e^t & 4e^{2t} \\ e^t & 3e^{2t} \end{bmatrix}.$$

If $\Phi(t)$ is a fundamental matrix for the system $X' = AX$, then $e^{At} =$

(a) $\begin{bmatrix} -3e^t + 4e^{2t} & e^t - e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

(b) $\begin{bmatrix} -3e^t + 4e^{2t} & 5e^t - 5e^{2t} \\ -2e^t + 2e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

(c) $\begin{bmatrix} -3e^t + 4e^{2t} & 4e^t - 4e^{2t} \\ e^t + 3e^{2t} & 4e^t - e^{2t} \end{bmatrix}$

(d) $\begin{bmatrix} 3e^t - 4e^{2t} & 4e^t - 4e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

(e) $\begin{bmatrix} -3e^t + 4e^{2t} & 4e^t - 4e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

20. If $y(x)$ is the solution of the initial-value problem $y'' + 4y = 2x$, $y(0) = 1$, $y'(0) = 2$, then $y(2\pi) =$

(a) $1 + \pi$

(b) $2 + \pi$

(c) π

(d) $1 - \pi$

(e) $2 - \pi$

21. A 2×2 real matrix A has an eigenvector $\begin{bmatrix} 2+2i \\ -1 \end{bmatrix}$ associated with the eigenvalue $\lambda = 2i$ of A. Then the general solution of the system $X' = AX$ is

(a) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(b) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -2\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(c) $X = c_1 \begin{bmatrix} 2\cos(2t) - \sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + \sin(2t) \\ -\sin(2t) \end{bmatrix}$

(d) $X = c_1 \begin{bmatrix} \cos(2t) - 2\sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} \cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(e) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -2\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -2\sin(2t) \end{bmatrix}$

22. The solution of the linear differential equation $x\dfrac{dy}{dx} - y = x^2 \sin x$ is given by

(a) $y = cx + x\sin x$

(b) $y = cx^2 + x\sin x$

(c) $y = cx - x\cos x$

(d) $y = cx^2 + x\cos x$

(e) $y = cx - x^2\cos x$

23. The general solution of the system $X' = \begin{bmatrix} 4 & 1 \\ 6 & -1 \end{bmatrix} X$ is given by

(a) $X = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} e^{-2t}$

(b) $X = c_1 \begin{bmatrix} 1 \\ -6 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t}$

(c) $X = c_1 \begin{bmatrix} 1 \\ 6 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t}$

(d) $X = c_1 \begin{bmatrix} 1 \\ -6 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{5t}$

(e) $X = c_1 \begin{bmatrix} 1 \\ 6 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t}$

24. If $V = \begin{bmatrix} \alpha \\ 1 \\ \beta \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 4 & -3 & 1 \\ 2 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ associated with the eigenvalue $\lambda = 1$, then $10\alpha + 6\beta =$

(a) 4

(b) 6

(c) 0

(d) 8

(e) 10

25. If the general solution of the system $X' = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix} X$ is given by

$$X = c_1 \begin{bmatrix} a \\ b \\ -2 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} \alpha \\ \beta \\ 1 \end{bmatrix} e^{\lambda t} + c_3 \begin{bmatrix} e \\ f \\ 13 \end{bmatrix}$$

then $a \cdot b \cdot \lambda =$

(a) -12
(b) 0
(c) 12
(d) -24
(e) 24

26. Let $t = (2, -7, 9)$, $u = (1, -2, 2)$, $v = (3, 0, 1)$, $w = (1, -1, 2)$ be four vectors in $\mathbb{R}^3$. If $t = au + bv + cw$, then $a^2 + b^2 + c^2 =$

(a) 17
(b) 15
(c) 16
(d) 14
(e) 18

27. The largest eigenvalue of the matrix

$$A = \begin{bmatrix} 4 & -3 & 1 \\ 2 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

is equal to

(a) 1

(b) 3

(c) 5

(d) -4

(e) 2

28. The matrix $A = \begin{bmatrix} 5 & 1 & 6 \\ -1 & 1 & -3 \\ 0 & 1 & 3 \end{bmatrix}$ has a defective eigenvalue $\lambda = 3$ of defect 2. Choosing $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A-3I)^3 V_3 = 0$ and $(A-3I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2+3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1+t+\frac{t^2}{2} \\ t \\ 1 \end{bmatrix} \right) e^{3t}$

