King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 208
Major Exam I
232
February 20, 2024
Net Time Allowed: 120 Minutes

USE THIS AS A TEMPLATE

Write your questions, once you are satisfied upload this file.

Question 23 / Section 1.1

1. If $y = \frac{1}{4}x^5 + bx^{-3}$ is a solution of the initial-value problem $x\frac{dy}{dx} + 3y = 2x^5$, $y(2) = 1$, then $b =$

   (a) -56
   (b) -57
   (c) -55
   (d) 54
   (e) 53

2. The order of the differential equation

   $$y^{(4)} + x^2y^{(3)} + x(y')^6 = \sin x$$

   is equal to

   (a) 4
   (b) 2
   (c) 6
   (d) 3
   (e) 7

Question 9 / Section 1.2

3. If $y = y(x)$ is the solution of the initial-value problem $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$, $y(0) = 0$, then $y\left(\frac{1}{2}\right) =$

(a) $\frac{\pi}{6}$

(b) $-\frac{\pi}{6}$

(c) $\frac{\pi}{3}$

(d) $-\frac{\pi}{3}$

(e) $\frac{\pi}{4}$

Question 17 / Section 1.2

4. If a particle is moving in a straight line with acceleration $a(t) = \frac{1}{(t+1)^3}$, initial-position $x(0) = 0$, and an initial velocity $v(0) = 0$, then the position of the particle at $t = \frac{1}{2}$ is

(a) $\frac{1}{12}$

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{3}{2}$

(e) $\frac{1}{14}$

Question 16 / Section 1.4

5. The general solution of the differential equation $(x^2+1)(\tan y)\,y' = x$ is given by

(a) $y(x) = \sec^{-1}\left(c\sqrt{1+x^2}\right)$
(b) $y(x) = \sec^{-1}\left(c\sqrt{2+x^2}\right)$
(c) $y(x) = \tan^{-1}\left(c\sqrt{1+x^2}\right)$
(d) $y(x) = \tan^{-1}\left(c\sqrt{2+x^2}\right)$
(e) $y(x) = \sec^{-1}\left(c\sqrt{x^2-1}\right)$

Similar to Question 43 / Section 1.4

6. A pitcher of buttermilk initially at $25°\,C$ is to be cooled by setting it on the front porch, where the temperature is $0°$. Suppose that the temperature of the buttermilk has dropped to $15°$ after 20 min. The temperature of the buttermilk will be $9°$ when $t =$

(a) 40
(b) 30
(c) 20
(d) 25
(e) 28

Similar to Question 18 / Section 1.5

7. The general solution of the linear differential equation $xy' = 3y + x^4 \cos x$ is given by

(a) $y = x^3 (\sin x + c)$
(b) $y = x^2 (\sin x + c)$
(c) $y = x^4 (\sin x + c)$
(d) $y = x^3 (\cos x + c)$
(e) $y = x^2 (\cos x + c)$

Question 43 / Section 1.6

8. The general solution of the differential equation $xy'' = y'$ is given by

(a) $y = Ax^2 + B$
(b) $y = Ax^3 + B$
(c) $y = Ax^4 + B$
(d) $y = Ax^2 + Bx$
(e) $y = Ax^3 + Bx$

Similar to Question 35 / Section 1.6

9. A constant K that makes the differential equation $\left(x^3+\frac{y}{x}\right)\,dx+(y^2+K\ln x)\,dy=0$ exact is

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

Question 41 / Section 1.6

10. A general solution of the exact differential equation

$$\left(\frac{2x}{y}-\frac{3y^2}{x^4}\right)dx+\left(\frac{2y}{x^3}-\frac{x^2}{y^2}+\frac{1}{\sqrt{y}}\right)dy=0$$

is

(a) $\frac{x^2}{y}+\frac{y^2}{x^3}+2\sqrt{y}=c$
(b) $\frac{x}{y}+\frac{y^2}{x^3}+4\sqrt{y}=c$
(c) $\frac{y}{x^2}+\frac{y^2}{x^3}+2\sqrt{y}=c$
(d) $\frac{x^2}{y}-\frac{x^3}{y^2}+3\sqrt{y}=c$
(e) $\frac{x^2}{y}+\frac{y^2}{x^3}-3\sqrt{y}=c$

Question 24 / Section 1.6

11. By making a suitable substitution, the differential equation $2xy' + y^3e^{-2x} = 2xy$ can be transformed into a linear differential equation