(b) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 2+3t \\ -1 \\ -t \end{bmatrix} + c_3 \begin{bmatrix} 1+2t+\frac{3t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

(c) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ -1 \\ -t \end{bmatrix} + c_3 \begin{bmatrix} 1+2t+\frac{t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

(d) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} \\ t \\ t^2 \end{bmatrix} \right) e^{3t}$

(e) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1+2t+\frac{3t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE04 **CODE04**

Math 208
Final Exam
231
December 19, 2023
Net Time Allowed: 180 Minutes

Name	
ID		**Sec**	

Check that this exam has 28 questions.

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5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The largest eigenvalue of the matrix

$$A = \begin{bmatrix} 4 & -3 & 1 \\ 2 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

is equal to

(a) 2
(b) -4
(c) 5
(d) 3
(e) 1

2. If $y = \sum_{n=0}^{\infty} C_n x^n$ is a power series solution about the ordinary point $x = 0$ of the differential equation $y'' - 2xy' + y = 0$, then the coefficients C_n satisfy

(a) $C_{n+2} = \dfrac{2n+1}{(n+2)(n+1)} C_n,\ n \geq 1$
(b) $C_{n+2} = \dfrac{2n}{(n+1)(n+2)} C_n,\ n \geq 1$
(c) $C_{n+2} = \dfrac{2n-1}{(n+2)(n+1)} C_n,\ n \geq 1$
(d) $C_{n+2} = \dfrac{3}{(n+1)(n+2)} C_{n-2},\ n \geq 1$
(e) $C_{n+2} = \dfrac{2}{(n+2)(n+1)} C_{n-1},\ n \geq 1$

3. The minimum radius of convergence of the power series solutions for the differential equation $(x^2 - 2x + 5)\, y'' + xy' - y = 0$ about the ordinary point $x = 0$ is

(a) $\sqrt{5}$
(b) $\sqrt{8}$
(c) ∞
(d) 2
(e) 0

4. If $y(x)$ is the solution of the initial-value problem $y'' + 4y = 2x$, $y(0) = 1$, $y'(0) = 2$, then $y(2\pi) =$

(a) π
(b) $1 - \pi$
(c) $1 + \pi$
(d) $2 - \pi$
(e) $2 + \pi$

5. The general solution of the system $X' = \begin{bmatrix} 4 & 1 \\ 6 & -1 \end{bmatrix} X$ is given by

(a) $X = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} e^{-2t}$

(b) $X = c_1 \begin{bmatrix} 1 \\ -6 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{5t}$

(c) $X = c_1 \begin{bmatrix} 1 \\ -6 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t}$

(d) $X = c_1 \begin{bmatrix} 1 \\ 6 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t}$

(e) $X = c_1 \begin{bmatrix} 1 \\ 6 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{5t}$

6. If $X = c_1 \begin{bmatrix} 5 \\ -6 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{4t}$ is the solution of the initial value problem

$$X' = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} X, \; X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

then $c_2 - c_1 =$

(a) 4

(b) 7

(c) 8

(d) 6

(e) 3

7. A 2×2 real matrix A has an eigenvector $\begin{bmatrix} 2+2i \\ -1 \end{bmatrix}$ associated with the eigenvalue $\lambda = 2i$ of A. Then the general solution of the system $X' = AX$ is

(a) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(b) $X = c_1 \begin{bmatrix} \cos(2t) - 2\sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} \cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(c) $X = c_1 \begin{bmatrix} 2\cos(2t) - \sin(2t) \\ -\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + \sin(2t) \\ -\sin(2t) \end{bmatrix}$

(d) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -2\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -\sin(2t) \end{bmatrix}$

(e) $X = c_1 \begin{bmatrix} 2\cos(2t) - 2\sin(2t) \\ -2\cos(2t) \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2t) + 2\sin(2t) \\ -2\sin(2t) \end{bmatrix}$

8. An eigenvector associated with eigenvalue $\lambda = 2i$ of the matrix $A = \begin{bmatrix} 2 & 8 \\ -1 & -2 \end{bmatrix}$ is

(a) $\begin{bmatrix} 2+2i \\ -4 \end{bmatrix}$

(b) $\begin{bmatrix} 2+2i \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} 2+2i \\ -3 \end{bmatrix}$

(d) $\begin{bmatrix} 2+2i \\ -5 \end{bmatrix}$

(e) $\begin{bmatrix} 2+2i \\ -2 \end{bmatrix}$

9. The solution of the homogeneous differential equation $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$ is given by

   (a) $\ln(x^2+y^2) + 5\tan^{-1}\left(\frac{y}{x}\right) = c$
   (b) $\ln(x^2+y^2) - 4\tan^{-1}\left(\frac{y}{x}\right) = c$
   (c) $\ln(x^2+y^2) + 2\tan^{-1}\left(\frac{y}{x}\right) = c$
   (d) $\ln(x^2+y^2) - \tan^{-1}\left(\frac{y}{x}\right) = c$
   (e) $\ln(x^2+y^2) + 3\tan^{-1}\left(\frac{y}{x}\right) = c$

10. The roots of the indicial equation at $x = 0$ for the differential equation $2xy'' - y' + 2y = 0$ are

   (a) $r_1 = 1,\ r_2 = \frac{3}{2}$
   (b) $r_1 = 0,\ r_2 = \frac{5}{2}$
   (c) $r_1 = 1,\ r_2 = \frac{5}{2}$
   (d) $r_1 = 0,\ r_2 = \frac{3}{2}$
   (e) $r_1 = 0,\ r_2 = -\frac{5}{2}$

11. The rank of the matrix $A = \begin{bmatrix} 1 & -3 & 0 & -5 \\ -1 & 4 & 1 & 7 \\ 2 & 1 & 7 & 4 \\ 2 & -2 & 4 & -2 \end{bmatrix}$ is equal to

(a) 4
(b) 2
(c) 3
(d) 0
(e) 1

12. If $y(x)$ is the solution of the initial value problem $e^y\,dx - e^{-x}\,dy = 0,\ y(0) = 0$, then $y(x) =$

(a) $y = -3\ln(2 - e^x)$
(b) $y = 3\ln(2 - e^x)$
(c) $y = 2\ln(2 - e^x)$
(d) $y = -\ln(2 - e^x)$
(e) $y = \ln(2 - e^x)$

13. Let $A = \begin{bmatrix} 6 & 4 \\ -9 & -6 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} 1+6t & -4t \\ 9t & 1+6t \end{bmatrix}$

(b) $\begin{bmatrix} 1-6t & 4t \\ 9t & 1-6t \end{bmatrix}$

(c) $\begin{bmatrix} 1+6t & -4t \\ -9t & 1-6t \end{bmatrix}$

(d) $\begin{bmatrix} 1+6t & 4t \\ -9t & 1-6t \end{bmatrix}$

(e) $\begin{bmatrix} 1+6t & 4t \\ 9t & 1+6t \end{bmatrix}$

14. An appropriate form of a particular solution of the differential equation

$$y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$$

is

(a) $y_p = Ax + Bx^2 + (Cx^2 + Dx^3)\, e^{-3x}$

(b) $y_p = Ax + Bx^2 + (Cx + Dx^2)\, e^{-3x}$

(c) $y_p = Ax + Bx^2 + (C + Dx^2)\, e^{-3x}$

(d) $y_p = Ax + Bx^3 + (Cx + Dx^2)\, e^{-3x}$

(e) $y_p = A + Bx + (Cx + Dx^2)\, e^{-3x}$

15. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 7 & 2 \\ 10 & -15 & -4 \end{bmatrix}$. A basis for the eigenspace of A associated with the eigenvalue $\lambda = 1$ of A is

(a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \right\}$

16. A basis for the subspace

$$W = \{(x, y, z) : x - 2y + 5z = 0\}$$

of $\mathbb{R}^3$ consist of

(a) $v_1 = (0, 1, 0)$ and $v_2 = (-5, 0, 1)$
(b) $v_1 = (2, 1, 0)$ and $v_2 = (-5, 1, 1)$
(c) $v_1 = (2, 1, 1)$ and $v_2 = (-5, 0, 1)$
(d) $v_1 = (2, 1, 0)$ and $v_2 = (-5, 0, 1)$
(e) $v_1 = (2, 1, 0)$ and $v_2 = (1, 0, 1)$