(a) $u' + 2u = \dfrac{e^{-2x}}{x}$

(b) $u' - 2u = \dfrac{e^{-2x}}{x}$

(c) $u' + 3u = \dfrac{e^{-2x}}{x}$

(d) $u' - 3u = \dfrac{e^{-2x}}{x}$

(e) $u' + 2u = \dfrac{e^{-2x}}{3x}$

Question 11 / Section 1.6

12. By making a suitable substitution, the differential equation $(x^2 - y^2)y' = 2xy$ can be transformed into a seperable differential equation

(a) $\dfrac{dy}{y} + \dfrac{2v}{v^2+1}\,dv = 0$

(b) $\dfrac{dy}{y} + \dfrac{v}{v^2+1}\,dv = 0$

(c) $\dfrac{dy}{y} + \dfrac{3v}{v^2+1}\,dv = 0$

(d) $\dfrac{dy}{y} - \dfrac{v}{v^2+1}\,dv = 0$

(e) $\dfrac{dy}{y} - \dfrac{2v}{v^2+1}\,dv = 0$

Question 8 / Section 1.5

13. A general solution of the differential equation $y' + \dfrac{1}{3x}\, y = 4$ is

(a) $y = 3x + \dfrac{c}{\sqrt[3]{x}}$

(b) $y = x + \dfrac{c}{\sqrt[3]{x}}$

(c) $y = 3x + \dfrac{c}{\sqrt{x}}$

(d) $y = x + \dfrac{c}{\sqrt{x}}$

(e) $y = 2x + \dfrac{c}{\sqrt{x}}$

Question 4 / Section 4.1

14. Let $\mathbf{a} = 2i - j$, $\mathbf{b} = j - 3k$, then $|3\mathbf{a} - 4\mathbf{b}| =$

(a) $\sqrt{229}$

(b) $\sqrt{227}$

(c) $\sqrt{231}$

(d) $\sqrt{226}$

(e) $\sqrt{224}$

Similar to Question 16 / Section 4.1

15. The value of k for which that vectors of $\mathbb{R}^3$

$\mathbf{u} = (1, -1, 2)$, $\mathbf{v} = (3k, 0, k+1)$,
$\mathbf{w} = (1, -2, 2)$ are linearly independent are

(a) $k \neq \frac{1}{5}$
(b) $k \neq \frac{1}{4}$
(c) $k \neq 0$
(d) $k \neq \frac{1}{6}$
(e) $k \neq 1$

Question 12 / section 4.1

16. Let $\mathbf{u} = (4, 1)$, $\mathbf{v} = (-2, -1)$, $\mathbf{w} = (2, -2)$ be vectors in $\mathbb{R}^2$. If $\mathbf{w} = a\mathbf{u} + b\mathbf{v}$, then $a + b =$

(a) 8
(b) 7
(c) 6
(d) 5
(e) 4

Question 30 / Section 4.1

17. Which one of the following statements is true about the subset V of $\mathbb{R}^3$ defined by

$$V = \{(x, y, z) : x + y + z = 0\}$$

(a) V is a subspace of $\mathbb{R}^3$

(b) V is not closed under addition

(c) V is not closed under multiplication by scalars

(d) V is closed under addition but not closed under multiplication by scalars

(e) V is not closed under addition but closed under multiplication by scalars

Question 22 / Section 4.2

18. The solution of the system

$$\begin{aligned} x_1 + 3x_2 + 3x_3 + 3x_4 &= 0 \\ 2x_1 + 7x_2 + 5x_3 - x_4 &= 0 \\ 2x_1 + 7x_2 + 4x_3 - 4x_4 &= 0 \end{aligned}$$

is the set of all scalars multiples of a vector $\mathbf{u}$ where $\mathbf{u} =$

(a) $(-6, 4, -3, 1)$

(b) $(-6, -4, -3, 1)$

(c) $(6, 4, -3, 1)$

(d) $(-6, 4, 3, 1)$

(e) $(-6, 4, -3, -1)$

Similar to Question 18 / Section 1.6

19. A general solution of the differential equation $(x+y)\,y' = 1$ is

(a) $y = \ln(x+y+1) + c$
(b) $y = \ln(x+y+2) + c$
(c) $y = \ln(x+y+1) + cx$
(d) $y = \ln(x+y+2) + cx$
(e) $y = \ln(x+y-1) + cx$

20. Which one of the following differential equations is linear

(a) $e^x y' + (\ln x)\,y = \tan x$
(b) $yy' + (\ln x)\,y = e^x$
(c) $(y')^2 + xy = x^2$
(d) $y' + xy^2 = x^3$
(e) $y' + x\,\sin y = e^x$