17. Let $t = (2, -7, 9)$, $u = (1, -2, 2)$, $v = (3, 0, 1)$, $w = (1, -1, 2)$ be four vectors in $\mathbb{R}^3$. If $t = au + bv + cw$, then $a^2 + b^2 + c^2 =$

(a) 15
(b) 14
(c) 17
(d) 16
(e) 18

18. The solution of the linear differential equation $x\dfrac{dy}{dx} - y = x^2 \sin x$ is given by

(a) $y = cx + x \sin x$
(b) $y = cx^2 + x \sin x$
(c) $y = cx - x \cos x$
(d) $y = cx^2 + x \cos x$
(e) $y = cx - x^2 \cos x$

19. Let $A = \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix}$. If P is diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 3 \\ 1 & -2 \end{bmatrix}$

20. Let $F(t) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, $e^{At} = \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}$. A particular solution for the system $X' = AX + F(t)$ is

(a) $X_p = \begin{bmatrix} 3 \\ \frac{1}{2} \end{bmatrix}$

(b) $X_p = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

(c) $X_p = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

(d) $X_p = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

(e) $X_p = \begin{bmatrix} -3 \\ \frac{1}{2} \end{bmatrix}$

21. If $V = \begin{bmatrix} \alpha \\ 1 \\ \beta \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 4 & -3 & 1 \\ 2 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ associated with the eigenvalue $\lambda = 1$, then $10\alpha + 6\beta =$

(a) 10
(b) 4
(c) 6
(d) 8
(e) 0

22. Using variation of parameters, the differential equation $y'' - 2y' + y = \frac{e^x}{1+x^2}$ has a particular solution $y_p = u_1(x)e^x + u_2(x)xe^x$, then $u_1(x) =$

(a) $3\ \ln(1 + x^2)$
(b) $\ln(1 + x^2)$
(c) $-3\ln(1 + x^2)$
(d) $\dfrac{3}{2}\ln(1 + x^2)$
(e) $-\dfrac{1}{2}\ \ln(1 + x^2)$

23. The general solution of the differential equation

$$(D+1)(D-2)^2(D^2-4D+13)y=0$$

is given by

(a) $y=c_1e^{-x}+c_2e^{2x}+c_3xe^{2x}+c_4e^{2x}\cos(3x)+c_5e^{2x}\sin(3x)$
(b) $y=c_1e^{-x}+c_2e^{2x}+c_3e^{2x}\cos(3x)+c_4e^{2x}\sin(3x)$
(c) $y=c_1e^{-x}+c_2e^{2x}+c_3xe^{2x}+c_4e^{3x}\cos(2x)+c_5e^{3x}\sin(3x)$
(d) $y=c_1e^{-x}+c_2e^{-2x}+c_3xe^{-2x}+c_4e^{2x}\cos(3x)+c_5e^{2x}\sin(3x)$
(e) $y=c_1e^{x}+c_2e^{2x}+c_3xe^{2x}+c_4e^{3x}\cos(2x)+c_5e^{3x}\sin(2x)$

24. A possible fundamental matrix for the system $X'=\begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix} X$ is

(a) $\Phi(t)=\begin{bmatrix} 1 & e^{4t} \\ -2 & -2e^{4t} \end{bmatrix}$

(b) $\Phi(t)=\begin{bmatrix} -1 & e^{4t} \\ 2 & -2e^{4t} \end{bmatrix}$

(c) $\Phi(t)=\begin{bmatrix} 1 & e^{4t} \\ 2 & 2e^{4t} \end{bmatrix}$

(d) $\Phi(t)=\begin{bmatrix} 1 & -e^{4t} \\ 2 & 3e^{4t} \end{bmatrix}$

(e) $\Phi(t)=\begin{bmatrix} 1 & e^{4t} \\ 2 & -2e^{4t} \end{bmatrix}$

25. The solution of the exact differential equation $(2xy^2 - 3)\,dx + (2x^2y + 4)\,dy = 0$ is given by

(a) $x^2y^3 + 3x + 4y = c$
(b) $x^2y^2 + 3x - 3y = c$
(c) $x^2y^3 - 3x - 4y = c$
(d) $x^2y^2 - 3x + 4y = c$
(e) $x^2y^2 + 4x + 3y = c$

26. If the general solution of the system $X' = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix} X$ is given by

$$X = c_1 \begin{bmatrix} a \\ b \\ -2 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} \alpha \\ \beta \\ 1 \end{bmatrix} e^{\lambda t} + c_3 \begin{bmatrix} e \\ f \\ 13 \end{bmatrix}$$

then $a \cdot b \cdot \lambda =$

(a) 12
(b) 0
(c) -12
(d) 24
(e) -24

27. Let

$$A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}, \ \Phi(t) = \begin{bmatrix} e^t & 4e^{2t} \\ e^t & 3e^{2t} \end{bmatrix}.$$

If $\Phi(t)$ is a fundamental matrix for the system $X' = AX$, then $e^{At} =$

(a) $\begin{bmatrix} -3e^t + 4e^{2t} & 4e^t - 4e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

(b) $\begin{bmatrix} 3e^t - 4e^{2t} & 4e^t - 4e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

(c) $\begin{bmatrix} -3e^t + 4e^{2t} & 4e^t - 4e^{2t} \\ e^t + 3e^{2t} & 4e^t - e^{2t} \end{bmatrix}$

(d) $\begin{bmatrix} -3e^t + 4e^{2t} & 5e^t - 5e^{2t} \\ -2e^t + 2e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

(e) $\begin{bmatrix} -3e^t + 4e^{2t} & e^t - e^{2t} \\ -3e^t + 3e^{2t} & 4e^t - 3e^{2t} \end{bmatrix}$

28. The matrix $A = \begin{bmatrix} 5 & 1 & 6 \\ -1 & 1 & -3 \\ 0 & 1 & 3 \end{bmatrix}$ has a defective eigenvalue $\lambda = 3$ of defect 2. Choosing $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A-3I)^3\,V_3 = 0$ and $(A-3I)^2\,V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2+3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1+t+\frac{t^2}{2} \\ t \\ 1 \end{bmatrix} \right) e^{3t}$

(b) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ -1 \\ -t \end{bmatrix} + c_3 \begin{bmatrix} 1+2t+\frac{t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

(c) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} \frac{t^2}{2} \\ t \\ t^2 \end{bmatrix} \right) e^{3t}$

(d) $X = \left(c_1 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 2+3t \\ -1 \\ -t \end{bmatrix} + c_3 \begin{bmatrix} 1+2t+\frac{3t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

(e) $X = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3t \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1+2t+\frac{3t^2}{2} \\ -t \\ -\dfrac{t^2}{2} \end{bmatrix} \right) e^{3t}$

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	C 5	E 7	A 14	A 12
2	A	B 27	A 26	C 7	C 21
3	A	D 13	E 16	E 4	A 22
4	A	E 15	E 5	C 16	C 10
5	A	B 11	A 2	B 18	B 23
6	A	A 4	D 9	D 25	B 24
7	A	A 24	A 23	A 11	A 26
8	A	C 21	A 3	A 9	B 14
9	A	C 10	D 27	D 21	C 4
10	A	B 20	B 6	E 8	D 16
11	A	D 2	A 4	C 6	B 6
12	A	B 26	D 18	B 15	D 1
13	A	E 8	A 15	C 3	D 19
14	A	A 6	C 22	C 22	B 9
15	A	D 14	B 21	D 20	E 15
16	A	D 12	A 10	A 19	D 7
17	A	B 7	C 13	A 24	B 5
18	A	C 1	E 24	A 1	C 2
19	A	B 9	E 20	E 27	B 25
20	A	A 17	B 11	A 10	E 18
21	A	B 25	C 25	A 26	A 13
22	A	C 19	D 19	C 2	E 11
23	A	D 18	D 1	D 23	A 8
24	A	A 22	D 12	E 13	E 20
25	A	C 3	C 17	D 17	D 3
26	A	B 16	B 8	D 5	E 17
27	A	A 23	D 14	E 12	A 27
28	A	B 28	C 28	B 28	D 28

Answer Counts

V	A	B	C	D	E
1	6	9	6	5	2
2	7	4	5	7	5
3	8	3	6	6	5
4	6	7	4	6	5